RADIATIVE CORRECTIONS TO THE SCATTERING OF μ MESONS ON ELECTRONS

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Formulas of the cross sections for the processes $\mu + e^{\pm} \to \mu + e^{\pm}$ and $e^+ + e^- \to \mu^+ + \mu^-$ are deduced with an accuracy to e^6.

INTRODUCTION

THE expected possibility of producing opposing beams of electrons and positrons in the near future, raises the question of an experimental investigation of the processes

$$e^- + e^+ \to \mu^- + \mu^+, \qquad e^- + e^+ \to \pi^- + \pi^+.$$

The cross section of the first process, without radiative corrections, was first obtained by Berestetskiĭ and Pomeranchuk.[1] The second process was considered by Aleksin (see the book by Akhiezer and Berestetskiĭ)[2] and by Afrikyan and Garibyan.[3] In the present article we give formulas for the radiative corrections to scattering and transformation processes, accurate to e^6. We show also that the exchange between two photons does not contribute to the total cross section (i.e., the cross section integrated over the angle) of the transformation $e^+ + e^- \to X + \overline{X}$, where X and $\overline{X}$ denote any particle and antiparticle.

1. ELASTIC SCATTERING OF NEGATIVE MUONS BY ELECTRONS

We shall calculate the scattering of a muon by an electron by a Feynman technique[4] similar to that used for the scattering of an electron by an electron.[5-6] We shall use a notation close to that of Redhead.[6]

Let p_1 and p_{1M} be the four-momenta of the colliding particles, and let p_2 and p_{2M} be the four momenta of the scattered particles with masses m = 1 and M respectively. We put:

$$q = p_2 - p_1, \qquad \xi = q^2 = 4\sinh^2 w = 4M^2 \sinh^2 w_M,$$

$$\eta = (p_{1M} - p_1)^2, \qquad \zeta = (p_{2M} - p_1)^2 = \eta - \xi$$

$$(ab = \mathbf{ab} - a_0 b_0).$$

The square of the matrix element for the process under consideration, averaged over the initial spin states and summed over the final ones, is

$$|\mathfrak{M}|^2 = \frac{1}{E_1 E_2 E_{1M} E_{2M} \xi^2} \operatorname{Re}\{Q + \frac{\alpha}{\pi}[2Q(A + A_M + W) + B^{(1)} + B^{(2)} + Z\xi(\xi - 2M^2) + Z_M M\xi(\xi - 2)]\}, \quad (1)$$

where

$$Q = \tfrac{1}{8}[2\eta^2 + \xi^2 - 2\xi\eta + 4\eta(M^2+1) - 4\xi(M^2+1) + 2(M^2+1)^2]; \quad (2)$$

$$A_M = \left(\ln\frac{M}{\lambda} - 1\right)(1 - 2w_M \coth 2w_M) - \frac{w_M}{2}\tanh w_M - 2\coth 2w_M \int_0^{w_M} \beta \tanh\beta\, d\beta;$$

$$A = \left(\ln\frac{1}{\lambda} - 1\right)(1 - 2w\coth 2w) - \frac{w}{2}\tanh w - 2\coth 2w \int_0^{w} \beta\tanh\beta\, d\beta; \quad (3)$$

$$Z_M = w_M / 4\sinh 2w_M, \qquad Z = w/4\sinh 2w; \quad (4)$$

$$W = \frac{1 - 2\sinh^2 w}{3\sinh^2 w}(1 - w\coth w) + \frac{1}{9}. \quad (5)$$

As usual, we have introduced the fictitious photon mass λ.

The terms $B^{(1)}$ and $B^{(2)}$ are due to the contribution from the exchange by two photons (interference of the main term with the terms from diagrams 1 and 2 of Fig. 1):

$$B^{(1)} = \tfrac{1}{8}\xi\{-8(\eta + M^2 + 1)Qb^{(1)} + (H^{(1)} - \tfrac{1}{2}\xi b^{(1)})\varphi_1^{(1)} + \overline{G}\varphi_2^{(1)} + G\varphi_3^{(1)} + \frac{\ln\xi}{2(\xi+4)}\varphi_4^{(1)} + \frac{\ln(\xi/M^2)}{2(\xi + 4M^2)}\varphi_5^{(1)} + \tfrac{1}{2}N^{(1)}\varphi_6^{(1)} + \tfrac{1}{2}K^{(1)}\varphi_7^{(1)}\}, \quad (6)$$

$$B^{(2)} = \tfrac{1}{8}\xi\{-8(\zeta + M^2 + 1)Qb^{(2)} + (H^{(2)} - \tfrac{1}{2}\xi b^{(2)})\varphi_1^{(2)} + \overline{G}\varphi_2^{(2)} + G\varphi_3^{(2)} + \frac{\ln\xi}{2(\xi+4)}\varphi_4^{(2)} + \frac{\ln(\xi/M^2)}{2(\xi+4M^2)}\varphi_5^{(2)} + \tfrac{1}{2}N^{(2)}\varphi_6^{(2)} + \tfrac{1}{2}K^{(2)}\varphi_7^{(2)}\}. \quad (7)$$

