ON THE THEORY OF THE MAGNETIC PROPERTIES OF A NONIDEAL FERMI GAS AT LOW TEMPERATURES

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The methods of quantum field theory are used to study the effect of the interaction between particles on the oscillations of the magnetic moment of a Fermi gas. The forces between the particles are assumed to be of the short-range type, and the calculations are made in the gas approximation. Values are found for the changes of period and amplitude of the oscillations of the magnetic moment that are caused by the interaction between particles. At not too low temperatures the amplitude of the oscillations contains a factor that decreases exponentially with decrease of the magnetic field.

1. In the study of the magnetic properties of a degenerate Fermi gas the interaction between particles is usually taken into account only by the introduction of a generalized dispersion law to replace the free-particle relation between energy and momentum (L. D. Landau,[1] I. M. Lifshitz and A. M. Kosevich[2]). A paper by Dingle[3] has been devoted to effects of the finite widths of the energy levels of the particles, which are also due to the interaction, but in this paper the width is introduced in a purely formal way and is assumed to be the same for all levels.

Therefore it is interesting to study the effects of the interaction between particles on the magnetic properties of a Fermi gas in the framework of the microscopic theory.

In the present paper we examine this problem for the simplest model, assuming that the interaction between the particles is of the short-range type. Our final results apply to the domain of not too low temperatures [cf. the condition (13)], at which the influence of the magnetic field on the collisions of the particles can be neglected. This condition, however, is used only at the last stage of the calculation as we have arranged it, so that it is possible to study the character of the approximation and find the region of applicability of the results.

2. The thermodynamic quantities that characterize a system of interacting particles can be obtained by means of the one-particle thermodynamic Green's function

$$G(\mathbf{r}_1, \mathbf{r}_2; t_1 - t_2) = \langle T\{\psi(\mathbf{r}_1, t_1)\psi^+(\mathbf{r}_2, t_2) S(\beta)\}\rangle \langle S(\beta)\rangle^{-1}, \quad (1)$$

where ψ^+, ψ are the operators for creation and annihilation of Fermi particles in the "thermodynamic interaction representation," $S(\beta)$ is the thermodynamic scattering matrix (β is the reciprocal of the temperature), and the averaging is carried out with the density matrix of noninteracting particles.

The quantity most simply expressed in terms of the Green's function is the density of the gas,

$$\mathfrak{N} = -\frac{\partial}{\partial\mu}\Omega = -\operatorname{Sp} G(\mathbf{r}, \mathbf{r}; -0), \quad (2)$$

where $\mu = p_0^2/2m$ is the chemical potential and Ω is the thermodynamic potential per unit volume of the gas. Knowing the Green's function G, we can use this formula to find Ω and then calculate the magnetic moment per unit volume, which is given by

$$M = -\frac{\partial}{\partial \mathcal{H}}\Omega = -\frac{e}{mc}\frac{\partial}{\partial\omega}\Omega, \quad (3)$$

where $\omega = e\mathcal{H}/mc$ is the Larmor frequency of a particle in the magnetic field $\mathcal{H}$.

Following the work of Abrikosov, Gor'kov, and Dzyaloshinskiĭ[4] and of Fradkin,[5] we expand the function $G(\mathbf{r}_1, \mathbf{r}_2; t)$ in Fourier series with respect to the variable t:

$$G(\mathbf{r}_1, \mathbf{r}_2; t) = \frac{1}{\beta}\sum_{p_4} G(\mathbf{r}_1, \mathbf{r}_2; p_4) e^{-ip_4 t}$$

$$\left(p_4 = \frac{2k+1}{\beta}\pi, \quad k = 0, \pm 1, \ldots\right).$$

Using the fact that the Green's function $G(\mathbf{r}_1, \mathbf{r}_2; t)$ has a discontinuity at the point $t = 0$,

$$G(\mathbf{r}_1, \mathbf{r}_2; +0) - G(\mathbf{r}_1, \mathbf{r}_2; -0) = \delta(\mathbf{r}_1 - \mathbf{r}_2),$$

we get:

$$\frac{1}{\beta}\sum_{p_4} G(\mathbf{r}_1, \mathbf{r}_2; p_4) = \frac{1}{2}\{G(\mathbf{r}_1, \mathbf{r}_2; +0) + G(\mathbf{r}_1, \mathbf{r}_2; -0)\}$$
$$= G(\mathbf{r}_1, \mathbf{r}_2; -0) + \frac{1}{2}\delta(\mathbf{r}_1 - \mathbf{r}_2).$$

