ELECTROMAGNETIC WAVES IN A HALF-SPACE FILLED WITH A PLASMA

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Submitted to JETP editor April 27, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) 39, 845-853 (September, 1960)

The propagation of electromagnetic waves across a magnetic field in half-space filled with a magnetoactive plasma is studied. It is assumed that the plasma is confined by a stationary magnetic field H, and the structure of this field is investigated for the case when the ratio of the plasma pressure to the magnetic pressure is small. It is demonstrated that, at large distances from the plasma boundary an electromagnetic wave with an electric vector parallel to the magnetic field H has the form of a plane wave with a propagation constant which is specified by the equation for an infinite plasma. The reflection and transmission coefficients are evaluated for a plane wave incident on the plasma from a vacuum.

IN the present paper we consider the problem of the penetration of an electromagnetic wave in a half-space filled with a plasma. Problems of this type have been investigated by many researchers.[1-6] A linearized kinetic equation together with Maxwell's equations were used to describe the process. The boundary condition at the plasma boundaries was that the electrons be specularly reflected.

Landau[1] considered the problem for a longitudinal electric field. A characteristic feature of the solution was that the field away from the boundary of the plasma was not a plane wave. With the aid of Landau's method, Silin[2] solved the problem for a plasma confined in a homogeneous magnetic field $\mathbf{H}_0$ perpendicular to its boundary (magnetoactive plasma). An investigation of the solution, carried out by Shafranov,[3] has shown that in this case the field is not a plane wave far away from the plasma boundary.

Shafranov has made an attempt to consider the penetration of the electromagnetic field into a plasma for an arbitrary direction of the field $\mathbf{H}_0$, but the solution has not been carried out in a consistent manner. While the equations and boundary conditions are rigorously formulated for the electromagnetic field, only general remarks are made regarding the electron distribution function. It is stated that the condition that the electrons be specularly reflected from the plasma boundary does not distort their distribution functions and therefore the kernel $K(\mathbf{r}, \mathbf{r}')$ of the integral relations between the field and the currents in the plasma depends, as previously, only on the quantity $R = |\mathbf{r} - \mathbf{r}'|$. This statement has not been proved. In our opinion, it is true only in the exceptional case when the field $\mathbf{H}_0$ is perpendicular to the plasma boundary. For such a direction of the magnetic field, the Larmor circles for the electrons do not intersect the plasma boundary, and therefore the distribution function is the same as in an unbounded plasma if the electrons are specularly reflected from the boundary. For any other direction of the magnetic field $\mathbf{H}_0$, the Larmor circles will intersect the plasma boundary and this should influence the distribution function.

In later investigations devoted to semi-bounded plasma,[4-6] the authors either referred to Shafranov's results,[3] or did not touch at all on problems related to the determination of the distribution function, confining themselves to unproved general remarks.

The problem can thus at present be considered as consistently solved only when the field $\mathbf{H}_0$ is perpendicular to the plasma boundary and is parallel to the direction of wave propagation.[2] In the present paper we solve this problem for the opposite case: the magnetic field is assumed parallel to the plasma boundary and perpendicular to the direction of wave propagation, and the electric vector is assumed polarized parallel to the magnetic field (ordinary wave). It is shown that the field away from the plasma boundary is in the form of a plane wave, the propagation constant of which is the root of the corresponding dispersion equation formulated for the infinite plasma. In solving the problem, an attempt is made to construct a more accurate model for the plasma boundary, so as to replace the artificial specular-reflection boundary condition.

1. FORMULATION OF THE PROBLEM

Let a plasma filling the half-space $x > 0$ be located in a stationary magnetic field $\mathbf{H}(x)$ parallel to the z axis $[\mathbf{H}(x) = \{0, 0, H(x)\}]$; assume a specified electric field on the boundary of the plasma in the plane $x = 0$

$$E_x = E_y = 0, \qquad E_z = E_z(0, y)\, e^{-i\omega t}. \tag{1}$$

It is required to determine the field in the plasma, in the form of an outgoing wave as $x \to \infty$.

In solving this problem, we assume, as usual, that 1) the plasma is neutral on the average, 2) the electromagnetic wave does not act on the ions. 3) the electromagnetic wave disturbs little the electronic component of the plasma, and 4) the effect of the magnetic field of the wave can be neglected compared with the electric field and with the stationary magnetic field.

