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THE ELASTIC SCATTERING OF γ RAYS BY DEUTERONS BELOW THE PION-PRODUCTION THRESHOLD

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Dispersion relations and the conditions for unitarity of the S matrix are used for the analysis of the elastic scattering of γ rays by deuterons below the threshold for pion production. The low-energy limit is examined for the scattering of γ rays by nuclei of arbitrary spin. The energy dependence of elastic γd scattering is deduced on the basis of the experimental data on the photodisintegration of the deuteron. The result differs decidely from that of the impulse approximation over a wide range of energies. It is found that it is not important to include the influence of photoproduction of pions from deuterons in the range of energy considered.

1. INTRODUCTION

THE scattering of γ rays by deuterons is an example of a process whose amplitude is decidedly affected by inelastic processes, such as the photodisintegration of the deuteron and the photoproduction of mesons. Inclusion of the influence of pion photoproduction, which is important at γ-ray energies near and above the photoproduction threshold, requires a rather detailed analysis of the processes

$$\gamma + d \rightarrow NN\pi, \qquad \gamma + d \rightarrow d + \pi^0$$

and is not dealt with here.

The influence of the photodisintegration of the deuteron on the elastic γd scattering near the threshold for photodisintegration, and the departures from monotonic variation with the energy, which lead to a sharp decrease of the cross section, have been considered previously.[1] The purpose of the present paper is to make an analysis of γd scattering on the basis of dispersion relations over a wider energy range, in which meson production still does not have much effect.

The experimental data[2] on the scattering of γ rays by deuterons in the energy range 50 — 100 Mev do not fit into the framework of the impulse approximation,[3] and this forces us to carry through an analysis that does not involve this approximation. On the other hand, the contribution to the scattering amplitude from meson-production processes falls off rapidly below the threshold for photoproduction of mesons.

We shall confine ourselves to the forward scattering. In the calculation of the dispersion integrals we take into account the cross sections for the electric-dipole and magnetic-dipole photodisintegrations. We begin with the phase-shift analysis, so as to express the imaginary parts of the scattering amplitudes in terms of the quantities that characterize the photodisintegration of the deuteron. We then consider the dispersion relations for forward scattering and the low-energy theorem. The dispersion integrals are evaluated in the range of γ-ray energies below ~ 100 Mev. The real and imaginary parts of the amplitudes are obtained, and the polarizabilities of the deuteron and of nucleons are discussed.

2. THE PHENOMENOLOGICAL ANALYSIS

As is well known, the formulas for the electric and magnetic multipole waves $\mathbf{Y}_{lm}^{(\lambda)}(\mathbf{k})$ of a photon are ($\lambda = 0, 1$)

$$\mathbf{Y}_{lm}^{(0)} = \sum_{\mu} C_{l\, m-\mu\, 1\mu}^{lm} Y_{l\, m-\mu}(\mathbf{k})\, \zeta_{\mu}, \tag{1a}$$

$$\mathbf{Y}_{lm}^{(1)} = -i\,[\mathbf{k} \times \mathbf{Y}_{lm}^{(0)}], \tag{1b}$$

where $\mathbf{k}$ is the unit vector along the momentum of the photon in the center-of-mass system, $Y_{lm}(\mathbf{k})$ are normalized spherical functions, and

$$\zeta_{+} = -(\mathbf{i} + i\mathbf{j})/\sqrt{2},$$

$$\zeta_0 = \mathbf{k}, \qquad \zeta_{-} = (\mathbf{i} - i\mathbf{j})/\sqrt{2} \tag{2}$$

— the eigenfunctions of the photon spin — satisfy the transversality condition

$$\mathbf{k}\mathbf{Y}^{(\lambda)}_{lm}(\mathbf{k}) = 0.$$

If we write

$$\eta_+ = -\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\\ 0\end{pmatrix}, \quad \eta_0 = \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}, \quad \eta_- = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\\ 0\end{pmatrix} \qquad (3)$$

for the spin functions of the deuteron, then for the γd system we can construct eigenfunctions of the total angular momentum J^2, the component J_z, and the parity from Eqs. (1) and (3):

$$\mathbf{Y}^{(\lambda)}_{jlM}(\mathbf{k}) = \sum_r C^{jM}_{lM-r1r}\mathbf{Y}^{(\lambda)}_{lM-r}(\mathbf{k})\,\eta_r. \qquad (4)$$

In the center-of-mass system (c.m.s.) all quantities in the final state are denoted by symbols with primes; for example, $\mathbf{k}'$ denotes the direction of the photon momentum in the final state.

By means of Eq. (4) we can write the scattering matrix T in the form

$$(\mathbf{e}'\mathbf{T}\mathbf{e}) = \sum_{\substack{jMll'\\ \lambda\lambda'}} Y^{(\lambda')*}_{jl'M}(\mathbf{k}')\,Y^{(\lambda)}_{jlM}(\mathbf{k})\,a^{\lambda\lambda'}_{jll'}, \qquad (5)$$

where $\mathbf{e}$ and $\mathbf{e}'$ are the respective polarization vectors of the photon in the initial and final states.