Here

$$\varphi_1^{(1)} = 2\eta^2 - \eta\xi + 6\eta(M^2+1) - \xi(M^2+1) + 4(M^2+1)^2,$$

$$\varphi_2^{(1)} = -2\eta^2 + \eta\xi + 2\xi(M^2+1) - 2\eta(2M^2+3) - 2(M^2+1)(M^2+2) - \xi[\xi^2 + 2\xi + 4\eta + 4(M^2+1)]/2(\xi+4),$$

$$\varphi_3^{(1)} = -2\eta^2 + \eta\xi + 2\xi(M^2+1) - 2\eta(3M^2+2) - 2(M^2+1)(2M^2+1) - \xi[\xi^2 + 2M^2\xi + 4M^2\eta + 4M^2(M^2+1)]/2(\xi+4M^2),$$

$$\varphi_4^{(1)} = \xi^2 - \xi\eta + 4\eta - \xi(M^2+1) + 4(M^2+1),$$

$$\varphi_5^{(1)} = \xi^2 - \xi\eta + 4\eta M^2 - \xi(M^2+1) + 4M^2(M^2+1), \qquad \varphi_6^{(1)} = \eta^2 - \xi\eta + 2\eta - \xi(3M^2+1) + 1 - M^4,$$

$$\varphi_7^{(1)} = \eta^2 - \xi\eta + 2M^2\eta - \xi(M^2+3) + M^4 - 1, \qquad \varphi_1^{(2)} = -2\eta^2 - \xi^2 + 3\xi\eta - 2\eta(M^2+1) + \xi(M^2+1),$$

$$\varphi_2^{(2)} = 2\eta^2 - \tfrac{7}{2}\xi\eta + \tfrac{3}{2}\xi^2 + 2\eta(2M^2+1) - \tfrac{1}{2}\xi(9M^2+5) + 2M^2(M^2+1) + \xi^2(\eta+M^2+3)/2(\xi+4),$$

$$\varphi_3^{(2)} = 2\eta^2 - \tfrac{7}{2}\xi\eta + \tfrac{3}{2}\xi^2 + 2\eta(M^2+2) - \tfrac{1}{2}\xi(5M^2+9) + 2(M^2+1) + \xi^2(\eta+3M^2+1)/2(\xi+4M^2),$$

$$\varphi_4^{(2)} = -\xi\eta - \xi(M^2+5) + 4\eta + 4(M^2+1), \qquad \varphi_5^{(2)} = -\xi\eta - \xi(5M^2+1) + 4M^2\eta + 4M^2(M^2+1),$$

$$\varphi_6^{(2)} = -\eta^2 + \xi\eta + \xi(5M^2+1) - 2\eta(2M^2+1) - (M^2+1)(3M^2+1),$$

$$\varphi_7^{(2)} = -\eta^2 + \xi\eta + \xi(M^2+5) - 2\eta(M^2+2) - (M^2+1)(M^2+3);$$

$$b^{(1,2)} = -(2\mu^{(1,2)}/\xi)\ln(\xi/\lambda^2), \qquad H^{(1,2)} - \tfrac{1}{2}\xi b^{(1,2)} = I^{(1,2)} + \mu^{(1,2)}\ln\xi. \qquad (8)$$

We denote for brevity

$$\alpha_1 = (M^2-1)/(\eta+2M^2+2), \qquad \alpha_2 = (M^2-1)/\zeta,$$

$$\beta_1 = [\alpha_1^2 + \eta/(\eta+2M^2+2)]^{1/2},$$

$$\beta_2 = [\alpha_2^2 + (\zeta+2M^2+2)/\zeta]^{1/2},$$

$$x_1 = \alpha_1 + \beta_1, \qquad y_1 = -\alpha_2 + \beta_2,$$

$$x_2 = \alpha_1 - \beta_1, \qquad y_2 = -\alpha_2 - \beta_2. \qquad (9)$$

Then

$$\mu^{(1)} = \frac{-1}{(\eta+2M^2+2)\,2\beta_1}\left\{\ln\left[\frac{1-x_1}{1-x_2}\,\frac{1+x_2}{1+x_1}\right] + 2\pi i\right\},$$

$$\mu^{(2)} = \frac{1}{\zeta 2\beta_2}\ln\left[\frac{y_1-1}{y_1+1}\,\frac{-y_2-1}{-y_2+1}\right]; \qquad (10)$$

$$K^{(1)} = \left(1 + \frac{M^2-1}{\eta+2M^2+2}\right)\mu^{(1)} + \frac{1}{\eta+2M^2+2}\ln M,$$

$$K^{(2)} = \left(1 - \frac{M^2-1}{\zeta}\right)\mu^{(2)} - \frac{1}{\zeta}\ln M; \qquad (11)$$

$$N^{(1)} = \left(1 - \frac{M^2-1}{\eta+2M^2+2}\right)\mu^{(1)} - \frac{1}{\eta+2M^2+2}\ln M,$$

$$N^{(2)} = \left(1 + \frac{M^2-1}{\zeta}\right)\mu^{(2)} + \frac{1}{\zeta}\ln M. \qquad (12)$$

The function $I^{(1)}$ is of the form

$$I^{(1)} = -\mu^{(1)}\ln\frac{\eta+2M^2+2}{4} + \frac{1}{(\eta+2M^2+2)\beta_1}\left\{u_2\ln\frac{\eta+2M^2+2}{4} + u_1\ln\frac{\eta+2M^2+2}{4M^2} - 2\int_0^{u_2}\beta\coth\beta d\beta - 2\int_0^{u_1}\beta\coth\beta d\beta + 2\pi i\ln 2\beta_1 + \pi^2\right\},$$

$$u_2 = \frac{1}{2}\ln\frac{1-x_2}{1-x_1}, \qquad u_1 = \frac{1}{2}\ln\frac{1+x_1}{1+x_2}. \qquad (13)$$