Let us proceed to discuss the conditions on the plasma boundary. We do not assume the usual hypothesis of specular reflection of the electrons from the boundary: this hypothesis necessitates an infinite magnetic field and is little justified physically in many cases. The model which we propose for the plasma boundary is based on the following premises: 1) there are no electrons in the region $x < 0$; 2) the electrons are confined in the region $x > 0$ by a stationary magnetic field $H(x)$, and 3) the stationary magnetic field becomes homogeneous away from the boundary, while the unperturbed electron distribution function is Maxwellian:

$$\lim_{x\to\infty} H(x) = H_0,$$

$$\lim_{x\to\infty} f_0 = N\,(m/2\pi T)^{3/2} \exp\left[-m\,(v_x^2 + v_y^2 + v_z^2)/2T\right]$$

(N is the electron density and T is the temperature in energy units).

Let us introduce the following notation: $\omega_0 = \sqrt{4\pi N e^2/m}$ is the plasma frequency, $\omega_H = eH_0/mc$ is the Larmor frequency, $x_0 = \sqrt{2T/m}\,\omega_H^{-1}$ is the average Larmor radius, and $\mu_0 = 2T\omega_0^2/mc^2\omega_H^2 = 8\pi NT/H_0^2$ is the ratio of the plasma pressure to the magnetic pressure. The problem so formulated is solved approximately under the assumption that $\mu_0 \ll 1$. Terms of order μ_0^2 are neglected.

2. UNPERTURBED DISTRIBUTION FUNCTION AND STATIONARY MAGNETIC FIELD

Let F be the electron distribution function, satisfying the kinetic equation in the Vlasov form

$$\frac{\partial F}{\partial t} + \mathbf{v}\nabla F + \frac{e}{m}\left(\mathbf{E} + \frac{1}{c}\,[\mathbf{v}\times\mathbf{H}],\ \nabla_v F\right) = 0. \tag{2}$$

We change over from variables x, v_x, v_y, v_z to dimensionless variables α, β, γ, and δ:

$$\xi = x/x_0, \qquad v_x = \sqrt{2T/m}\,\beta\cos\delta,$$

$$v_y = \sqrt{2T/m}\,\beta\sin\delta, \qquad v_z = \sqrt{2T/m}\,\gamma$$

and represent the function F in the form

$$F = f_0(\xi, \beta, \gamma, \delta) + f(t, \xi, y, \beta, \gamma, \delta), \tag{3}$$

where f_0 is the unperturbed distribution function, while f is a small perturbation due to the electromagnetic wave. From the kinetic equation (2) and from Maxwell's equations we obtain a system for the determination of the function f_0 and of the stationary magnetic field $H(x)$

$$\beta\cos\delta\,\partial f_0/\partial\xi - (1 + g(\xi))\,\partial f_0/\partial\delta = 0; \tag{4}$$

$$\frac{dg}{d\xi} = -\frac{4\pi x_0 j_y}{cH_0} = -\frac{4\pi e x_0}{cH_0}\left(\frac{2T}{m}\right)^2 \int_0^{2\pi} \sin\delta\, d\delta \int_0^{\infty} \beta^2 d\beta \int_{-\infty}^{\infty} f_0\, d\gamma, \tag{5}$$

$$H(\xi) = H_0(1 + g(\xi)). \tag{6}$$

The unperturbed distribution function f_0 for the solution of (4) is chosen as

$$f_0 = N\left(\frac{m}{2\pi T}\right)^{3/2} e^{-(\beta^2+\gamma^2)}\, \eta\left(\xi + \int_0^{\xi} g(\sigma)\, d\sigma + \beta(\sin\delta - 1)\right), \tag{7}$$

where $\eta(\xi)$ is the step function, $\eta(\xi) = 1$ when $\xi > 0$ and $\eta(\xi) = 0$ when $\xi < 0$. The function (7) satisfies the conditions formulated above for confining the electrons in the half-space $x > 0$. The discontinuity line of the function (7)

$$\xi + \int_0^{\xi} g(\sigma)\, d\sigma + \beta(\sin\delta - 1) = 0$$

is the characteristic of Eq. (4).