Parity conservation requires that

$$a^{\lambda\lambda'}_{jll'} = 0 \text{ for } (-1)^{l+\lambda} \neq (-1)^{l'+\lambda'}. \qquad (6)$$

Time-reversal invariance leads to the symmetry condition

$$a^{\lambda\lambda'}_{jll'} = a^{\lambda'\lambda}_{jl'l}. \qquad (7)$$

The usual arguments show that for forward scattering the spin dependence of the matrix T is of the form

$$(\mathbf{e}'\mathbf{T}\mathbf{e}) = A(\mathbf{e}'\mathbf{e}) + iB(\mathbf{S}[\mathbf{e}'\times\mathbf{e}]) + \tfrac{1}{2}C[(\mathbf{S}\mathbf{e})(\mathbf{S}\mathbf{e}') + (\mathbf{S}\mathbf{e}')(\mathbf{S}\mathbf{e})] + \tfrac{1}{2}D[(\mathbf{S}[\mathbf{k}\times\mathbf{e}])(\mathbf{S}[\mathbf{k}'\times\mathbf{e}']) + (\mathbf{S}[\mathbf{k}'\times\mathbf{e}'])(\mathbf{S}[\mathbf{k}\times\mathbf{e}])]. \qquad (8)$$

Here $\mathbf{S}$ is the operator of the spin vector of the deuteron; its components S_i satisfy the Duffin-Kemmer commutation relations:

$$[S_i, S_j] = i\varepsilon_{ijk}S_k, \qquad S_iS_jS_k + S_kS_jS_i = \delta_{ij}S_k + \delta_{jk}S_i. \quad (9)$$

Using the Stokes parameters to describe the polarization of the photon, as was done in our paper on γN scattering,[4] we get without difficulty from Eqs. (2) and (8):

$$(\zeta^*_\pm \mathbf{T}\zeta_\pm) = A \mp B(\mathbf{S}\mathbf{k}) + \tfrac{1}{2}(D+C)[2-(\mathbf{S}\mathbf{k})^2],$$
$$(\zeta^*_\pm \mathbf{T}\zeta_\mp) = \tfrac{1}{2}(D-C)\{(\mathbf{S}\mathbf{i})^2 - (\mathbf{S}\mathbf{j})^2 \mp i[(\mathbf{S}\mathbf{i})(\mathbf{S}\mathbf{j}) + (\mathbf{S}\mathbf{j})(\mathbf{S}\mathbf{i})]\}. \qquad (10)$$

By means of Eq. (10) and the method developed previously[4] we can construct the density matrix of the final state and calculate all observable quantities. The unpolarized forward scattering cross section is given by

$$\sigma_0(0^\circ) = |A + \tfrac{2}{3}(C+D)|^2 + \tfrac{1}{18}|C + D|^2 + \tfrac{2}{3}|B|^2 + \tfrac{1}{3}|D-C|^2, \qquad (11)$$

and we have

$$4\pi\,\mathrm{Im}\,(A + \tfrac{2}{3}C + \tfrac{2}{3}D) = q\sigma_t, \qquad (11')$$

where σ_t is the total interaction cross section, including both elastic and inelastic interactions; $\mathbf{q} = |q|\mathbf{k}$.

Let us turn to the phase-shift analysis. We include the amplitudes for electric-dipole and magnetic-dipole transitions. The magnetic-dipole transition is characterized by the matrix

$$F^0_j = \sum Y^{(0)}_{j1M}(\mathbf{k})\,Y^{(0)*}_{j1M}(\mathbf{k}) = \frac{3}{4\pi}\sum C^{jM}_{1M-r\,1r}C^{jM}_{1M-r'1r'}C^{1M-r}_{10\,1M-r}C^{1M-r'}_{10\,1M-r'}\eta_r\eta^*_{r'}\zeta_{M-r}\zeta^*_{M-r'}, \qquad (12)$$

where we have used the fact that for forward scattering

$$Y_{lm} = \delta_{lm}\sqrt{(2l+1)/4\pi}.$$

From Eqs. (12) we easily obtain

$$(\zeta^*_+ F\zeta_+) + (\zeta^*_- F\zeta_-) = \frac{3}{8\pi}[\alpha_j + \beta_j S_z^2],$$
$$(\zeta^*_+ F\zeta_+) - (\zeta^*_- F\zeta_-) = \frac{3}{8\pi}\gamma_j S_z,$$
$$(\zeta^*_+ F\zeta_-) = -\frac{3}{8\pi}\beta_j T_{2,-2} = -\frac{3}{8\pi}\beta_j\frac{1}{2}(S_x - iS_y)^2,$$
$$(\zeta^*_- F\zeta_+) = -\frac{3}{8\pi}\beta_j T_{2,2} = -\frac{3}{8\pi}\beta_j\frac{1}{2}(S_x + iS_y)^2. \qquad (13)$$

For j = 0, 1, 2 the quantities α_j, β_j, γ_j are

$j =$	2	1	0
$\alpha_j =$	1	1	0
$\beta_j =$	$1/6$	$-1/2$	$1/3$
$\gamma_j =$	$5/6$	$-1/2$	$-1/3$

(14)