Therefore the formula (2) can be rewritten in the form

$$\mathfrak{N} = -\lim_{\mathbf{r}_2\to\mathbf{r}_1} \mathrm{Sp}\left\{\frac{1}{\beta}\sum_{p_4} G(\mathbf{r}_1, \mathbf{r}_2; p_4) - \frac{1}{2}\delta(\mathbf{r}_1 - \mathbf{r}_2)\right\}. \quad (4)$$

To calculate the Green's function G we shall use the Feynman-diagram method, using a solid line for the Green's function of noninteracting particles in the magnetic field, $G_0(\mathbf{r}_1, \mathbf{r}_2; t_1 - t_2)$, and a dashed line for the interaction potential $V(\mathbf{r}_1 - \mathbf{r}_2)\,\delta(t_1 - t_2)$. The function $G_0(\mathbf{r}_1, \mathbf{r}_2; p_4)$ is of the following form:

$$G_0(\mathbf{r}_1, \mathbf{r}_2; p_4) = \sum_\alpha \frac{\psi_\alpha(\mathbf{r}_1)\psi_\alpha^*(\mathbf{r}_2)}{\varepsilon_\alpha - \mu + ip_4} \equiv \sum_\alpha \psi_\alpha(\mathbf{r}_1)\, G_0(\alpha; p_4)\, \psi_\alpha^*(\mathbf{r}_2), \quad (5)$$

where $\alpha \equiv (n, p_z, q)$ is a set of quantum numbers for the particle in the magnetic field*

$$\varepsilon_\alpha = \omega\left(n + \tfrac{1}{2}\right) + p_z^2/2m,$$

$$\psi_\alpha(\mathbf{r}) = (2\pi)^{-1}\exp\{i(p_z z + qy)\}\,\Phi_n(x - q/m\omega),$$

and $\Phi_n(x)$ are the wave functions of an oscillator with the frequency ω.

Noting that

$$\int_{-\infty}^{\infty} d\xi e^{i\xi m\omega y}\Phi_n\left(\xi + \frac{x}{2}\right)\Phi_n\left(\xi - \frac{x}{2}\right)$$
$$= \exp\left\{-\frac{m\omega}{4}(x^2 + y^2)\right\} L_n\left\{\frac{m\omega}{2}(x^2 + y^2)\right\}$$

[$L_n(x)$ are the Laguerre polynomials], we bring Eq. (5) into the form

$$G_0(\mathbf{r}_1, \mathbf{r}_2; p_4) = \exp\left\{i\frac{x_1 + x_2}{2}(y_1 - y_2)m\omega\right\}\frac{m\omega}{(2\pi)^2}$$
$$\times \sum_{n=0}^{\infty}\exp\left\{-\frac{m\omega}{4}[(x_1 - x_2)^2 + (y_1 - y_2)^2]\right\} L_n\left\{\frac{m\omega}{2}[(x_1 - x_2)^2 + (y_1 - y_2)^2]\right\}\int_{-\infty}^{\infty} dp_z \frac{\exp\{ip_z(z_1 - z_2)\}}{\omega(n + 1/2) + p_z^2/2m - \mu + ip_4}.$$

By considering an arbitrary diagram for the one-particle Green's function, one can verify without difficulty that $G(\mathbf{r}_1, \mathbf{r}_2; p_4)$ is of the form

$$G(\mathbf{r}_1, \mathbf{r}_2; p_4) = \exp\left\{\frac{1}{2}i(x_1 + x_2)(y_1 - y_2)m\omega\right\} G(\mathbf{r}_1 - \mathbf{r}_2; p_4), \quad (6)$$

where $G(\mathbf{r}_1 - \mathbf{r}_2; p_4)$ is a function that involves only the difference of the coordinates, $\mathbf{r}_1 - \mathbf{r}_2$.