Substituting (7) into (5) and carrying out several transformations, we obtain the following equation for the function $g(\xi)$, which characterizes the inhomogeneity of the magnetic field

$$g(\xi) = \mu_0 \frac{1}{\pi} \int_0^{2\pi} (1 + \sin\delta)\, d\delta \int_{\xi}^{\infty} e^{-\Phi^2}\, \Phi^3 ds,$$

$$\Phi = \frac{1}{1 - \sin\delta}\left(s + \int_0^{s} g(\sigma)\, d\sigma\right). \tag{8}$$

A solution of this equation can be constructed in the form of a series in the parameter μ_0:

$$g(\xi) = \sum_{n=1}^{\infty} \mu_0^n g_n(\xi).$$

In the first approximation this yields

$$g(\xi) = \mu_0 g_1(\xi) + O(\mu_0^2), \tag{9}$$

where

$$g_1(\xi) = \frac{1}{2\pi}\int_0^{2\pi} \exp\left\{-\left(\frac{\xi}{1-\sin\delta}\right)^2\right\}\left[\left(\frac{\xi}{1-\sin\delta}\right)^2+1\right]\cos^2\delta d\delta. \tag{10}$$

Thus, the unperturbed distribution function f_0 and the stationary magnetic field $H(x)$ are given by (7) and (6), in which $g(\xi)$ is given by (9) and (10). We see that the inhomogeneity of the magnetic field is of order μ_0 and manifests itself only in a boundary zone of width $l \sim x_0$ ($\xi \sim 1$). Upon further increase of x, the additional term tends to zero exponentially.

3. DETERMINATION OF THE ELECTRIC FIELD OF THE WAVE IN THE PLASMA

Let us proceed now to determine the alternating electromagnetic field excited in the plasma by the boundary mode (1). We consider first the case when

$$E_z|_{x=0} = E(0)e^{i(h_y y-\omega t)}. \tag{11}$$

The general case can be reduced to that given by expansion of the boundary field in a Fourier integral with respect to the variable y. We shall seek the functions f and E_z in the form

$$f(t, \xi, y, \beta, \gamma, \delta) = e^{i(h_y y-\omega t)} f(\xi, \beta, \gamma, \delta),$$

$$E_z(t, \xi, y) = e^{i(h_y y-\omega t)} E(\xi).$$

From the kinetic Eq. (2) and from Maxwell's equations we obtain the following linearized system of equations for the functions $f(\xi, \beta, \gamma, \delta)$ and $E(\xi)$:

$$\beta\cos\delta\frac{\partial f}{\partial\xi} - (1+g(\xi))\frac{df}{d\delta} + i\left(x_0 h_y \beta\sin\delta - \frac{\omega}{\omega_H}\right)f = -\frac{e}{m\omega_H}\left(\frac{m}{2T}\right)^{1/2}\frac{\partial f_0}{\partial\gamma}E(\xi), \tag{12}$$

$$\frac{d^2E}{d\xi^2} + x_0^2(k^2-h_y^2)E = -ik^2x_0^2\frac{4\pi e}{\omega_H}\left(\frac{2T}{m}\right)^2\int_0^{2\pi}d\delta\int_0^\infty\beta d\beta\int_{-\infty}^{\infty} f\gamma d\gamma, \tag{13}$$

where $k = \omega/c$. From (12), taking into account the expression (7) for the function f_0, we see that the function f is proportional to $(Ne/m\omega_H)\times(m/2T)^2$. Consequently, the integral term in Eq. (13) is of order μ_0. Thus, to determine the electric field accurate to terms of order μ_0 inclusive, it is sufficient to obtain the function f in the zero approximation with $g(\xi) = 0$. In other words, the effect of the inhomogeneity of the stationary magnetic field can be neglected in determining the electric field with the degree of accuracy which we require. Taking this into account, we rewrite (12) as

$$\beta\cos\delta\,\partial f/\partial\xi - \partial f/\partial\delta + i(x_0h_y\beta\sin\delta - \omega/\omega_H)f = (Ne/m\omega_H)(2T/m)^2E(\xi)2\gamma e^{-(\beta^2+\gamma^2)}\eta(\xi+\beta(\sin\delta-1)). \tag{14}$$