In obtaining the relations (13) we have used the relations

$$\eta_+\eta_+^* = \tfrac{1}{2}(S_z^2 + S_z), \quad \eta_-\eta_-^* = \tfrac{1}{2}(S_z^2 - S_z), \quad \eta_0\eta_0^* = 1 - S_z^2,$$
$$\eta_+\eta_-^* = T_{2,2} = \tfrac{1}{2}[S_x^2 - S_y^2 + i(S_xS_y + S_yS_x)],$$
$$\eta_-\eta_+^* = T_{2,-2} = \tfrac{1}{2}[S_x^2 - S_y^2 - i(S_xS_y + S_yS_x)]. \quad (15)$$

The case of the electric-dipole transition is obtained from the relations (13) by replacing $\boldsymbol{\zeta}_+$ by $i[\boldsymbol{\zeta}_+ \times \mathbf{k}] = -\boldsymbol{\zeta}_+$ and $\boldsymbol{\zeta}_-$ by $i[\boldsymbol{\zeta}_- \times \mathbf{k}] = \boldsymbol{\zeta}_-$. Comparing Eqs. (13) and (14) with Eq. (10), we get

$$2A = \frac{3}{8\pi}\sum_j(\alpha_j + 2\beta_j)(a_j^{(m)} + a_j^{(e)}),$$
$$2B = -\frac{3}{8\pi}\sum_j\gamma_j(a_j^{(m)} + a_j^{(e)}),$$
$$C = -\frac{3}{8\pi}\sum_j\beta_j a_j^{(e)}, \qquad D = -\frac{3}{8\pi}\sum_j\beta_j a_j^{(m)}. \qquad (16)$$

The condition for unitarity of the S matrix reduces to the relation

$$2\pi i\,[T^*(-\mathbf{k}', -\mathbf{k}, -\mathbf{e}', -\mathbf{e}, -\mathbf{S})_{\gamma d\to\gamma d} - T(\mathbf{k}', \mathbf{k}, \mathbf{e}', \mathbf{e}, \mathbf{S})_{\gamma d\to\gamma d}] = q\int d\Omega_{n+p}\, T^+_{\gamma d\to np} T_{\gamma d\to np}, \quad (17)$$

where q is the relative momentum of the γd system. Let us represent $T_{\gamma d\to np}$ in the form

$$T_{\gamma d\to np} = \sum Y^s_{je'M}(\mathbf{n})\, Y^{(\lambda)*}_{jlM}(\mathbf{k})\, d^{s\lambda}_{jl'l}, \quad (18)$$

where j is the total angular momentum, l' is the orbital angular momentum in the final state, and s is the total spin of the np system.

Parity conservation requires that

$$(-1)^{l+\lambda+1} = (-1)^{l'}.$$

The quantities $d^{s\lambda}_{jl'l}$ are connected with the partial cross sections for photodisintegration:

$$24\pi\sigma^{s(m)}_{jl'} = (2j+1)\,|d^{s0}_{jl'1}|^2, \qquad 24\pi\sigma^{s(e)}_{jl'} = (2j+1)\,|d^{s1}_{jl'1}|^2. \quad (19)$$

The total cross section for photodisintegration is given by

$$24\pi\sigma_{\gamma d\to np} = \sum_{jl's}(2j+1)\,[\,|d^{s0}_{jl'1}|^2 + |d^{s1}_{jl'1}|^2]. \quad (20)$$

Substitution of Eqs. (18) and (5) in Eq. (17) gives

$$4\pi\,\mathrm{Im}\, a^{\lambda\lambda'}_{jll'}(q) = q\sum_{l''s}(d^{s\lambda}_{jl''l})^* d^{s\lambda'}_{jl''l'}, \quad (21)$$

and using Eqs. (16) and (14) we get the results

$$\mathrm{Im}\left[A + \tfrac{2}{3}(C + D)\right] = \frac{3}{16\pi}\sum_j\left(\alpha_j + \tfrac{2}{3}\beta_j\right)(\mathrm{Im}\, a^{(m)}_j + \mathrm{Im}\, a^{(e)}_j) = \frac{q}{4\pi}\sigma_{\gamma d\to np};$$

$$\mathrm{Im}\, A = (3\sigma_0 + \tfrac{6}{5}\sigma_2)\, q/4\pi,$$

$$\mathrm{Im}\, B = (\tfrac{3}{2}\sigma_0 + \tfrac{3}{4}\sigma_1 - \tfrac{3}{4}\sigma_2)\, q/4\pi,$$

$$\mathrm{Im}\, C = (-3\sigma^{(e)}_0 + \tfrac{3}{2}\sigma^{(e)}_1 - \tfrac{3}{10}\sigma^{(e)}_2)\, q/4\pi,$$

$$\mathrm{Im}\, D = (-3\sigma^{(m)}_0 - \tfrac{3}{10}\sigma^{(m)}_2)\, q/4\pi, \quad (22)$$

where σ_j denotes the partial cross section for photodisintegration in the state j, including the factor (2j + 1).