$I^{(2)}$ has different forms for $\zeta > 0$ and $\zeta < 0$. When $\zeta > 0$

$$I^{(2)} = -\mu^{(2)}\ln\frac{\zeta}{4} + \frac{1}{\zeta\beta_2}\left\{-\varphi_2\ln\frac{\zeta}{4} - \varphi_1\ln\frac{\zeta}{4M^2} + 2\int_0^{\varphi_2}\beta\tanh\beta d\beta + 2\int_0^{\varphi_1}\beta\tanh\beta d\beta\right\},$$

$$\varphi_2 = \frac{1}{2}\ln\frac{1-y_2}{-1+y_1}, \qquad \varphi_1 = \frac{1}{2}\ln\frac{1+y_1}{-1-y_2}, \qquad (14)$$

and when $\zeta < 0$

$$I^{(2)} = -\mu^{(2)}\ln\frac{|\zeta|}{4} + \frac{1}{|\zeta|\beta_2}\left\{-\Omega_2\ln\frac{|\zeta|}{4} + \Omega_1\ln\frac{|\zeta|}{4M^2} + 2\int_0^{\Omega_2}\beta\coth\beta d\beta - 2\int_0^{\Omega_1}\beta\coth\beta d\beta\right\},$$

$$\Omega_2 = \frac{1}{2}\ln\frac{-1+y_1}{-1+y_2}, \qquad \Omega_1 = \frac{1}{2}\ln\frac{1+y_1}{1+y_2}. \qquad (15)$$

At the point $\zeta = 0$, the function $I^{(2)}$ is continuous: $I^{(2)}(\zeta) \rightarrow M^{-2}\ln^2 M$ as $\zeta \rightarrow \pm 0$.

Finally

$$\bar{G} = \frac{1}{2\sqrt{\xi(\xi+4)}}\left[\ln^2\frac{1-\beta}{\beta} - \ln^2\frac{\alpha-1}{\alpha} - 2\Phi\left(\frac{\beta}{\beta-1}\right) + 2\Phi\left(\frac{\alpha}{\alpha-1}\right) + \pi^2\right], \qquad (16)$$

$$\alpha = \tfrac{1}{2}[\xi + 2 + \sqrt{\xi(\xi+4)}]; \quad \beta = \tfrac{1}{2}[\xi + 2 - \sqrt{\xi(\xi+4)}];$$

$$G = \frac{1}{2\sqrt{\xi(\xi+4M^2)}}\left[\ln^2\frac{1-\beta'}{\beta'} - \ln^2\frac{\alpha'-1}{\alpha'} - 2\Phi\left(\frac{\beta'}{\beta'-1}\right) + 2\Phi\left(\frac{\alpha'}{\alpha'-1}\right) + \pi^2\right], \qquad (17)$$

$$\alpha' = \tfrac{1}{2}\left[2 + \frac{\xi}{M^2} + \sqrt{\frac{\xi}{M^2}\left(\frac{\xi}{M^2}+4\right)}\right],$$

$$\beta' = \tfrac{1}{2}\left[2 + \frac{\xi}{M^2} - \sqrt{\frac{\xi}{M^2}\left(\frac{\xi}{M^2}+4\right)}\right],$$

where

$$\Phi(x) = -\int_0^x \ln|1-y|\,y^{-1}dy$$

is the Spence function, tabulated in the paper by Mitchell.[7]

2. INELASTIC-SCATTERING CROSS SECTION

We now add to the elastic-scattering cross section the cross section of the scattering that is accompanied by the emission of soft quanta (with total energy $\Delta\epsilon$). Here, as is well known, the photon mass λ, which we have introduced, drops out. The inelastic scattering cross section is

$$d\sigma_{\text{inel}} = d\sigma_{\text{el}} \frac{\alpha}{\pi}\left[L - L_0 \ln\frac{2\Delta\varepsilon}{\lambda}\right]$$

$$(\lambda \ll \Delta\varepsilon \ll E_1,\ E_2, E_{1M},\ E_{2M}), \tag{18}$$

where

$$L_0 = 4 + 2K_0(p_1,\ p_2) + 2K_0(p_{1M},\ p_{2M}) - 4K_0(p_1,\ p_{1M}) + 4K_0(p_1,\ p_{2M}), \tag{19}$$

$$L = K(p_1,\ p_1) + K(p_2,\ p_2) + K(p_{1M},\ p_{1M}) + K(p_{2M},\ p_{2M}) - 2K(p_1,\ p_2) - 2K(p_{1M},\ p_{2M}) - 2K(p_1,\ p_{2M}) - 2K(p_2,\ p_{1M}) + 2K(p_1,\ p_{1M}) + 2K(p_2,\ p_{2M});$$

$$K_0(P_1,\ P_2) = -\tfrac{1}{2}(P_1P_2)\int_{-1}^{1} P_z^{-2}\,dz,$$

$$K(P_1,\ P_2) = \tfrac{1}{4}(P_1P_2)\int_{-1}^{1} P_z^{-2}\frac{E_z}{|\mathbf{P}_z|}\ln\frac{E_z+|\mathbf{P}_z|}{E_z-|\mathbf{P}_z|}\,dz, \tag{20}$$

$$P_z = \tfrac{1}{2}P_1(1+z) + \tfrac{1}{2}P_2(1-z);$$

$$K_0(p_1,\ p_2) = -2w\coth 2w,$$

$$K_0(p_{1M},\ p_{2M}) = -2w_M\coth 2w_M, \tag{21}$$

$$K_0(p_1,\ p_{1M}) = -(\eta + M^2 + 1)\,\mathrm{Re}\,\mu^{(1)},$$

$$K_0(p_1,\ p_{2M}) = (\zeta + M^2 + 1)\,\mu^{(2)}.$$

(P stands for p_1, p_{1M}, . . .).