Let us consider the system (13) and (14). It is easy to write down a solution of (14) which is periodic in δ with period 2π. Such a solution is unique and has the form

$$f = i\frac{Ne}{m\omega_H}\left(\frac{m}{2T}\right)^2\varkappa\left(\frac{\omega}{\omega_H}\right)2\gamma e^{-(\beta^2+\gamma^2)}\eta(\xi+\beta(\sin\delta-1)) \times\int_{\delta-2\pi}^{\delta}\exp\left\{i\left[\frac{\omega}{\omega_H}(\alpha-\delta+\pi)+x_0h_y\beta(\cos\alpha-\cos\delta)\right]\right\} \times E(\xi+\beta(\sin\delta-\sin\alpha))d\alpha, \tag{15}$$

$$\varkappa\left(\frac{\omega}{\omega_H}\right) = \frac{\omega}{\omega_H}\Big/2\pi\sin\frac{\omega}{\omega_H}\pi.$$

Substituting (15) and (13), we obtain an integro-differential equation for the field $E(\xi)$:

$$\frac{d^2E}{d\xi^2} + x_0^2(k^2-h_y^2)E = \mu_0\varkappa\left(\frac{\omega}{\omega_H}\right)\int_0^{2\pi}d\delta\int_{\delta-2\pi}^{\delta} \times d\alpha\int_0^\infty E(\xi+\beta(\sin\delta-\sin\alpha))\times\eta(\xi+\beta(\sin\delta-1)) \times\exp\left\{-\beta^2+i\left[\frac{\omega}{\omega_H}(\alpha-\delta+\pi) + x_0h_y\beta(\cos\alpha-\cos\delta)\right]\right\}\beta d\beta. \tag{16}$$

We are interested in a solution of (16) which assumes a specified value $E(0)$ when $\xi = 0$ and behaves like an outgoing wave as $\xi \to \infty$. We shall seek such a solution in the form

$$E(\xi) = E_\infty(e^{ix_0h_x\xi} + u(\xi)). \tag{17}$$

Here $u(\xi)$ is a new unknown function, which must approach zero at infinity, while h_x and E_∞ are constants to be determined. According to (16), the function $u(\xi)$ should satisfy the following equation:

$$\frac{d^2u}{d\xi^2} + x_0^2(k^2-h_y^2)u = \mu_0\varkappa\left(\frac{\omega}{\omega_H}\right)\int_0^{2\pi}d\delta \times\int_{\delta-2\pi}^{\delta}d\alpha\int_0^\infty u(\xi+\beta(\sin\delta-\sin\alpha)) \times\eta(\xi+\beta(\sin\delta-1))\exp\left\{-\beta^2+i\left[\frac{\omega}{\omega_H}(\alpha-\delta+\pi) + x_0h_y\beta(\cos\alpha-\cos\delta)\right]\right\}\times\beta d\beta - e^{-ih_xx_0\xi}\mu_0\varkappa\left(\frac{\omega}{\omega_H}\right) \times\int_0^{2\pi}d\delta\int_{\delta-2\pi}^{\delta}d\alpha\int_{\xi'}^{\infty}\exp\left\{-\beta^2+i\left[\frac{\omega}{\omega_H}(\alpha-\delta+\pi) + x_0h_y\beta(\cos\alpha-\cos\delta)+x_0h_x\beta(\sin\delta-\sin\alpha)\right]\right\} \times\beta d\beta - x_0^2D\left(\sqrt{h_x^2+h_y^2}\right)e^{-ix_0h_x\xi},$$

where $\xi' = \xi/(1-\sin\delta)$, and

$$D(h) = k^2 - h^2 - \pi \frac{\omega_0^2}{c^2} \varkappa \left(\frac{\omega}{\omega_H}\right) \int_0^{2\pi} \exp\left\{-\frac{x_0^2 h^2}{2}(1-\cos\varepsilon)\right\} \times \cos\frac{\omega}{\omega_H}(\varepsilon - \pi)\, d\varepsilon.$$