3. CROSSING SYMMETRY AND THE DISPERSION RELATIONS

The retarded amplitude for γd forward scattering can be written in the form

$$\langle\mu'\,|\,\mathbf{e}'\mathbf{N}^{ret}\mathbf{e}\,|\,\mu\rangle = -2\pi^2 i\int d^4z e^{-iqz}\,\langle\mathbf{p}',\mu'\,|\,\Theta(z_0)\,[\mathbf{e}'\mathbf{j}(z/2), \mathbf{e}\mathbf{j}(-z/2)]\,|\,\mathbf{p},\mu\rangle, \quad (23)$$

where μ and μ' are the spin indices of the deuteron.

For the advanced amplitude we have the analogous expression

$$\langle\mu'\,|\,\mathbf{e}'\mathbf{N}^{adv}\mathbf{e}\,|\,\mu\rangle = 2\pi^2 i\int d^4z e^{-iqz}\,\langle\mathbf{p}',\mu'\,|\,\Theta(-z_0)\,[\mathbf{e}'\mathbf{j}(z/2), \mathbf{e}\mathbf{j}(-z/2)]\,|\,\mathbf{p},\mu\rangle. \quad (24)$$

For the case of forward scattering the deuteron momentum **p** can be set equal to zero.

Considering the relations complex conjugate to Eqs. (23) and (24), we get

$$\langle\mu'\,|\,\mathbf{e}'\mathbf{N}^{ret(adv)}(q)\,\mathbf{e}\,|\,\mu\rangle^* = \langle\mu\,|\,\mathbf{e}'\mathbf{N}^{ret(adv)}(-q)\,\mathbf{e}\,|\,\mu'\rangle. \quad (25)$$

Interchanging the order of $(\mathbf{e}'\mathbf{j}(z/2))$ and $(\mathbf{e}\mathbf{j}(-z/2))$ in Eqs. (23) and (24) and changing the sign of the variable z, we arrive at the relation

$$\langle\mu'\,|\,\mathbf{e}'\mathbf{N}^{ret(adv)}(q)\,\mathbf{e}\,|\,\mu\rangle = \langle\mu'\,|\,\mathbf{e}\mathbf{N}^{adv(ret)}(-q)\,\mathbf{e}'\,|\,\mu\rangle. \quad (26)$$

Let us represent $N^{ret(adv)}$ in the form (8). The conditions (25) and (26) reduce to the following symmetry properties of the scalar functions A, B, C, D:

$$A^{ret(adv)}(\nu)^* = A^{ret(adv)}(-\nu), \quad B^{ret(adv)}(\nu)^* = -B^{ret(adv)}(-\nu),$$
$$C^{ret(adv)}(\nu)^* = C^{ret(adv)}(-\nu), \quad D^{ret(adv)}(\nu)^* = D^{ret(adv)}(-\nu); \quad (27)$$

$$A^{adv}(\nu) = A^{ret}(-\nu), \qquad B^{adv}(\nu) = -B^{ret}(-\nu),$$
$$C^{adv}(\nu) = C^{ret}(-\nu), \qquad D^{adv}(\nu) = D^{ret}(-\nu). \quad (28)$$

Denoting hereafter the quantities $A(\nu)$, $C(\nu)$, and $D(\nu)$ by $L_1(\nu)$, $L_2(\nu)$, and $L_3(\nu)$, respectively, and $B(\nu)$ by $L_4(\nu)$, we write the dispersion relations for the scalar functions in the form

$$\mathrm{Re}\, L_{1,2,3}(\nu_0) - \mathrm{Re}\, L_{1,2,3}(0) = \frac{2\nu_0^2}{\pi}\, P\int_{\nu_d}^{\infty}\frac{d\nu\,\mathrm{Im}\, L_{1,2,3}(\nu)}{\nu(\nu^2-\nu_0^2)}, \quad (29)$$

$$\mathrm{Re}\, L_4(\nu_0) - \nu_0\,\mathrm{Re}\, L_4'(0) = \frac{2\nu_0^3}{\pi}\, P\int_{\nu_d}^{\infty}\frac{d\nu\,\mathrm{Im}\, L_4(\nu)}{\nu^2(\nu^2-\nu_0^2)}, \quad (30)$$

where ν_d is the threshold for photodisintegration of the deuteron, approximately equal to the binding energy of the deuteron.

In order for it to be possible to use the relations (29) and (30) for an actual analysis, it is necessary to know $L_{1,2,3}(0)$ and $L_4'(0)$, i. e., to calculate the γd scattering amplitude in the energy region close to zero. The result of the calculations carried out in the following section is that

$$\mathrm{Re}\, L_1(0) = -e^2/M, \qquad \mathrm{Re}\, L_{2,3}(0) = 0,$$

$$\mathrm{Re}\, L_4'(0) = (\mu_0 - e/M_d)^2, \quad (31)$$

where μ_0 is the magnetic moment and M_d the mass of the deuteron.

4. THE LOW-ENERGY LIMIT FOR γd SCATTERING

Thirring, Low, Gell-Mann, and Goldberger[5] have shown that the limiting values at $\nu_0 = 0$ of the scattering amplitude and of its derivative with respect to the photon frequency are determined by the statistical properties of the system, for systems with spin ½. Following a method developed by Low, we shall show that analogous results are also valid for systems with arbitrary spin.