Let us expand the Green's function $G(\mathbf{r}_1, \mathbf{r}_2; p_4)$ in terms of the functions $\psi_\alpha(\mathbf{r})$. We get:

$$G(\mathbf{r}_1, \mathbf{r}_2; p_4) = \sum_{\alpha_1, \alpha_2}\psi_{\alpha_1}(\mathbf{r}_1)\psi_{\alpha_2}^*(\mathbf{r}_2)\, G(\alpha_1, \alpha_2; p_4).$$

It follows from Eq. (6) that the Green's function in the α representation is diagonal in the variables p_z, q:

$$G(\alpha_1, \alpha_2; p_4) = \delta(p_{z1} - p_{z2})\,\delta(q_1 - q_2)\, G(n_1, n_2; p_{z1}; p_4),$$

where the function $G(n_1, n_2; p_z; p_4)$ does not depend on the variable q.

The density of the gas, defined by the formula (4), is expressed in terms of the function $G(n_1, n_2; p_z; p_4)$ in the following way:

$$\mathfrak{N} = -\frac{m\omega}{(2\pi)^2}\mathrm{Sp}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty} dp_z\left\{\frac{1}{2}\sum_{p_4} G(n, n; p_z; p_4) - \frac{1}{2}\right\}. \quad (7)$$

3. The Green's function $G(\mathbf{r}_1, \mathbf{r}_2; p_4)$ satisfies the well known equation

$$(H_0 - \mu + ip_4)\, G(\mathbf{r}_1, \mathbf{r}_2; p_4) + \int d\mathbf{r}'\Sigma(\mathbf{r}_1, \mathbf{r}'; p_4)\, G(\mathbf{r}', \mathbf{r}_2; p_4) = \delta(\mathbf{r}_1 - \mathbf{r}_2),$$

where H_0 is the Hamiltonian of a free particle in the magnetic field and Σ is the mass operator which takes account of the interaction between the particles. In the α representation this equation takes the form

$$(\varepsilon_{\alpha_1} - \mu + ip_4)\, G(\alpha_1, \alpha_2; p_4) + \sum_{\alpha'}\Sigma(\alpha_1, \alpha'; p_4)\, G(\alpha', \alpha_2; p_4) = \delta_{\alpha_1\alpha_2}. \quad (8)$$

We shall calculate the mass operator Σ in the gas approximation (cf. the paper by Galitskiĭ[6]). Then, as can be seen from the diagrams shown in Fig. 1 (which have weight factors − 2 and 1, respectively), the quantity Σ can be connected with the four-fermion vertex function Γ:

$$\Sigma(\alpha_1, \alpha_2; p_4) = \frac{1}{\beta}\sum_{p_4'; \alpha_3} G_0(\alpha_3; p_4')$$
$$\times\{\Gamma(\alpha_1, \alpha_3; \alpha_3, \alpha_2; p_4 + p_4') - 2\Gamma(\alpha_1, \alpha_3; \alpha_2, \alpha_3; p_4 + p_4')\}. \quad (9)$$

(The function Γ corresponds to the shaded squares in the diagrams.)

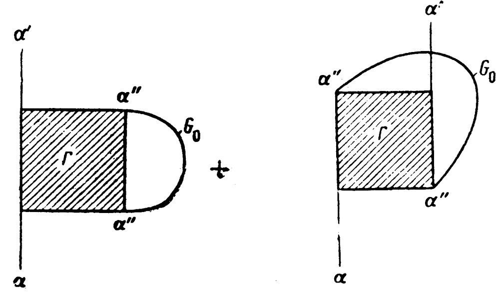

FIG. 1

*Here and hereafter ħ = 1.

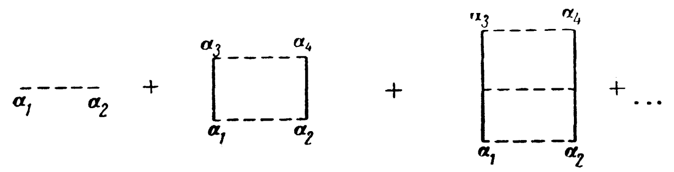

FIG. 2

In the gas approximation the quantity Γ corresponds to the set of diagrams shown in Fig. 2. As was stated above, the solid lines in the diagrams correspond to the function $G_0(\mathbf{r}_1, \mathbf{r}_2; p_4)$ and the dashed lines to the function $V(\mathbf{r}_1 - \mathbf{r}_2)$. Applying the method developed by Galitskiĭ,[6] we can get the equation satisfied by the operator $\Gamma(p_4)$:

$$\Gamma(p_4) = \Gamma_0(p_4) + \Gamma_0(p_4)\frac{\mathfrak{N}_1 + \mathfrak{N}_2}{H_1^0 + H_2^0 - 2\mu - ip_4}\Gamma(p_4),$$

$$\Gamma_0(p_4) = V - V[H - 2\mu - ip_4]^{-1}V. \quad (10)$$

Here H_1^0 and H_2^0 are the Hamiltonians of the first and second particles in the magnetic field in the absence of interaction; $H = H_1^0 + H_2^0 + V$; and

$$\mathfrak{N}_{1,2} = [1 + \exp\{\beta(H_{1,2}^0 - \mu)\}]^{-1}.$$

The quantity $\Gamma(\alpha_1, \alpha_2; \alpha_3, \alpha_4; p_4)$ is the matrix element of the operator $\Gamma(p_4)$ in the α representation:

$$\Gamma(\alpha_1, \alpha_2; \alpha_3, \alpha_4; p_4) = \langle\alpha_1, \alpha_2|\Gamma(p_4)|\alpha_3, \alpha_4\rangle.$$

We now note that in the Schrödinger equation for two interacting particles in a constant magnetic field one can separate the variables belonging to the center of mass and to the relative motion of the particles. Therefore it is convenient to go over to a representation in which both the Hamiltonian that describes the motion of the center of mass, and the Hamiltonian that describes the relative motion of two noninteracting particles, are diagonal. In this representation the matrix element of the operator $\Gamma_0(p_4)$ has the form

$$\langle\alpha A|\Gamma_0(p_4)|\alpha' A'\rangle = \delta_{AA'}\Gamma_0(\alpha, \alpha'; p_4),$$

where $\alpha \equiv (n, p_z, q)$ is a set of quantum numbers for a particle with the mass $m/2$ and the charge $e/2$, and $A \equiv (N, P_z, Q)$ is a set of quantum numbers for a particle with the mass $2m$ and the charge $2e$, in the magnetic field $\mathcal{H}$.

The quantity $\Gamma_0(\alpha, \alpha'; p_4)$ can be connected with the amplitude $T_{\alpha\alpha'}$ for the scattering of particles with the mass $m/2$ and the charge $e/2$ by the short-range potential $V(\mathbf{r})$ in the magnetic field $\mathcal{H}$:

$$\Gamma_0(\alpha, \alpha'; p_4) = T^*_{\alpha'\alpha} + \sum_{\alpha''} T_{\alpha\alpha''}T^*_{\alpha'\alpha''}\left\{\frac{1}{\varepsilon_{\alpha''} - \varepsilon_\alpha + i0} - \frac{1}{\varepsilon_{\alpha''} + \varepsilon_A - 2\mu - ip_4}\right\},$$

$$\varepsilon_\alpha = \omega\left(n + \tfrac{1}{2}\right) + p_z^2/m, \qquad \varepsilon_A = \omega\left(N + \tfrac{1}{2}\right) + P_z^2/4m. \quad (11)$$

4. The scattering amplitude $T_{\alpha\alpha'}$, which has been calculated in a paper by Skobov,[7] is of the form

$$T_{\alpha\alpha'} = \frac{f_0}{\pi m}\varphi_n\left(-\frac{2q}{m\omega}\right)\varphi_{n'}\left(-\frac{2q'}{m\omega}\right)\frac{1}{1 + if_0K(\varepsilon_\alpha)}, \quad (12)$$

where $\varphi_n(x)$ is a wave function of an oscillator with the mass $m/2$ and the frequency ω, f_0 is the scattering amplitude of particles of zero energy in the absence of the magnetic field, and

$$K(\varepsilon) = \frac{m\omega}{2}\sum_{n=0}^{n_0}[m\omega(n_0 - n - \xi)]^{-1/2}$$

(the quantities n_0 and ξ are connected with the energy ϵ by the relation $\epsilon = \omega(n_0 + 1/2 - \xi)$, $0 < \xi \leq 1$).