After adding the elastic and inelastic scattering cross sections, the term $L - L_0 \ln(2\Delta\epsilon/\lambda)$ in the inelastic-scattering cross section [see formula (18)] is replaced by $L - L_0 \ln(2\Delta\epsilon)$.

The overall cross section for elastic and inelastic scattering, is symmetrical in the variables M^2 and m^2, as it should be. When $M = 1$, it is identical with the corresponding expression for the case of equal masses [see the first curly bracket of Eq. (2.16) in the paper by Redhead[6]]. As $M \to \infty$ we obtain Schwinger's formula with $Z = -1$ (the charge of the scattering center coincides with the charge of the incoming particle).[8,9]

3. SCATTERING OF POSITIVE MUONS BY ELECTRONS

Let us consider now the scattering of particles of different sign

$$\mu^+ + e^- \to \mu^+ + e^-, \qquad \mu^- + e^+ \to \mu^- + e^+.$$

The matrix element of this process is obtained from the initial element (1) by making the substitution

$$p_1 \to -p_{2+}, \qquad p_2 \to -p_{1+},$$

$$\xi \to \xi, \qquad \eta \to -\zeta - 2M^2 - 2.$$

The new ξ, η, and ζ are defined as

$$\xi = (p_{2+} - p_{1+})^2 = 4\sinh^2 w = 4M^2\sinh^2 w_M,$$

$$\eta = (p_{1M} - p_{1+})^2, \qquad \zeta = (p_{2M} - p_{1+})^2 = \eta - \xi.$$

Under the aforementioned substitution, $B^{(1)}$ and $B^{(2)}$, are transformed into one another with signs reversed, as they should:

$$\varphi_i^{(1)} \rightleftarrows -\varphi_i^{(2)}, \qquad Q \to Q,$$

$$\mu^{(1)} \leftrightarrows \mu^{(2)}, \qquad H^{(1)} \rightleftarrows H^{(2)} \text{ etc.}$$

The reason for this is that the main term in the matrix element is proportional to ee_M, and the corrections due to diagrams 1 and 2 of Fig. 1 are

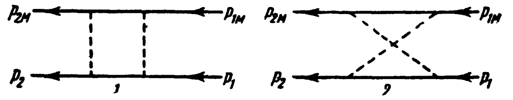

FIG. 1

proportional to $e^2e_M^2$. Their interference with the main term is proportional to $e^3e_M^3$, i.e., it reverses sign when one of the particles is replaced by an antiparticle. This property of the contribution from the two-photon exchange is also conserved when one of the particles is nuclear-active. For example, the total cross section of the processes $\mu^- + p \to \mu^- + p$ and $\mu^+ + p \to \mu^+ + p$ does not contain a contribution from the exchange of two photons (for more details see Appendix 1).

4. CONVERSION OF AN ELECTRON PAIR INTO A MESON PAIR

For the process $e^+ + e^- \to \mu^+ + \mu^-$ we must make the following substitution in the initial matrix element [formula (1)]

$$p_1 \to p_-, \qquad p_2 \to -p_+;$$

$$p_{1M} \to -p_{M+}, \quad p_{2M} \to p_{M-},$$

or, what is the same,

$$\xi \to -(\eta + 4),$$

$$\eta \to \xi - (\eta + 4) = -\zeta - 2M^2 - 2, \qquad \zeta \to \xi,$$

where ξ, η, and ζ are defined as

$\xi = (p_{M-} - p_-)^2, \qquad \eta = (p_+ - p_-)^2 = 4\sinh^2 u,$

$\eta_M = (p_{M+} - p_{M-}) = 4M^2 \sinh^2 u_M,$

$\zeta = (p_{M+} - p_-)^2 = \eta - \xi - 2M^2 + 2,$

$\eta + 4 = \eta_M + 4M^2.$

For the real part of the function A_M we obtain instead of (3) (for more details see the paper by Redhead[6])

$$\operatorname{Re} A_M = \left(\ln\frac{M}{\lambda} - 1\right)(1 - 2u_M \coth 2u_M) - \frac{u_M}{2}\coth u_M + \frac{\pi^2}{2}\coth 2u_M - 2\coth 2u_M \int_0^{u_M} \beta \coth\beta d\beta.$$

The functions with index 1, which depend on η, become functions with index 2, which depend on ζ:

$$\mu^{(1)}(\eta) \to \mu^{(2)}(\zeta), \qquad H^{(1)}(\eta) \to H^{(2)}(\zeta),\ldots$$

Analogously

$$\mu^{(2)}(\zeta) \to \mu^{(2)}(\xi), \qquad H^{(2)}(\zeta) \to H^{(2)}(\xi),\ldots$$