The S matrix for the scattering of photons from the state (q, e) into the state (q′, e′) is given by the expression

$$S' = -e'_i q_{ij} e_j (4q_0 q'_0)^{-1/2}, \quad (32)$$

$$q_{ij} = \int P[j_i(x),\ j_j(y)]\, e^{iqx - iq'x}\, dx dy. \quad (33)$$

Using the technique of Low, it is not hard to get the results*

$$g_{ij} = g_{ij}^{(0)} + A\delta_{ij} + B\varepsilon_{ijk} S_k + D(S_i S_j + S_j S_i), \quad (34)$$

$$A(\mathbf{q}'\mathbf{q}) + B(\mathbf{S}[\mathbf{q}'\times\mathbf{q}]) + D[(\mathbf{S}\mathbf{q}')(\mathbf{S}\mathbf{q}) + (\mathbf{S}\mathbf{q})(\mathbf{S}\mathbf{q}')] = q_0 q'_0 C, \quad (35)$$

$$C = \frac{(2\pi)^4}{i}\delta^{(4)}(p' + q' - p - q)\sum\Big[\frac{\langle \mathbf{q}-\mathbf{q}' | j_0 | \mathbf{q}\rangle\langle \mathbf{q} \mid j_0 \mid 0\rangle}{E(q) - E(0) - q_0} + \frac{\langle \mathbf{q}-\mathbf{q}' \mid j_0 \mid -\mathbf{q}'\rangle\langle -\mathbf{q}' \mid j_0 \mid 0\rangle}{E(q') - E(0) + q'_0}\Big], \quad (36)$$

$$g_{ij}^{(0)} = \frac{(2\pi)^4}{i}\delta^{(4)}(p' + q' - p - q)\sum\Big[\frac{\langle \mathbf{q}-\mathbf{q}' \mid j_i \mid \mathbf{q}\rangle\langle \mathbf{q} \mid j_j \mid 0\rangle}{E(q) - E(0) - q_0} + \frac{\langle \mathbf{q}-\mathbf{q}' \mid j_j \mid -\mathbf{q}'\rangle\langle -\mathbf{q}' \mid j_i \mid 0\rangle}{E(q') - E(0) + q'_0}\Big]. \quad (37)$$

The summation in Eqs. (36) and (37) is taken over the spins of the particles involved in the reaction.

Let us consider the case in which the states $|\mathbf{q}\rangle$ and so on are eigenstates of a system with spin S. For the calculation of Eqs. (36) and (37) we need the expression for the current matrix $\langle \mathbf{p}_2 | \mathbf{j} | \mathbf{p}_1\rangle$ in the low-energy region to accuracy v/c, and for $\langle \mathbf{q}' | j_0 | \mathbf{q}\rangle$ to accuracy v^2/c^2. It turns out that these matrix elements can be determined with the required accuracy on the basis of general principles.

Since $\mathbf{j}$ and j_0 are Hermitian operators and the interaction is invariant under three-dimensional rotations and time reversal, the most general form of the matrix element of the current is, in the approximation in question,

$$\langle p_2 | \mathbf{j} | p_1\rangle = (e/2M)(\mathbf{p}_1 + \mathbf{p}_2) + i\mu \mathbf{S}\times[\mathbf{p}_2 - \mathbf{p}_1] + c\{\mathbf{S}(\mathbf{S}, \mathbf{p}_1 + \mathbf{p}_2) + (\mathbf{S}, \mathbf{p}_1 + \mathbf{p}_2)\mathbf{S}\}, \quad (38)$$

$$\langle \mathbf{p}_2 | j_0 | \mathbf{p}_1\rangle = a + b(p_1^2 + p_2^2) + d(\mathbf{p}_1\mathbf{p}_2) + if(\mathbf{S}[\mathbf{p}_2\times\mathbf{p}_1]) + h[(\mathbf{S}\mathbf{p}_1)(\mathbf{S}\mathbf{p}_1) + (\mathbf{S}\mathbf{p}_2)(\mathbf{S}\mathbf{p}_2) + g[(\mathbf{S}\mathbf{p}_1)(\mathbf{S}\mathbf{p}_2) + (\mathbf{S}\mathbf{p}_2)(\mathbf{S}\mathbf{p}_1)], \quad (39)$$

where e is the total charge, $\mu s = \mu_0$ is the total magnetic moment, and the quantities a, b, c, d, f, h, g are invariant constants.