Thus to determine the Green's function in the gas approximation it is necessary to calculate the quantity $\Gamma_0(\alpha, \alpha', p_4)$, using the expression (12) for $T_{\alpha\alpha'}$, and then, in accordance with Eqs. (9) and (10), to find the mass operator Σ that appears in (8). It follows from the expression for $K(\epsilon)$ that the quantity $f = f_0(1 + if_0K)^{-1}$ differs from f_0 only in the regions $0 < \xi < \xi_0$ and $1 - \xi_0 < \xi < 1$, where $\xi_0 = 1/4\, m\omega f_0^2$. In the formula (9) for the mass operator Σ the main contribution for $\beta^{-1} \ll \mu$ obviously comes from the energy range $\mu - 1/2\beta^{-1} \lesssim \epsilon_\alpha \lesssim \mu + 1/2\beta^{-1}$; therefore if the conditions $\xi_0 \ll 1$ and $\beta^{-1} \gg \omega\xi_0$ are satisfied we can neglect the difference between f and f_0. In what follows we shall assume that these conditions are satisfied.

For $\beta^{-1} \lesssim \omega\xi_0$ the difference between the quantities f and f_0 can be important, since the region in which the step of the Fermi function is smeared out can enter one of the intervals $(0, \xi_0)$ or $(1 - \xi_0, 1)$. In the present paper, however, we shall not consider this low a range of temperatures.

As is customary in the study of the magnetic properties of a Fermi gas, we shall assume the magnetic field weak enough so that $\omega \ll \mu$. Thus we are assuming that the following conditions are satisfied:

$$p_0f_0 \ll 1, \qquad \omega/\mu \ll 1, \qquad (\omega/\mu)^2(p_0f_0)^2 \ll (\beta\mu)^{-1} \ll 1. \quad (13)$$

The scattering amplitude then takes the form

$$T_{\alpha\alpha'} = \frac{f_0}{\pi m}\varphi_n\left(-\frac{2q}{m\omega}\right)\varphi_{n'}\left(-\frac{2q'}{m\omega}\right).$$

Since $\omega \ll \mu$, large quantum numbers n are important; therefore in Eq. (11) we replace the summation over n″ by an integration over k_t, where $k_t^2/m = \omega(n'' + 1/2)$, $\mathbf{k} \equiv (\mathbf{k}_t, k_z)$:

$$\Gamma_0(\alpha, \alpha'; p_4) = \frac{1}{(2\pi)^2}\varphi_n\left(-\frac{2q}{m\omega}\right)\varphi_{n'}\left(-\frac{2q'}{m\omega}\right)$$

$$\times\left\{\frac{f_0}{m} + \frac{f_0^2}{m^2}\int\frac{d\mathbf{k}}{(2\pi)^3}\left[\frac{1}{\frac{1}{m}k^2 - \varepsilon_\alpha + i0}\right.\right.$$

$$\left.\left.-\frac{1}{\frac{1}{m}k^2 + \varepsilon_A - 2\mu - ip_4}\right]\right\}.$$

The fractional error so introduced is of the order of $(\omega/\mu)^{1/2}$, and by the conditions (13) it is small. In fact, let us use the Poisson summation formula

$$\sum_{n=0}^{\infty} F(n+1/2) = \int_0^\infty F(x)\,dx + 2\sum_{r=1}^{\infty}(-1)^r\int_0^\infty F(x)\cos 2\pi rx\,dx. \quad (14)$$

In the case in question, by Eqs. (11) and (12), the function F(x) is

$$F(x) = f_0^2\int_{-\infty}^{\infty}dk_z\left\{\frac{1}{k_z^2/m + \omega x - \varepsilon_\alpha + i0}\right.$$

$$\left.-\frac{1}{k_z^2/m + \omega x + \varepsilon_A - 2\mu - ip_4}\right\}\int_{-\infty}^{\infty}dq''\,\frac{[\varphi_n(-2q''/m\omega)]^2}{(2\pi)^2}$$

$$= \frac{m\omega}{2(2\pi)^2}f_0^2\int_{-\infty}^{\infty}\left\{\frac{1}{k_z^2/m + \omega x - \varepsilon_\alpha + i0}\right.$$

$$\left.-\frac{1}{k_z^2/m + \omega x + \varepsilon_A - 2\mu - ip_4}\right\}dk_z.$$

Making the change of variable $\omega x = k_t^2/m$ and integrating over the angles of the vector $\mathbf{k} = (\mathbf{k}_t, k_z)$, one easily verifies that the ratio of the sum of the oscillating terms in Eq. (14) to the first term, which we are keeping, is of the order of $(\omega/\mu)^{1/2}$.