L and L_0 of (18) now become

$$L = K(p_-, p_-) + K(p_+, p_+) + K(p_{M+}, p_{M+}) + K(p_{M-}, p_{M-}) - 2K(p_-, p_+) + 2K(p_-, p_{M+}) - 2K(p_-, p_{M-}) - 2K(p_+, p_{M+}) + 2K(p_+, p_{M-}) - 2K(p_{M+}, p_{M-}),$$

$$L_0 = 4 + 2K_0(p_-, p_+) + 2K_0(p_{M-}, p_{M+}) + 4K_0(p_-, p_{M-}) - 4K_0(p_-, p_{M+}).$$

$K(P_1, P_2)$ is defined in (20), and instead of (21) we have

$K_0(p_-, p_+) = -2u\coth 2u,$

$K_0(p_{M-}, p_{M+}) = -2u_M \coth 2u_M,$

$K_0(p_-, p_{M-}) = (\xi + M^2 + 1)\mu^{(2)}(\xi),$

$K_0(p_-, p_{M+}) = (\zeta + M^2 + 1)\mu^{(2)}(\zeta).$

If we consider the process in the c.m.s., then the substitution $\vartheta \to \pi - \vartheta$ will yield $\xi \rightleftarrows \zeta$ and $B^{(1)} \rightleftarrows -B^{(2)}$ (ϑ is the angle between $\mathbf{p}_-$ and $\mathbf{p}_{M-}$). Thus, the exchange of two photons not only does not change the total cross section for the conversion of the particles, but does not even change the angular distribution of the reaction products, if we disregard the sign of the particle charge. This property of the two-photon contribution is conserved also in a process with nuclear-active particles, for example, for $p + \bar{p} \to \mu^+ + \mu^-$ (see Appendix 1).

5. CASE OF HIGH ENERGIES. NUMERICAL RESULTS

Let us consider now the most interesting particular case, when $\xi \gg 1$ and η, $\zeta \geq -0.8M^2$. We make the following remark concerning $d\sigma_{inel}$. The main contribution to terms of the type $K(P_1, P_2)$ [formula (20)] is made by the integration region near 1 and (or) −1, because the poles of the function $(P_Z^2 - E_Z^2)^{-1}$ are close to each other. If P_1 refers to a light particle and P_2 to a heavy one, only the region near one is significant in (20). If, however, the two momenta P_1 and P_2 refer to the light particle, both regions (near 1 and −1) become significant. If both P_1 and P_2 refer to the particle with mass M, then $K(P_1, P_2)$ is small. Naturally, when $\xi \gg m^2$ both regions are significant in any case, but we do not consider this circumstance since it will be apparently a long time before experiments can be performed on it.

The fact that the poles are close together makes it possible to obtain relatively simple approximate expressions for $K(P_1, P_2)$. By way of an example, let us give the value of $K(p, p_M)$ in the c.m.s.:

$$K(p, p_M) \approx \frac{E_M(1 - \beta_M \cos\vartheta)}{8[E_M - E + 2E\sin^2(\vartheta/2)]} \ln^2 4E^2.$$

Here β_M is the velocity of the particle M and ϑ is the angle between $\mathbf{p}$ and $\mathbf{p}_M$.

The most cumbersome expressions in the formula for the radiative corrections are due to the contribution of the irreducible diagrams [see Fig. 1 and formulas (6) and (7) for $B^{(1)}$ and $B^{(2)}$]. Allowance for the irreducible diagrams, and also for the emission of the soft quanta, does not entail much difficulty.

Let us pay principal attention now to $B^{(1)} + B^{(2)}$ and write out for the given particular case the expression for this sum without terms that depend on λ, assuming that the latter are included in $d\sigma_{inel}$. This inclusion [together with the corresponding terms from A and A_M as given by (3)] eliminates the dependence on λ. The latter reduces to the substitution $\lambda \to 1$ in (18). Denoting by B the quantity

$$\frac{\alpha}{\pi}\frac{B^{(1)} + B^{(2)}}{Q},$$

in which the terms proportional to $\ln\lambda$ are left out, and retaining only the double-logarithmic terms, which make the principal contribution, we obtain for the scattering of a negative muon by an electron

$$B = \frac{\alpha}{\pi} 2\ln\xi \ln\frac{\eta + M^2}{\zeta + M^2} + \operatorname{Re}\frac{\alpha}{\pi}\frac{\xi}{8Q}\{(I^{(1)} + \mu^{(1)}\ln\xi)\varphi_1^{(1)} + (I^{(2)} + \mu^{(2)}\ln\xi)\varphi_1^{(2)} + \bar{G}(\varphi_2^{(1)} + \varphi_2^{(2)}) + G(\varphi_3^{(1)} + \varphi_3^{(2)})\}; \quad (22)$$

$Q = \frac{1}{8}(2\eta^2 + \xi^2 - 2\xi\eta + 4\eta M^2 - 4\xi M^2 + 2M^4),$

$\varphi_1^{(1)} = -\xi\eta - \xi M^2 + 2(\eta^2 + 3\eta M^2 + 2M^4),$

$\varphi_1^{(2)} = -\xi^2 + \xi(3\eta + M^2) - 2\eta(\eta + M^2),$

$\varphi_2^{(1)} + \varphi_2^{(2)} = \xi^2 - 2\xi(\eta + M^2), \qquad \varphi_3^{(1)} + \varphi_3^{(2)} = \frac{3}{2}\xi^2 - \frac{1}{2}\xi(5\eta + M^2) - 4\eta M^2 - 4M^4 + [-\xi^3 + \xi(\xi - 4M^2)(\eta + M^2)]/2(\xi + 4M^2),$

$$\mu^{(1)} = \frac{1}{\eta + M^2}\left\{\ln\frac{\eta + M^2}{M} - i\pi\right\}, \qquad \mu^{(2)} = \frac{1}{\zeta + M^2}\ln\frac{M}{\zeta + M^2}.$$

