ON THE PSEUDOVECTOR CURRENT AND LEPTON DECAYS OF BARYONS AND MESONS

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By the use of the analytic properties of a certain matrix element it is shown that the result of Goldberger and Treiman regarding the decay $\pi \rightarrow \mu + \nu$ is valid for wider classes of strong interactions than those found by Feynman, Gell-Mann, and Levy, and in particular for the ordinary pseudoscalar theory with pseudoscalar coupling. A formula is obtained which can be used for an experimental test of the assumptions that are made. Lepton decays of hyperons and K mesons are also discussed.

1. INTRODUCTION

At the present time the theory of the universal V — A interaction given by Feynman and Gell-Mann and by Sudarshan and Marshak is in good agreement with all the experimental data on β decay and the decay of the μ meson.[1] The experimental ratio of the probabilities for the two types of π-meson decay, $R(\pi \rightarrow e + \nu)/R(\pi \rightarrow \mu + \nu)$, also agrees with the theoretical prediction. It may therefore be supposed that the universal V — A theory is also valid for processes of capture of μ mesons in nuclei.

One of the most important problems is the calculation of the probability of the decay $\pi \rightarrow \mu + \nu$ according to the universal V — A theory. This problem has been studied in detail in a paper by Goldberger and Treiman (G. T.)[2] by means of the technique of dispersion theory. Despite the fact that G. T. made many crude approximations, the numerical result of their work agrees almost exactly with the experimental result.

Quite recently Feynman, Gell-Mann, and Levy (F.G.L.) have reexamined this problem in a very interesting paper.[3] They have shown that the G.T. result can be obtained rigorously in certain models. Namely, let us write the Hamiltonian for β decay and μ-meson capture in the form

$$H = (g_0/\sqrt{2})(P_\alpha + V_\alpha) L_\alpha + \text{Herm. adj}. \quad (1)$$

where

$$L_\alpha = \bar{\nu}\gamma_\alpha (1 + \gamma_5) e + \bar{\nu}\gamma_\alpha (1 + \gamma_5) \mu. \quad (2)$$

P_α and V_α are the pseudovector and vector currents for the weak interactions. F.G.L. succeeded in finding three models of the strong interactions in which the following equation holds:

$$\partial_\alpha P_\alpha (x) = ia\pi(x)/\sqrt{2},$$

where a is a constant parameter and $\pi(x)$ is the pion-field operator. By using the equation (3), F.G.L. obtained the G.T. result in a simple and elegant way.

F.G.L. stated that their results would be extended later to any theory of the strong interactions. In their new theory, it is said in reference 3, there appears a form factor $\varphi(s)$, which is very complicated in the usual theory. In the opinion of F.G.L. it is only in the case of their models that it is reasonable to assume that $\varphi(s)$ is slowly varying.

After studying reference 3 we have come to the conclusion that the G.T. result is a good approximation for a wider class of strong interactions. In the present paper this conclusion is examined under the following assumptions:

1. The matrix element $\langle n | \partial_\alpha P_\alpha(0) | p \rangle$ is an analytic function of the variable $s = -(p_n - p_p)^2$.
2. If the matrix element of the commutator for equal times is equal to zero, then we can write a dispersion relation without subtraction.
3. The contribution of the nearest singularities predominates in the dispersion relation.

From our point of view the form factor $\varphi(s)$ actually is slowly varying in any theory in which there is a dispersion relation without subtraction for a certain matrix element.

In Secs. 2 and 3 a derivation of the G.T. result is presented in the most general form. It is shown that the G.T. result is also a good approximation for the ordinary pseudoscalar theory with pseudoscalar coupling. A relation is obtained between the pseudovector constant g_A for μ capture, g_A for β decay, and the pseudoscalar coupling constant f for μ capture. Since we can measure $g_{A\mu}$,

$g_{A\beta}$, and f separately, a test of this relation between the constants gives a sensitive criterion for the correctness of the assumptions made about the universality of the pseudovector coupling in the weak interaction and the analyticity of a certain matrix element.

In Sec. 4 the lepton decays of hyperons and K mesons are treated in an analogous way. From the data on the lifetime of K mesons the result is obtained that the pseudovector coupling constant g_{AY} for the β decay of hyperons is smaller than the coupling constant g_A for the β decay of neutrons.[9]

2. THE RESULT OF GOLDBERGER AND TREIMAN

Let us write

$$i\partial_\alpha P_\alpha(x) \equiv O(x). \tag{4}$$

Applying this equation to the decay $\pi \rightarrow \mu + \nu$, we get

$$\langle 0|O(0)|\pi\rangle = -q_\alpha \langle 0|P_\alpha(0)|\pi\rangle, \tag{5}$$

where q_α is the four-momentum of the pion. The matrix element $\langle 0|P_\alpha(0)|\pi\rangle$ can be expressed in the form

$$\langle 0|P_\alpha(0)|\pi\rangle = -q_\alpha F(m^2)/\sqrt{2q_0}, \tag{6}$$

where m is the mass of the pion and $F(m^2)$ is a constant parameter, which is determined by the lifetime of the pion.