Writing the operator equation (10) in the α representation and replacing the summation over the indices n, n′, N, n_3 in this equation and in the relation (9) by integration over the variables p_t, p_t', P_t, p_{t3}, with

$$p_t^2/m = \omega(n + 1/2), \quad p_t'^2/m = \omega(n' + 1/2),$$
$$P_t^2/4m = \omega(N + 1/2), \quad p_{t3}^2/2m = \omega(n_3 + 1/2)),$$

we get

$$\Sigma(\alpha_1, \alpha_2; p_4) = \delta(p_{z1} - p_{z2})\,\delta(q_1 - q_2)\,\delta_{n_1n_2}\Sigma(\mathbf{p}_1, p_4),$$

$$\Sigma(\mathbf{p}, p_4) = -\frac{f_0^2}{m^2}\int\frac{d\mathbf{p}'d\mathbf{k}}{(2\pi)^6}\left\{1 - \mathfrak{N}\left(\frac{\mathbf{p}+\mathbf{p}'}{2} + \mathbf{k}\right)\right.$$

$$\left.-\mathfrak{N}\left(\frac{\mathbf{p}+\mathbf{p}'}{2} - \mathbf{k}\right)\right\}\left\{\mathfrak{N}(\mathbf{p}') + \left[\exp\beta\left(\frac{k^2}{m} + \frac{[\mathbf{p}+\mathbf{p}']^2}{4m} - 2\mu\right)\right.\right.$$

$$\left.\left.-1\right]^{-1}\right\}\frac{1}{k^2/m - (\mathbf{p}-\mathbf{p}')^2/4m - \mu + p^2/2m + ip_4} + \frac{f_0}{m}\int\frac{d\mathbf{p}'}{(2\pi)^3}$$

$$\times\,\mathfrak{N}(\mathbf{p}') + \frac{f_0^2}{m^2}\,\mathrm{P}\int\frac{d\mathbf{p}'d\mathbf{k}}{(2\pi)^6}\,\mathfrak{N}(\mathbf{p}')\,\frac{1}{k^2/m - (\mathbf{p}-\mathbf{p}')^2/4m}, \quad (15)$$

where

$$\mathfrak{N}(\mathbf{p}) = [1 + \exp\{\beta(p^2/2m - \mu)\}]^{-1}$$

and the quantum number n_1 is connected with the variable p_1 by the relation $p_{t1}^2/2m = \omega(n_1 + 1/2)$.

By making estimates as we did for the calculation of $\Gamma_0(\alpha, \alpha'; p_4)$, we can verify that the terms omitted in the function $\Sigma(\alpha_1, \alpha_2; p_4)$ do not exceed $(\omega/\mu)^{1/2}(p_0f_0)^2\mu$. Also it follows from Eq. (15) that

$$\mathrm{Re}\,\Sigma(\mathbf{p}, p_4) = \mathrm{Re}\,\Sigma(\mathbf{p}, -p_4);$$

$$\mathrm{Im}\,\Sigma(\mathbf{p}; p_4) = -\mathrm{Im}\,\Sigma(\mathbf{p}; -p_4). \quad (16)$$

We note that the quantity $\Sigma(p, p_4)$ does not depend on the angles of the vector **p** and has different analytical forms in different ranges of variation of the variables p and p_4. It is important that in the first approximation in the gas parameter p_0f_0 this quantity is real and positive; the difference between the analytical expressions for Σ in the different ranges of p, p_4 arises only in the second gas approximation.

In what follows we shall need the value of the mass operator* $\Sigma(p, p_4)$ for $ip_4 = \mu - p^2/2m$ in the region of negative and nearly zero values of the quantity $p - p_0$ ($|p - p_0| \ll p_0$):

$$\Sigma\left(p,\ i\frac{p^2}{2m} - i\mu\right) = -\Delta\mu + \frac{m^* - m}{m}\left(\frac{p^2}{2m} - \mu\right)$$

$$-\frac{i}{\tau}\left[1 + \frac{2}{\pi^2}\beta^2\left(\frac{p^2}{2m} - \mu\right)^2\right]; \quad (17)$$

$$-\frac{\Delta\mu}{\mu} = \frac{4}{3\pi}(p_0f_0)\left[1 + \frac{11 - 2\ln 2}{5\pi}(p_0f_0)\right],$$

$$\frac{m^*}{m} = 1 + \frac{8}{15\pi^2}(p_0f_0)^2(7\ln 2 - 1), \quad \tau^{-1} = \frac{\pi}{4}\,\frac{1}{\mu\beta^2}(p_0f_0)^2. \quad (18)$$