$I^{(1,2)}$ are determined by (13), (14), and (15), where we can now put

$$(\eta + 2M^2 + 2)\beta_1 \to \eta + M^2,$$

$$u_2 = \frac{1}{2}\ln\frac{(\eta + M^2)^2}{\eta + 2M^2}, \qquad u_1 = \frac{1}{2}\ln\frac{\eta + 2M^2}{M^2}$$

in (13) and

$$|\zeta|\beta_2 \to \zeta + M^2,$$

$$\varphi_2 = \Omega_2 = \frac{1}{2}\ln\frac{(\zeta + M^2)^2}{|\zeta|}, \qquad \varphi_1 = -\Omega_1 = \frac{1}{2}\ln\frac{|\zeta|}{M^2},$$

$$\overline{G} = \frac{1}{2\xi}\left[\ln^2\xi + \frac{4}{3}\pi^2\right]$$

in (14) and (15); G is defined in (17).

All these formulas can be readily obtained from the general equations if we write down (9) for this case in the form

$$x_1 = 1 - 2/(\eta + M^2), \qquad x_2 = -\eta/(\eta + 2M^2);$$

$$y_1 = 1 + 2/(\zeta + M^2), \qquad y_2 = -(\zeta + 2M^2)/\zeta \quad \text{for} \quad \zeta > 0,$$

$$y_1 = -(\zeta + 2M^2)/\zeta, \qquad y_2 = 1 + 2/(\zeta + M^2) \quad \text{for} \quad \zeta < 0.$$

We note, to avoid errors, that the changeover to other processes must be made in the complex matrix element, without first leaving out the imaginary parts in the individual terms.

We give now the numerical results for the c.m.s. The curves of Fig. 2 give the percentage contributions from the irreducible diagrams to the uncorrected cross section [i.e., the ordinates represent the quantity 100 B, see (22)]. For the scattering of particles of different signs, the values of B given on Fig. 2 must be taken with a minus sign. The corresponding corrections for the case of the conversion $e^+ + e^- \longleftrightarrow \mu^+ + \mu^-$ are given in Fig. 3.

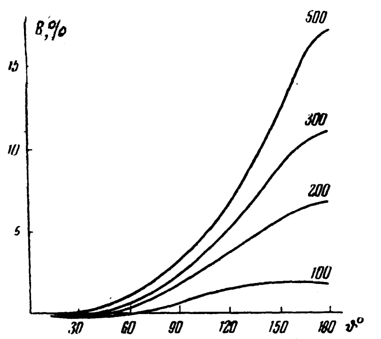

FIG. 2. Percentage of the contribution from the irreducible diagrams to the uncorrected cross section, for the scattering of particles of equal sign. ϑ – scattering angle in the c.m.s. The numbers on the curves indicate the energy of the incoming electron in the c.m.s. in mc^2 units.

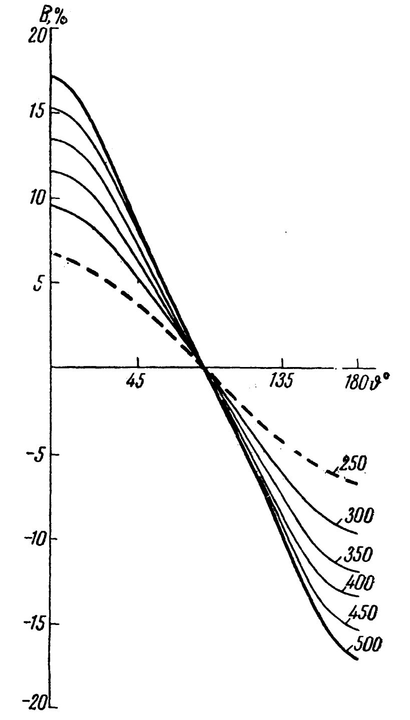

FIG. 3. Percentage contribution from irreducible diagrams to the uncorrected cross section, for the case $e^+ + e^- \leftrightarrow \mu^+ + \mu^-$. ϑ is the angle in the c.m.s. between the incoming electron and the outgoing μ^- meson. The numbers on the curves denote the energy of the incoming electron in mc^2 units.

Finally, Fig. 4 shows the total contribution, in percent, to the uncorrected cross section for the case when the c.m.s. electron energy is 300 mc^2 and the total loss to emission of soft quanta is $\leq 30\ mc^2 = \Delta\epsilon$.

In conclusion, I am grateful to I. L. Rozental' for interest in the work and to Z. S. Maksimova for making the numerical calculations.