Under Lorentz transformations j_1 behaves like a component of a four-vector. Being an irreducible representation of the inhomogeneous Lorentz group, the wave function $|\mathbf{p}, \mu\rangle$ transforms in the following way:

$$|\mathbf{p}, \mu\rangle \xrightarrow{L} |\mathbf{p}, \mu\rangle' = R_{\mu\mu'}(L, \mathbf{p})\, |L^{-1}\mathbf{p}, \mu'\rangle, \quad (40)$$

where $R_{\mu\mu'}(L, \mathbf{p})$ is the rotation of the spin in the Lorentz transformation, which has been treated by a number of authors.[6,7]

Let us consider two coordinate systems. In one

$$\mathbf{p}_1 = 0, \qquad \mathbf{p}_2 = \mathbf{p},$$

and in the other

$$\mathbf{p}_1 = \mathbf{q}, \qquad p_{10} = E_q = \sqrt{q^2 + M^2},$$

$$\mathbf{p}_2 = \mathbf{p} + \frac{\mathbf{q}}{E_q}\Big[\frac{(\mathbf{p}\mathbf{q})}{q^2} E_q\Big(1 - \frac{E_q}{M}\Big) + \frac{E_q}{M}E_p\Big],$$

$$p_{20} = \frac{E_q}{M}\Big[E_p + \frac{(\mathbf{p}\mathbf{q})}{E_q}\Big]. \quad (41)$$

The second system moves with the velocity $-\mathbf{q}/E_q$ relative to the first. For the Lorentz transformation from the first system to the second we have to accuracy v^2/c^2

$$R(L, \mathbf{p}) = 1 + i(\mathbf{S}[\mathbf{p}\times\mathbf{q}])/2M^2. \quad (42)$$

We also have to accuracy v/c

$$\langle \mathbf{p}+\mathbf{q} | \mathbf{j} | \mathbf{q}\rangle' = e(\mathbf{p} + 2\mathbf{q})/2M + i\mu[\mathbf{S}\times\mathbf{p}] + c\{\mathbf{S}(\mathbf{S}, \mathbf{p} + 2\mathbf{q}) + (\mathbf{S}, \mathbf{p} + 2\mathbf{q})\mathbf{S}\}$$
$$= \langle \mathbf{p} | \mathbf{j} | 0\rangle + (\mathbf{q}/M)\langle \mathbf{p} | \mathbf{j} | 0\rangle$$
$$= e\mathbf{p}/2M + i\mu[\mathbf{S}\times\mathbf{p}] + c[\mathbf{S}(\mathbf{S}\mathbf{p}) + (\mathbf{S}\mathbf{p})\mathbf{S}] + a\mathbf{q}/M \quad (43)$$

and to accuracy v^2/c^2

$$\langle \mathbf{p}+\mathbf{q} | j_0 | \mathbf{q}\rangle' = a + b(p^2 + 2\mathbf{p}\mathbf{q} + 2q^2) + d(\mathbf{p}\mathbf{q} + q^2) + if(\mathbf{S}[\mathbf{p}\times\mathbf{q}]) + h\{(\mathbf{S}, \mathbf{p}+\mathbf{q})(\mathbf{S}, \mathbf{p}+\mathbf{q}) + (\mathbf{S}\mathbf{q})(\mathbf{S}\mathbf{q})\}$$
$$+ g\{(\mathbf{S}, \mathbf{p}+\mathbf{q})(\mathbf{S}\mathbf{q}) + (\mathbf{S}\mathbf{q})(\mathbf{S}, \mathbf{p}+\mathbf{q})\}$$
$$= [1 - i(\mathbf{S}[\mathbf{p}\times\mathbf{q}])/2M^2]\, E_q M^{-1}[\langle \mathbf{p} | j_0 | 0\rangle + (\mathbf{q}M^{-1}, \langle \mathbf{p} | \mathbf{j} | 0\rangle)] = a - ia(\mathbf{S}[\mathbf{p}\times\mathbf{q}])/2M^2 + aq^2/2M^2$$
$$+ bp^2 + h(\mathbf{S}\mathbf{p})(\mathbf{S}\mathbf{p}) + e(\mathbf{p}\mathbf{q})/2M^2 + i\mu(\mathbf{S}[\mathbf{p}\times\mathbf{q}])/M. \quad (44)$$

From Eq. (43) it follows that

$$a = e, \qquad c = 0,$$

and from Eq. (44) that

$$d + 2b = e/2M^2, \qquad f = \mu/M - e/2M^2, \qquad g = h = 0.$$

*This is the most general expression for an arbitrary S, if we are not concerned with terms with energy dependence higher than linear.

Finally we have the covariant expressions

$$\langle p_2 | \mathbf{j} | p_1 \rangle = e(\mathbf{p}_1 + \mathbf{p}_2)/2M + i\mu S \times [\mathbf{p}_2 - \mathbf{p}_1],$$

$$\langle p_2 | j_0 | p_1 \rangle = e + i(\mu/M - e/2M^2)(S[\mathbf{p}_2\, \mathbf{p}_1]) + e(\mathbf{p}_1 \mathbf{p}_2)/2M^2 + 2b(\mathbf{p}_1 - \mathbf{p}_2)^2, \qquad (45)$$

and, as must be so, the first of these is the same as the matrix element of the current of a nonrelativistic particle interacting with a magnetic field (cf., e. g., the book of Landau and Lifshitz[8]). It turns out that the term contained in the expression (45) makes no contribution to the final result.