Substituting the expression (15) for Σ in Eq. (8), we find the Green's function

$$G(\alpha_1, \alpha_2; p_4) = \frac{\delta_{\alpha_1\alpha_2}}{\varepsilon_{\alpha_1} - \mu + ip_4 + \Sigma(\mathbf{p}, p_4)};$$

$$\frac{p_t}{2m} = \omega\left(n_1 + \frac{1}{2}\right). \quad (19)$$

5. Applying the Poisson summation formula (14), we rewrite the relation (7) in the form

$$\mathfrak{N} = \mathfrak{N}_0 + 2\sum_{r=1}^{\infty}(-1)^r\mathfrak{N}_r,$$

where

$$\mathfrak{N}_r = -\frac{2m\omega}{(2\pi)^2}\int_{-\infty}^{\infty}dp_z\int_0^\infty dx\left\{\frac{1}{\beta}\sum_{p_4}G\left(x - \frac{1}{2},\ x - \frac{1}{2};\ p_z;\ p_4\right)\right.$$

$$\left.-\frac{1}{2}\right\}\cos 2\pi rx.$$

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*The expression for the mass operator at temperature equal to zero has been obtained in a paper by V. Galitskiĭ.[6]

Making the change of variable $\omega x = p_t^2/2m$ and using the formula (19) for G, we get

$$\mathfrak{N}_r = -\frac{2}{(2\pi)^3}\int d\mathbf{p}\left\{\frac{1}{\beta}\sum_{p_4}\frac{1}{p^2/2m-\mu+ip_4+\Sigma(p,\ p_4)} - \frac{1}{2}\right\}\cos\frac{\pi r p_t^2}{m\omega}. \qquad (20)$$

The quantity $\mathfrak{N}_r$ is real in virtue of the relations (16).

We note that by neglecting the oscillating terms in Σ we incur a fractional error of the order $(p_0f_0)^2$ in $\mathfrak{N}_r$.

Integrating over the angles of the vector $\mathbf{p}$ in the formula (20) and dropping the nonoscillating terms, we rewrite $\mathfrak{N}_r$ in the form

$$\mathfrak{N}_r^{osc} = -\frac{m}{2\pi^2}\left(\frac{m\omega}{r}\right)^{1/2}\mathrm{Re}\exp\left(\frac{2\pi i r\mu}{\omega} - \frac{\pi i}{4}\right)\frac{1}{\beta}\sum_{p_4}\int_{-\infty}^{\infty}\frac{e^{2\pi i r z/\omega}}{z+ip_4+\Sigma(z,\ p_4)}\,dz, \qquad (21)$$

where $z = p^2/2m - \mu$.

To calculate the integral that appears in Eq. (21), we introduce an analytic function $\tilde{\Sigma}(z,\ p_4)$ that coincides with the function $\Sigma(z,\ p_4)$ in the region of negative values of z close to zero. Using the fact that for $\beta\mu \gg 1$ the main contribution to the integral in question comes from the region of small negative values of z, to accuracy $(p_0f_0)^2$ we can replace the integration of the function $e^{2\pi i r z/\omega}[z + ip_4 + \Sigma(z,\ p_4)]^{-1}$ along the real axis by integration of the function $e^{2\pi i r z/\omega}[z + ip_4 + \tilde{\Sigma}(z,\ p_4)]^{-1}$ around the contour shown in Fig. 3. According to Eq. (17) the pole of the integrand is given to accuracy $(p_0f_0)^2$ by the expression

$$z_0(p_4) = -ip_4 m^*/m + \Delta\mu - i\tau^{-1}(1 - 2\beta^2 p_4^2/\pi^2),$$

and to accuracy $(p_0f_0)^2$ the residue at the point z_0 is unity. Therefore the final expression for $\mathfrak{N}_r^{osc}$ is

$$\mathfrak{N}_r^{osc} = \frac{m}{\pi}\left(\frac{m\omega}{r}\right)^{1/2}\frac{1}{\beta}\sin\left[\frac{2\pi r}{\omega}(\mu+\Delta\mu) - \frac{\pi}{4}\right] \times \sum_{k=0}^{\infty}\exp\left\{-\frac{2\pi^2 r}{\beta\omega}\frac{m^*}{m}(2k+1) - \frac{4\pi r}{\omega\tau}\left[(2k+1)^2 - \frac{1}{2}\right]\right\}. \qquad (22)$$