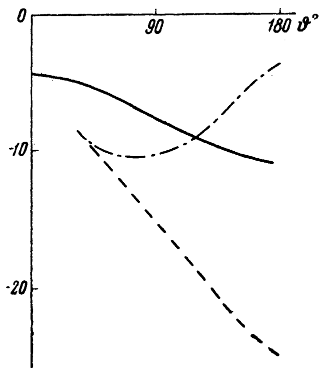

FIG. 4. Total percentage contribution to the uncorrected cross section, for an electron energy $E = 300\ mc^2$ and $\Delta\varepsilon = 30\ mc^2$. Solid curves – corrections for the process $e^- + e^+ \leftrightarrow \mu^+ + \mu^-$; dashed curve – corrections for the process $e^- + \mu^+ \to e^- + \mu^+$; dash-dot curve – corrections for $e^- + \mu^- \to e^- + \mu^-$.

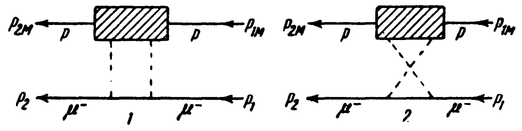

FIG. 5

APPENDIX 1

General Properties of Corrections Due to the Two-Photon Exchange

We denote the contributions to the scattering cross section from the irreducible diagrams 1 and 2 of Fig. 5 by $B^{(1)}$ and $B^{(2)}$. These functions are not independent. For example, the matrix element for the process $\mu^- + p \to \mu^- + p$ can be written in the form

$$|\mathfrak{M}|^2_{\mu-} = |\mathfrak{M}|^2_1 + B^{(1)}(\xi, \eta) + B^{(2)}(\xi, \eta),$$

where $B^{(1)}$ and $B^{(2)}$ corresponds to contributions from diagrams 1 and 2 of Fig. 5, and $|\mathfrak{M}_1|^2$ is the matrix element in the approximation of the one-photon exchange (see reference 10). Analogously, we have for $\mu^+ + p \to \mu^+ + p$ (see Fig. 6):

$$|\mathfrak{M}|^2_{\mu+} = |\mathfrak{M}|^2_1 - B^{(1)}(\xi, \eta) - B^{(2)}(\xi, \eta).$$

On the other hand, $|\mathfrak{M}|^2_{\mu+}$ can be obtained from $|\mathfrak{M}|^2_{\mu-}$ by the substitution

$$\xi \to \xi, \quad \eta \to -\zeta - 2M^2 - 2.$$

The diagrams 1 and 2 of Fig. 5 go in this case into the respective diagrams 2 and 1 of Fig. 6. Hence

$$B^{(1)}(\xi, \eta) = -B^{(2)}(\xi, -\zeta - 2M^2 - 2),$$

$$|\mathfrak{M}|^2_{\text{scat}} = |\mathfrak{M}|^2_{1\,\text{scat}} + B^{(2)}(\xi, \eta) - B^{(2)}(\xi, -\zeta - 2M^2 - 2).$$

For particle conversion we have

$$|\mathfrak{M}|^2_{\text{conv}} = |\mathfrak{M}|^2_{1\,\text{conv}} + B^{(2)}(-\eta - 4, -\zeta - 2M^2 - 2) - B^{(2)}(-\eta - 4, -\xi - 2M^2 - 2).$$

Thus, the contribution from the two-photon exchange is antisymmetrical in the variables ζ and ξ. This means that the charge acquires a tendency of retaining the direction of the initial motion after the particle conversion.

APPENDIX 2

We give here the results of the calculations of the principal integrals encountered in the matrix element for the scattering of muons by electrons. They can be used to calculate the polarization effects in μe interactions.

These integrals have the following form (for notation see reference 6).

$$b^{(1)}_{1,\sigma,\sigma\tau} = \frac{1}{\pi^2 i}\int \frac{(1, K_\sigma, K_\sigma K_\tau)\, d^4K}{(0)\,(q)\,(1)\,(2)},$$

where we put for brevity

$$(0) = K^2 + 2Kp_1, \qquad (q) = (K - q)^2 + \lambda^2,$$
$$(1) = K^2 - 2Kp_{M1}, \qquad (2) = K^2 + \lambda^2.$$

We need also

$$H^{(1)}_{1,\sigma} = \frac{1}{\pi^2 i}\int \frac{(1, K_\sigma)}{(0)\,(1)\,(2)} d^4K, \qquad F^{(1)}_{1,\sigma} = \frac{1}{\pi^2 i}\int \frac{(1, K_\sigma)}{(0)\,(1)\,(q)} d^4K,$$

$$G^{(1)}_{1,\sigma} = \frac{1}{\pi^2 i}\int \frac{(1, K_\sigma)}{(1)\,(q)\,(2)} d^4K, \qquad \overline{G}_{1,\sigma} = \frac{1}{\pi^2 i}\int \frac{(1, K_\sigma)}{(0)\,(q)\,(2)} d^4K.$$

We have written out only the quantities with index 1. All the quantities with index 2 [for example, $b_1^{(2)}{}_{\sigma,\sigma\tau}$] are obtained from the corresponding quantities with index 1 by the substitution $p_{1M} \to -p_{2M}$.

As a result of the calculation we obtain

$$H^{(1)}_1 = F^{(1)}_1 = H^{(1)} = I^{(1)} + \mu^{(1)} \ln \lambda^2,$$

where $I^{(1)}$ and $\mu^{(1)}$ are given by Eqs. (13) and (10) of the text. Furthermore

$$G^{(1)}_1 = G^{(2)}_1 = G$$

[See Eq. (17)].

