ESTIMATE OF THE ROLE OF THE LAW OF CONSERVATION OF ANGULAR MOMENTUM IN THE STATISTICAL THEORY OF PARTICLE PRODUCTION

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Submitted to JETP editor July 4, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) 40, 180-182 (January, 1961)

It is shown that the predictions of the statistical theory of multiple meson production remain almost unaltered if the conservation of the z component of the angular momentum is taken into account in the simplest classical way. The resulting anisotropy in the angular distribution of the particles is apparently smaller than that observed in reality.

It is asserted in several papers on the statistical theory of multiple production of particles that the anisotropy in the angular distribution is due to the restrictions connected with the conservation of the initial angular momentum by the particles in the final state.[1]

However, there is a certain inconsistency in this argument. Indeed, when we replace, in the statistical theory, the quantum mechanical expression for the probability of the process

$$W_n = \int \mathfrak{M}\, \delta(E - \Sigma E_k)\, \delta(\mathbf{P} - \Sigma \mathbf{p}_k)\, d\mathbf{p}_1 \ldots d\mathbf{p}_n \qquad (1)$$

($\mathfrak{M}$ is the square of the modulus of the matrix element) simply by the statistical weight[2]

$$S_n = V^n \int \delta(E - \Sigma E_k)\, \delta(\mathbf{P} - \Sigma \mathbf{p}_k)\, d\mathbf{p}_1 \ldots d\mathbf{p}_n, \qquad (2)$$

we assume that the matrix element is constant, i.e., it conserves angular momentum in a trivial way (invariance under rotations).

The essence of the above-mentioned assertion seems to be that we require an expression for the matrix element which, while being invariant under rotations, depends on the angle in the way prescribed by experiment. Such an expression has not yet been found.

Since a consistent quantum mechanical treatment is impossible, it is of interest to investigate a certain classical model, in which the behavior of the probability W_n is imitated by the statistical weight of a rotating system:[3]

$$S_n = \int \delta(E - \Sigma E_k)\, \delta(\mathbf{P} - \Sigma \mathbf{p}_k)\, \delta(\mathbf{M} - \Sigma [\mathbf{r}_k \times \mathbf{p}_k])\, d\mathbf{p}_1 \ldots d\mathbf{p}_n d\mathbf{r}_1 \ldots d\mathbf{r}_n; \qquad (3)$$

this procedure is consistent with the statistical theory of Fermi and introduces in the simplest classical way angular dependences which are connected with the conservation of angular momentum.

The comparison of (3) and (1) shows that this is equivalent to the replacement

$$\mathfrak{M} \to \mathfrak{M}_{\text{cl}} = \int \delta(\mathbf{M} - \Sigma [\mathbf{r}_k \times \mathbf{p}_k])\, d\mathbf{r}_1 \ldots d\mathbf{r}_n. \qquad (4)$$

This replacement presupposes that the quantity $\mathfrak{M}_{\text{cl}}$ retains some of the characteristic features of the quantum mechanical expression $\mathfrak{M}$.

The calculations with the formula (3) were done by using the Monte Carlo method (method of random stars).[4] Since only one axis — the z axis of the interaction — is fixed experimentally, an average being taken over the other two axes, we must integrate over all values of the transverse components of the angular momentum M_x and M_y in formula (4). Then only a single δ function containing the z component of the angular momentum remains in that formula. Since we have chosen the z axis along the momentum of the incident particle, $M_z = 0$, so that no new parameters appear in our model.

Integrating formula (4) over the volume of the Fermi ellipsoid,[2] we obtain

$$\mathfrak{M}_{\text{cl}} = \int_{(V)} \delta(\Sigma [\mathbf{r}_k \times \mathbf{p}_k])\, d\mathbf{r}_1 \ldots d\mathbf{r}_n = \frac{V^n}{2\pi} \int_{-\infty}^{\infty} d\lambda \prod_{k=1}^{n} \frac{1}{V} \int_{(V)} \exp\{i\lambda [\mathbf{r} \times \mathbf{p}_k]_z\}\, d\mathbf{r} = \frac{V^n}{\pi} \int_0^{\infty} d\lambda \sqrt{9\pi/2} \prod_{k=1}^{n} \frac{J_{3/2}(R\lambda p_{\perp k})}{(R\lambda p_{\perp k})^{3/2}}$$

(J is a Bessel function, $p_{\perp k}$ is the transverse momentum of the k-th particle, and R is the radius of the ellipsoid, i.e., the effective radius of the nucleon). Substituting this expression in (1), we can calculate the probability for the production of various systems of particles occurring in the collision.

The method of random stars also allows us to obtain simultaneously the angular and energy distributions and the correlations between the directions of emission of the produced particles. The computations were carried out for collisions of nucleons with energies of 10 Bev; together with the direct creation of the mesons, we also considered the production mechanism going through an isobar. We obtained the following results (for more detail, see reference 5):

1. The statistical weights, after normalization, are not very different from their values in the statistical theory of Fermi, so that the multiplicity is almost unaltered (instead of $\bar{n}_s = 3.68$, one obtains 3.70). The same holds for the momentum distributions. As the energies approach the reaction threshold, the cross sections decrease more slowly than in the Fermi theory.

2. The angular distribution is anisotropic. The anisotropy becomes weaker as the number of particles increases. If we characterize the anisotropy by the ratio of the numbers of particles emitted into the equal solid angles $0° \leq \theta \leq 60°$ and $60° \leq \theta \leq 90°$, respectively, we find for 2, 3, 4, ... secondary particles the anisotropy 2.5, 1.29, 1.22, ... (averaged over mesons and nucleons). The experimental values of this quantity are contradictory and display a wide spread; but they are probably higher than our values.[6]

3. The angular correlations between the particles are also almost unchanged. We call attention to the following fact observed by us: the average value of the angle $\bar{\theta}_{ik}$ agrees with the average value of the angle in the plane of the target $\bar{\varphi}_{ik}$; both depend only on the multiplicity. This agreement is also observed in the Fermi model.

Our result that the physical quantities are insensitive to modifications of the statistical weight is evidently due to the fact that our corrected weight $\mathfrak{M}_{cl}$ depends only on the transverse components of the momentum, which vary only little. If, instead of the Fermi ellipsoid, we choose a volume of a different shape (flat disk, the peripheral part of an ellipsoid, etc.), M_{cl} changes very little, retaining all its qualitative features.

These calculations led us to the conclusion that it is futile to try to explain the anisotropy quantitatively using the classical analogy. It appears to us that this effect can only be explained by a quantum mechanical computation of the matrix elements, even if it be of the most approximate nature.

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Translated by R. Lipperheide
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