By means of Eq. (45) one easily gets

$$S = i(2\pi)^4 \delta^{(4)}(p' + q' - p - q)(4q_0 q_0')^{-1/2} \{e^2(\mathbf{e}'\mathbf{e})/M - 2ie(S[\mathbf{e}' \times \mathbf{e}])\, M(\mu - e/2M) - (i\mu^2/q_0)(S[[\mathbf{e} \times \mathbf{q}] \times [\mathbf{e}' \times \mathbf{q}']]) - (ie\mu/Mq_0)[(\mathbf{e}\mathbf{q}')(S[\mathbf{q}' \times \mathbf{e}']) - (\mathbf{e}'\mathbf{q})(S[\mathbf{q} \times \mathbf{e}])]\} \qquad (46)$$

or for the matrix T

$$-T = e^2(\mathbf{e}'\mathbf{e})/M - 2i(e/M)\nu(\mu - e/2M)(S[\mathbf{e}' \times \mathbf{e}] - i(\mu^2/\nu)(S[[\mathbf{e} \times \mathbf{q}] \times [\mathbf{e}' \times \mathbf{q}']]) - (ie\mu/M\nu)[(\mathbf{e}\mathbf{q}')(S[\mathbf{q}' \times \mathbf{e}']) - (\mathbf{e}'\mathbf{q})(S[\mathbf{q} \times \mathbf{e}])]. \qquad (47)$$

For forward scattering, in particular, we have

$$-T = e^2(\mathbf{e}'\mathbf{e})/M - i\nu(S[\mathbf{e}' \times \mathbf{e}])[\mu_0/S - e/M]^2, \qquad (48)$$

from which the result (31) indeed follows for S = 1.

In the energy range below the threshold for photoproduction of mesons from deuterons the terms that depend on the spin make an insignificant contribution to the cross section, since the mass of the nucleus is doubled in comparison with that of a nucleon, and the magnetic moment is much smaller.

5. RESULTS OF THE ANALYSIS. DISCUSSION

Experimental data on the photodisintegration of the deuteron are available right up to ~ 500 Mev.[9] The results of the calculation of Re $[L_1 + (2/3)(L_2 + L_3)]$, for which it is sufficient to know the total cross sections, are shown in Fig. 1, where the values of the real part of the amplitude are represented as fractions of $e^2/M_p c^2$. The photon energy is measured as a multiple of the threshold for photodisintegration of the deuteron: $\nu_0/\nu_d = \gamma_0$. The diagram also shows the energy dependence of

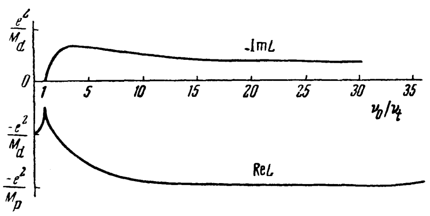

FIG. 1

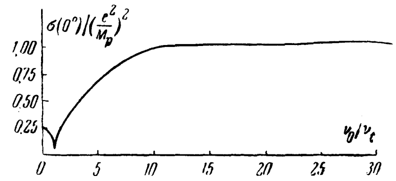

FIG. 2

the imaginary part of the quantity $L_1 + (2/3)(L_2 + L_3)$.

In the energy range considered, $\nu_0 \lesssim 100$ Mev, the dominant contribution is that of photodisintegration with $\nu \lesssim 75$ Mev. For the other amplitudes a more detailed analysis of the photodisintegration is required. If we assume that the contribution from photodisintegration with $\nu_0 \lesssim 80$ Mev is also decisive for the other amplitudes and use the analysis of de Swart and Marshak,[10] it is possible to estimate the dispersion parts of all the scalar amplitudes. But for γd scattering the spin-dependent amplitudes play a much smaller part as compared with the case of γN scattering. Figure 2 shows the energy dependence of the γd forward scattering. With increase of the γ-ray energy the γd scattering cross section at first shows a marked decrease as compared with the Thomson limit, and then ($\nu \gtrsim 4$ Mev) rapidly rises, and in the range $20 < \nu < 80$ Mev reaches values larger than $(e^2/M_d c^2)^2$ by a factor four.

Inclusion of the magnetic-dipole absorption, especially near the threshold, leads to an additional sharp "dip" of the cross section,[1] with a total half-width ~ 200 — 300 kev. The width of the total decrease of the cross section is considerably larger.

The large influence of inelastic processes that involve the deuteron as a whole, in addition to the processes involving the individual nucleons of the deuteron, makes it impossible to apply the impulse approximation to elastic γd scattering over a wide range of energies.* The presence of the inelastic process of photodisintegration of the deuteron has an especially strong effect on the polarizability of the deuteron. If, as A. M. Baldin has shown, the polarizability of nucleons is entirely due to the process of meson production, on the other hand the main contribution to the polarizability of the deuteron, and of nuclei in general, comes from photonuclear processes at much smaller energies.

*In a preprint received very recently, Schult and Capps have made a new examination of the corrections to the impulse approximation for γd scattering and have come to a similar conclusion.

It follows from Eq. (29) that the polarizability of the deuteron is given by

$$\alpha_d = \frac{d}{d\nu^2}[\mathrm{Re}(L_1 + {}^2/_3 L_2 + {}^2/_3 L_3)]_{\nu=0} = \frac{\hbar c}{2\pi^2} \mathrm{P}\int_{\nu_d}^{\infty} \frac{\sigma_t(\nu)\, d\nu}{\nu^2}. \quad (49)$$

An analogous formula is also valid for other nuclei. Dipole absorption plays the fundamental role in the total interaction cross section $\sigma_t(\nu)$. When this is taken into account Eq. (49) goes over into the well known formula of Migdal[11] (cf. also reference 12).