The quantity G is given in (16), and

$$H^{(1)}_\sigma = K^{(1)} p_{1\sigma} - N^{(1)} p_{1\sigma M}, \quad F^{(1)}_\sigma = K^{(1)} p_{2\sigma} - N^{(1)} p_{2\sigma M} + q_\sigma H^{(1)},$$

where $K^{(1)}$ and $N^{(1)}$ are given in (11) and (12).

Finally

$$G^{(1)}_\sigma = \frac{1}{\xi + 4M^2}\left\{\xi G - 2\ln\frac{\xi}{M^2}\right\} p_{1\sigma M} + \left(2M^2 G + \ln\frac{\xi}{M^2}\right) q_\sigma,$$

$$\overline{G}^{(1)}_\sigma = -\frac{1}{\xi + 4}\{\xi \overline{G} - 2\ln\xi\} p_{1\sigma} + (2\overline{G} + \ln\xi)\, q_\sigma,$$

where $b_1^{(1)} = b^{(1)}$ is given by (8).

The expressions for $b^{(1)}_\sigma$ and particularly for $b^{(1)}_{\sigma\tau}$ are quite cumbersome. We give therefore only their "projections," for they alone are used.

$$2p_{1\sigma} b^{(1)}_\sigma = G - F^{(1)}_1, \qquad 2p_{2\sigma M} b^{(1)}_\sigma = H^{(1)}_1 - \overline{G} - \xi b^{(1)}_1,$$
$$2p_{1\sigma} b^{(1)}_{\sigma\tau} = G^{(1)}_\tau - F^{(1)}_\tau, \qquad 2p_{2\sigma M} b^{(1)}_{\sigma\tau} = H^{(1)}_\tau - \overline{G}_\tau - \xi b^{(1)}_\tau,$$
$$2p_{1\sigma M} b^{(1)}_{\sigma\tau} = F^{(1)}_\tau - \overline{G}_\tau, \qquad 2p_{2\sigma} b^{(1)}_\sigma = G_1 - H^{(1)}_1 + \xi b^{(1)}_1,$$
$$2p_{1\sigma M} b^{(1)}_\sigma = F^{(1)}_1 - \overline{G}, \qquad 2p_{2\sigma} b^{(1)}_{\sigma\tau} = G^{(1)}_\tau - H^{(1)}_\tau + \xi b^{(1)}_\tau.$$

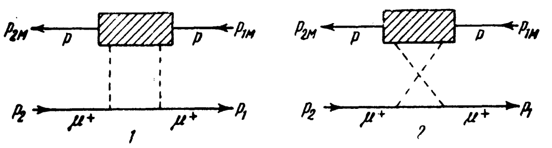

FIG. 6

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Translated by J. G. Adashko
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Errata

Volume	No.	Author	page	col.	line	Reads	Should read
10	5	Bogachev et al.	872	1	21	± 0.3 cm	± 0.7 cm
11	6	Gol'danskii et al.	1229	r	Eq. (13)	$\frac{1}{4\pi^2}\ \frac{h}{Mc}$	$\frac{1}{4\pi^2}\ \frac{h}{Mc}$
			1331	r	4	$\ldots + \frac{1}{4} + \frac{\gamma_a}{2}$	$\ldots + \frac{1}{4}\cos\quad + \frac{\lambda}{2}$
12	2	Moroz and Fedorov	210	1	Eq. (7)	$\ldots \frac{\sin k_0 x_0}{k_0} e^{i\mathbf{kx}} d^3k,$	$\ldots \frac{ik_0 \delta(k^2)}{\lvert k_0 \rvert} e^{ikx}\, d^4k.$
			212	1	Eq. (39)	$\ldots = 4\pi\hbar c \ldots$	$\ldots = -4\pi\hbar c \ldots$
			212-3	r-1	Eqs. (44) and (39)	$\ldots + \frac{1}{2} iel\, \nabla_k \psi_4(x) \ldots$	$\ldots + \frac{1}{2} iel \nabla_k \psi_4''(x) \ldots$
			213	r	Eq. (51), line 2	$\ldots \frac{iel}{2} \int \nabla_m \psi_4(x) \ldots$	$\ldots \frac{iel}{2} \int \nabla_m \psi_4''(x) \ldots$
			213	r	Eq. (53)	$\ldots e^{-ik_0 \lvert x_0 - x_0' \rvert} e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')} \frac{d^3k}{k_0}.$	$\ldots e^{ik(x-x')} \frac{d^4k}{2\pi i (k^2 - i\varepsilon)}$
12	3	Nikishov	530	1	Eq. (10)	—	$\mu^{(2)} = \frac{1}{2\beta_2\zeta} \ln\left[\frac{y_1 - 1}{y_1 + 1} \cdot \frac{-y_2 - 1}{-y_2 + 1}\right]$
			533	r	Fig. 4	The dashed curve of Fig. 4 has been incorrectly calculated (corrections to μ^+ scattering on electrons). Its value ranges from − 6 to − 8.	
12	1	Anisovich	72, 75		Eqs. (4a), (4b), (11)	$\sigma(\pi^+ + p \to n + \pi^+ + \pi^+)$	$2\sigma(\pi^+ + p \to n + \pi^+ + \pi^+)$
	5	"	948		Eq. (6)	$\sigma(\pi^- + p \to n + \pi^0 + \pi^0)$	$2\sigma(\pi^- + p \to n + \pi^0 + \pi^0).$