Let us put $h_x = (h^2 - h_y^2)^{-1/2}$, where h is the root of the equation

$$D(h) = 0. \quad (19)$$

This equation is a dispersion equation for waves of the type considered in an unbounded plasma. With this choice of h_x, the term that does not vanish at infinity drops out from Eq. (18). The amplitude E_∞ can be determined from the boundary condition at $\xi = 0$:

$$E(0) = E_\infty(1 + u(0)). \quad (20)$$

The solution of the integro-differential equation (18), which tends to zero at infinity, must satisfy the following integral equation

$$u(\xi) = \mu_0 L[u] + \mu_0 w(\xi), \quad (21)$$

where

$$L[u] = \varkappa\left(\frac{\omega}{\omega_H}\right) \int_0^{2\pi} d\delta \int_{\delta-2\pi}^{\delta} d\alpha \int_0^\infty \exp\left\{-\beta^2 + i\left[\frac{\omega}{\omega_H}(\alpha - \delta + \pi) + x_0 h_y \beta(\cos\alpha - \cos\delta)\right]\right\}\beta d\beta \times \int_\xi^\infty \frac{\sin[x_0\sqrt{k^2 - h_y^2}(\sigma - \xi)]}{x_0\sqrt{k^2 - h_y^2}} \eta(\sigma + \beta(\sin\delta - 1))\, u(\sigma + \beta(\sin\delta - \sin\alpha))\, d\sigma, \quad (22)$$

$$w(\xi) = -\varkappa\left(\frac{\omega}{\omega_H}\right) \int_0^{2\pi} d\delta \int_{\delta-2\pi}^{\delta} d\alpha \int_\xi^\infty \frac{\sin[x_0\sqrt{k^2 - h_y^2}(\sigma - \xi)]}{x_0\sqrt{k^2 - h_y^2}} e^{ix_0 h_x \sigma} d\sigma \times \int_{\sigma'}^\infty \exp\left\{-\beta^2 + i\left[\frac{\omega}{\omega_H}(\alpha - \delta + \pi) + x_0 h_y \beta(\cos\alpha - \cos\delta) + x_0 h_x \beta(\sin\delta - \sin\alpha)\right]\right\}\beta d\beta \quad (23)$$

[here $\sigma' = \sigma/(1 - \sin\delta)$]. Equation (21) contains the small parameter μ_0 ahead of the integral term, and it is therefore natural to solve it by successive approximations. We shall seek the solution of (21) in the form

$$u(\xi) = \sum_{n=1}^\infty \mu_0^n u_n(\xi), \quad (24)$$

and determine the functions $u_n(\xi)$ from the recurrence formulas

$$u_1(\xi) = w(\xi), \qquad u_{n+1}(\xi) = L[u_n].$$

It is easy to show that (24) converges, and consequently that (21) is solvable at sufficiently small values of the parameter μ_0. To obtain a solution of (21) with the required degree of accuracy, it is sufficient to use the zeroth approximation

$$u(\xi) = \mu_0 w(\xi) + O(\mu_0^2). \quad (25)$$

The amplitude E_∞ of the wave at infinity is determined from Eq. (20):

$$E_\infty = E(0)(1 - \mu_0 w(0) + O(\mu_0^2)). \quad (26)$$

Thus, the electric field in the plasma, under boundary condition (11), is of the form

$$E_z(x, y) = E_\infty e^{i(h_x x + h_y y)}\{1 + \mu_0 e^{-ih_x x} w(x/x_0) + O(\mu_0^2)\}. \quad (27)$$

As $x \to \infty$, this field behaves like a plane wave with a propagation constant $h = (h_x^2 + h_y^2)^{1/2}$ which is determined by the dispersion equation for the unbounded plasma. The function $w(x/x_0)$ describes the distortion of the field at the plasma boundary. It differs noticeably from zero only in a boundary zone of width x_0, and tends exponentially to zero with increasing distance from the boundary.

If we neglect the motion of the electrons and assume that their temperature is zero, then Eq. (27) goes automatically into the elementary-theory equation for the interaction between the electromagnetic wave and the plasma in terms of the dielectric-constant tensor.

4. CASE OF LARGE WAVELENGTHS

In this section we shall consider in greater detail the case when

$$\mu = 2T\omega^2/mc^2\omega_H^2 = (kx_0)^2 \ll 1,$$

i.e., when the average Larmor radius of the electrons is much smaller than the wave length in vacuum.