Substitution into Eq. (49) of the expressions (41) and (46) of reference 1 gives for the sum of the electric and magnetic polarizabilities of the deuteron

$$\alpha_e + \alpha_m = \alpha_d = \frac{e^2}{M_p c^2}\left(\frac{\hbar c}{\varepsilon}\right)^2 \left\{\frac{3}{64} + \frac{1}{12}\left(1 + \sqrt{\frac{\varepsilon'}{\varepsilon}}\right)^2 \frac{\varepsilon}{M_p c^2} \times (\mu_p - \mu_n)^2\right\} = 0.64 \cdot 10^{-39}\,\mathrm{cm}^3, \quad (50)$$

which agrees with the result of Levinger and Rustgi (cf. reference 12).

The presence of sizable contributions from photodisintegration in the γd elastic scattering amplitude and in the polarizability of the deuteron prevents our obtaining reliable conclusions about the polarizability of neutrons from the experimental data on the scattering of low-energy γ rays by deuterons.

To get information on the magnetic polarizability of the deuteron one must evidently have a much more detailed analysis of the photodisintegration of the deuteron and of processes of photoproduction of mesons from deuterons.*

Strictly speaking, the treatment carried out in the present paper is valid only for forward scattering. In the dipole approximation, however, the main results remain valid for other scattering angles also. But we have not made a direct comparison with the experimental data, since in the experiments[2] inelastic scattering of γ rays by deuterons,

$$\gamma + d \rightarrow n + p + \gamma,$$

was observed along with the elastic scattering. Recently A. M. Baldin (private communication) has examined the corrections to the impulse approximation in the inelastic scattering of γ rays and has arrived at the conclusion that for this process also there are appreciable corrections associated with the photodisintegration.

Thus we can evidently conclude that the results of an analysis that takes into account the photodisintegration of the deuteron (and the production of mesons), and the experimental data on the scattering of γ rays by deuterons in the energy range $\sim 50 - 100$ Mev are in agreement with each other. For a more reliable comparison of calculated results with experiment one first needs an analysis of the inelastic processes over a wider range of energies.

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*All conclusions concerning the magnetic polarizability of the proton are very sensitive to the assumptions that have to be made in the analysis of the photoproduction of pions. When one uses the analysis of Watson it follows from the results[4] that the magnetic polarizability of the proton is small. (In the case of the analysis of Watson it goes to zero.) This conclusion evidently is not in contradiction with the experimental data.[13]

[1] L. I. Lapidus and Chou Kuang-Chao, JETP **39**, 112 (1960), Soviet Phys. JETP **12**, 82 (1961).

[2] Hyman, Ely, Frisch, and Wahlig, Phys. Rev. Letters **3**, 93 (1959).

[3] R. H. Capps, Phys. Rev. **106**, 1031 (1957); **108**, 1032 (1957).

[4] L. I. Lapidus and Chou Kuang-Chao, JETP **37**, 1714 (1959), Soviet Phys. JETP **10**, 1213 (1960); JETP **38**, 201 (1960), Soviet Phys. JETP **11**, 147 (1960).

[5] F. E. Low, Phys. Rev. **96**, 1428 (1954). M. Gell-Mann and M. L. Goldberger, Phys. Rev. **96**, 1453 (1954).

[6] Chou Kuang-Chao and M. I. Shirokov, JETP **34**, 1230 (1958), Soviet Phys. JETP **7**, 851 (1958).

[7] L. Zastavenko and Chou Kuang-Chao, JETP **35**, 1417 (1958), Soviet Phys. JETP **8**, 990 (1959).

[8] L. Landau and E. Lifshitz, Quantum Mechanics, Pergamon, 1958.

[9] Barnes, Carver, Stafford, and Wilkinson, Phys. Rev. **86**, 359 (1952). J. Halpern and E. V. Weinstock, Phys. Rev. **91**, 934 (1953). Lew Allen, Jr., Phys. Rev. **98**, 705 (1955). J. C. Keck and A. V. Tollestrup, Phys. Rev. **101**, 360 (1956). Whalin, Schriever, and Hanson, Phys. Rev. **101**, 377 (1956). D. R. Dixon and K. C. Bandtel, Phys. Rev. **104**, 1730 (1956). Aleksandrov, Delone, Slovokhotov, Sokol, and Shtarkov, JETP **33**, 614 (1957), Soviet Phys. JETP **6**, 472 (1958). Tatro, Palfrey, Whaley, and Haxby, Phys. Rev. **112**, 932 (1958).

[10] J. J. de Swart, Physica **25**, 233 (1959). J. J. de Swart and R. E. Marshak, Physica **25**, 1001 (1959).

[11] A. B. Migdal, JETP **15**, 81 (1948).

[12] J. S. Levinger, Phys. Rev. **107**, 554 (1957).

[13] G. Bernardini, Report at the International Conference on the Physics of High-Energy Particles, Kiev, 1959.

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