Integrating the expression (22) for the density of the Fermi gas with respect to μ and neglecting terms proportional to the gas parameter p_0f_0, we get the following expression for Ω, the thermodynamic potential per unit volume:

$$\Omega = \Omega_0 + 2\sum_{r=1}^{\infty}(-1)^r\Omega_r,$$

$$\Omega_r^{osc} = \frac{1}{2\pi^2}\left(\frac{m\omega}{r}\right)^{3/2}\frac{1}{\beta}\cos\left[\frac{2\pi r}{\omega}(\mu+\Delta\mu) - \frac{\pi}{4}\right] \times \sum_{k=0}^{\infty}\exp\left\{-\frac{2\pi^2 r}{\beta\omega}\frac{m^*}{m}(2k+1) - \frac{4\pi r}{\omega\tau}\left[(2k+1)^2 - \frac{1}{2}\right]\right\}. \qquad (23)$$

Using the formula (3), we find the oscillating part of the magnetic moment

$$M_r^{osc} = -\frac{1}{\pi}\frac{e}{c}\left(\frac{m}{r\omega}\right)^{1/2}\frac{\mu}{\beta}\sin\left[\frac{2\pi r}{\omega}(\mu+\Delta\mu) - \frac{\pi}{4}\right] \times \sum_{k=0}^{\infty}\exp\left\{-\frac{2\pi^2 r}{\beta\omega}\frac{m^*}{m}(2k+1) - \frac{4\pi r}{\omega\tau}\left[(2k+1)^2 - \frac{1}{2}\right]\right\}. \qquad (24)$$

6. If the condition $\omega\tau \gg 1$ is satisfied, as it is in the temperature range

$$\mu(\omega/\mu)^2(p_0f_0)^2 \ll \beta^{-1} \ll \mu(\omega/\mu)^{1/2}(p_0f_0)^{-1},$$

the formula (24) takes the form

$$M_r^{osc} = -\frac{1}{2\pi}\frac{e}{c}\left(\frac{m}{r\omega}\right)^{1/2}\frac{\mu}{\beta}\sin\left[\frac{2\pi r}{\omega}(\mu + \Delta\mu) - \frac{\pi}{4}\right]\sinh^{-1}\frac{2\pi^2 r}{\beta\omega}\frac{m^*}{m}. \qquad (25)$$

This same sort of expression for the oscillating part of the magnetic moment is obtained, according to a paper by I. M. Lifshitz and Kosevich,[2] if one starts from a dispersion law which is of the form

$$\varepsilon(p) = \frac{1}{2m}p^2 - \Delta\mu + \frac{m^* - m}{m}\frac{p_0}{m}(p - p_0)$$

near the Fermi surface.

In the temperature range $\mu \gg \beta^{-1} \gtrsim \mu(\omega/\mu)^{1/2} \times (p_0f_0)^{-1}$ we can confine ourselves to the first term of the sum in Eq. (24). This gives a formula for M_r^{osc} which is analogous to that obtained in a paper by Dingle,[3]

$$M_r^{osc} = -\frac{1}{\pi}\frac{e}{c}\left(\frac{m}{r\omega}\right)^{1/2}\frac{\mu}{\beta}\sin\left[\frac{2\pi r}{\omega}(\mu + \Delta\mu) - \frac{\pi}{4}\right]\exp\left\{-\frac{2\pi^2 r}{\beta\omega}\frac{m^*}{m} - \frac{2\pi r}{\omega\tau}\right\}, \qquad (26)$$

where $\Delta\mu$, m^*, and τ are defined by the formulas (18).

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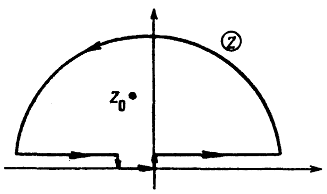

FIG. 3

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