Let us compare the parameters μ_0 and $\mu = \mu_0\omega^2/\omega_0^2$. In a cold plasma (T = 0), the ordinary wave can propagate transversely to the magnetic field if $\omega_0/\omega < 1$. Consequently, in a plasma with low temperature ($T \ll mc^2$) the parameter μ_0 for propagating waves should either be smaller than μ or of the same order. In this section we shall expand all the quantities in powers of the parameters μ, and consider at the same time that $O(\mu_0) \lesssim O(\mu)$.

Let us consider first the function $w(\xi)$, given by (23), which can be represented in the form [we use $\xi' = \xi/(1 - \sin\delta)$]

$$w(\xi) = w_0(\xi) + \sqrt{\mu}\, w_1(\xi) + O(\mu);$$

$$w_0(\xi) = -\frac{1}{4\pi}\int_0^{2\pi}\left[(1-\sin\delta)^2 e^{-\xi'^2} - 2\xi(1-\sin\delta)\int_{\xi'}^\infty e^{-\beta^2}d\beta\right]d\delta,$$

$$w_1(\xi) = -\frac{i}{8\pi}\int_0^{2\pi}\left\{\frac{(h_y - ih_x\omega/\omega_H)\cos\delta - (h_x + ih_y\omega/\omega_H)\sin\delta}{k(1-\omega^2/\omega_H^2)} \times \left[\xi(1-\sin\delta)e^{-\xi'^2} - (2\xi^2 + 3(1-\sin\delta)^2)\int_{\xi'}^\infty e^{-\beta^2}d\beta\right] + \frac{2h_x}{k}(1-\sin\delta)^2\int_{\xi'}^\infty e^{-\beta^2}d\beta\right\}d\delta. \quad (28)$$

In particular

$$w(0) = -\frac{3}{4} - \frac{i}{8}\left[5\frac{h_x}{k} - 3\frac{h_x + ih_y\omega/\omega_H}{k(1-\omega^2/\omega_H^2)}\right]\sqrt{\mu} + O(\mu). \quad (29)$$

Substituting (28) and (29) in (26) and (27), we get

$$E_\infty = E(0)\left\{1 + \frac{3}{4}\mu_0 + \frac{i}{8}\mu_0\sqrt{\mu}\left[5\frac{h_x}{k} - 3\frac{h_x + ih_y\omega/\omega_H}{k(1-\omega^2/\omega_H^2)}\right] + O(\mu^2)\right\}, \quad (30)$$

$$E_z(x, y) = E_\infty e^{i(h_x x + h_y y)}\left\{1 + \mu_0 e^{-ih_x x}\left[w_0\left(\frac{x}{x_0}\right) + \sqrt{\mu}\, w_1\left(\frac{x}{x_0}\right)\right] + O(\mu^2)\right\}. \quad (31)$$

With the aid of (30) and (31) we readily can construct a solution of the problem with the general boundary condition (1). Confining ourselves to terms of order μ, we obtain

$$E_z(x, y) = \frac{ih}{2}\left(1 + \frac{3}{4}\mu_0\right)\int_{-\infty}^{\infty} H_0^{(1)}\left(h\sqrt{x^2 + (y-\eta)^2}\right) \times \frac{xE_z(0, \eta)\, d\eta}{\sqrt{x^2 + (y-\eta)^2}} + \mu_0 w_0\left(\frac{x}{x_0}\right)E_z(0, y) + O(\mu^{3/2}). \quad (32)$$

The simplicity of formula (32) is due to the fact that E_∞ and $u(x/x_0)$ are independent of h_y at the degree of accuracy indicated above.

In conclusion let us consider the incidence of a plane electromagnetic wave with wave vector $\mathbf{k} = (k_x, k_y, 0)$ from vacuum, $x < 0$, on a half-space filled with plasma. We assume that the electric vector of the incident wave is polarized along the z axis, and obtain the solution of this problem accurate to terms of order μ inclusive.

In the region $x < 0$ the electric field has the form

$$E = E_0 e^{i(k_x x + k_y y)} + E_1 e^{i(-k_x x + k_y y)},$$

where E_0 is the specified amplitude of the incident wave. The field in the plasma is determined by the formula (31), where $h_y = k_y$. Using the conditions of the continuity of the functions $E_z(x, y)$ and $\partial E_z(x, y)/\partial x$ in the plane $x = 0$, we obtain the following system of equations for E_1 and E_∞:

$$E_0 + E_1 = E_\infty\left[1 - \frac{3}{4}\mu\omega_0^2/\omega^2 + O(\mu^{3/2})\right]$$

$$(E_0 - E_1)k_x = E_\infty h_x\left[1 - i\frac{\sqrt{\mu}}{2}\frac{\omega_0^2}{\omega^2}\frac{k}{h_x} + \mu\frac{\omega_0^2}{\omega^2}\left(\frac{3}{4} - \frac{1}{2}\frac{1 + ik_y\omega/h_x\omega_H}{1-\omega^2/\omega_H^2}\right) + O(\mu^{3/2})\right].$$

Hence

$$\frac{E_1}{E_0} = \frac{k_x - h_x}{k_x + h_x}\left\{1 - \mu\left[\frac{\pi}{2}\frac{k_x^2 - k_x h_x}{k^2} + \frac{k_x h_x}{k^2}\frac{2 - 3\omega^2/\omega_H^2}{1-\omega^2/\omega_H^2}\right] + i\left[\sqrt{\mu}\frac{\sqrt{\pi}h_x}{k} + \mu\frac{k_x h_y}{k^2}\frac{\omega_0^2}{\omega\omega_H}\frac{1}{1-\omega^2/\omega_H^2}\right] + O(\mu^{3/2})\right\} \quad (33)$$

$$\frac{E_\infty}{E_0} = \frac{2k_x}{k_x + h_x}\left\{1 + \mu\left[\frac{3}{4}\frac{\omega_0^2}{\omega^2} - \frac{(k_x - h_x)h_x}{2k^2}\frac{2 - 3\omega^2/\omega_H^2}{1-\omega^2/\omega_H^2} - \frac{\pi}{4}\frac{(k_x - h_x)^2}{k^2}\right] + i\left[\sqrt{\mu}\frac{\sqrt{\pi}}{2}\frac{k_x - h_x}{k} + \mu\frac{(k_x - h_x)k_y}{2k^2}\frac{1}{1-\omega^2/\omega_H^2}\frac{\omega_0^2}{\omega\omega_H}\right] + O(\mu^{3/2})\right\}. \quad (34)$$

In deriving these formulas we used the relations

$$k_x^2 - h_x^2 = k^2 - h^2 = k^2\omega_0^2\omega^{-2}(1 + O(\mu)).$$

If $|k_y| = |h_y| < h$, i.e., if h_x is real, we have

$$\left|\frac{E_1}{E_0}\right|^2 = \left(\frac{k_x - h_x}{k_x + h_x}\right)^2\left\{1 + \mu\frac{k_x h_x}{k^2}\left[\pi - 2\frac{2 - 3\omega^2/\omega_H^2}{1-\omega^2/\omega_H^2}\right] + O(\mu^{3/2})\right\}, \quad (35)$$

$$\left|\frac{E_\infty}{E_0}\right|^2 = \left(\frac{2k_x}{k_x + h_x}\right)^2\left\{1 + \mu\left[\frac{3}{2}\frac{\omega_0^2}{\omega^2} - \frac{h_x(k_x - h_x)}{k^2}\frac{(2 - 3\omega^2/\omega_H^2)}{1-\omega^2/\omega_H^2} - \frac{\pi}{4}\frac{(k_x - h_x)^2}{k^2}\right] + O(\mu^{3/2})\right\}, \quad (36)$$

$$\arg E_1 = \sqrt{\mu}\frac{\sqrt{\pi}k_x}{k} + \mu\frac{k_x k_y}{k^2}\frac{\omega_0^2}{\omega\omega_H}\frac{1}{1-\omega^2/\omega_H^2} + O(\mu^{3/2}), \quad (37)$$

$$\arg E_\infty = \sqrt{\mu}\frac{\pi}{2}\frac{k_x - h_x}{k} + \mu\frac{(k_x - h_x)k_y}{k^2}\frac{\omega_0^2}{\omega\omega_H}\frac{1}{2(1-\omega^2/\omega_H^2)} + O(\mu^{3/2}). \quad (38)$$

When $\mu = 0$, Eqs. (33) — (38) coincide with the corresponding formulas of elementary theory. It must be noted that the phases of the transmitted and reflected waves are more sensitive to variation of the electron temperature than the amplitudes.

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Translated by J. G. Adashko
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