Substituting Eq. (6) in Eq. (5), we get

$$\langle 0|O(0)|\pi\rangle = -m^2F/\sqrt{2q_0}. \tag{7}$$

Let us now turn to the consideration of β decay and μ capture. In the general case the matrix element $\langle n|P_\alpha(0)|p\rangle$ is of the form

$$\langle n|P_\alpha(0)|p\rangle = \bar{u}_n\{g_A\gamma_\alpha\gamma_5 + if(p_p - p_n)_\alpha\gamma_5\}u_p, \tag{8}$$

where g_A and f are invariant functions of $s = -(p_p - p_n)^2$.

Applying the relation (4) to β decay and μ capture, we get

$$\langle n|O(0)|p\rangle = -(p_p - p_n)_\alpha\langle n|P_\alpha(0)|p\rangle. \tag{9}$$

Substituting Eq. (8) in Eq. (9), we have

$$\langle n|O(0)|p\rangle = i[2Mg_A + fs]\bar{u}_n\gamma_5 u_p. \tag{10}$$

The central problem is to find the connection between the matrix elements $\langle n|O(0)|p\rangle$ and $\langle 0|O(0)|\pi\rangle$. This can be done if we use the analyticity properties of the matrix element

$$\langle n|O(0)|p\rangle = i\bar{u}_n\gamma_5 u_p T(s), \tag{11}$$

$$T(s) = -\sqrt{2}GFm^2/(-s+m^2) + T'(s), \tag{12}$$

where G is the renormalized constant of the strong interactions of pions with nucleons, and $T'(s)$ is a function analytic in the region

$$|s| < 9m^2. \tag{13}$$

The derivation of (11) — (14) is given later, in Sec. 3. It is also shown there that $T'(s)$ is in fact a slowly varying function for small s.

In the region $|s| < m^2$ the function $T'(s)$ is approximated with good accuracy by a constant.

Let us rewrite (12) in the form

$$T(s) = -\sqrt{2}GF\varphi(s)m^2/(-s+m^2), \tag{14}$$

where

$$\varphi(s) = 1 + \alpha(s - m^2)/m^2. \tag{15}$$

Comparing (11) with (10), we get

$$2Mg_A + fs = -\sqrt{2}GF\varphi(s)m^2/(-s+m^2). \tag{16}$$

A very important fact is that the relation (16) holds for all s. Setting s = 0, we get

$$F = -2Mg_{A\beta}/\sqrt{2}G\varphi(0), \qquad g_{A\beta} = g_A(0). \tag{17}$$

This is the fundamental result of F.G.L., which was first obtained in the paper of Goldberger and Treiman.[2]

For μ capture

$$s_\mu = -Mm_\mu^3/(M+m_\mu) = -0.9m_\mu^2.$$

From (15), (16), and (17) we get*

$$2Mg_{A\mu} + f_\mu s_\mu = m^2 2Mg_{A\beta}/(-s_\mu + m^2). \tag{18}$$

Equation (18) is an exact relation between g_A, f for μ capture and g_A for β decay, which can be tested experimentally. It must be pointed out that the derivation of (18) has been carried out in the most general way, for an arbitrary value of α. As will be shown in Sec. 3, this holds for almost any theory in which the matrix element $\langle n|\partial_\alpha P_\alpha \times (0)|p\rangle$ is analytic.

Substituting the experimental values of G, $g_{A\beta}$ and F in (17), we get

$$\varphi(0) = 0.8. \tag{19}$$

From this and Eq. (15) we find

$$\alpha = 0.2. \tag{20}$$

We emphasize that the G.T. result is valid only for those theories in which the condition $\alpha \ll 1$ holds. This question is discussed in the following section.

*This relation is contained implicitly in a formula of Goldberger and Treiman.[4]

3. THE ANALYTICITY OF THE MATRIX ELEMENTS

Let us now turn to the calculation of the matrix element $\langle n|O(0)|p\rangle$. Using the standard method,[5] we write

$$\langle n|O(0)|p\rangle = -i\bar{u}_n \int d^4z e^{-ip_n z} \langle 0|T(\eta(z)O(0))|p\rangle$$

$$-u_n^* \int d^4z e^{-ip_n z} \delta(z_0) \langle 0|[\psi_n(z), O(0)]|p\rangle, \quad (21)$$

where $\eta(z) = iS^+\delta S/\delta\bar{\psi}_n(z)$ is the current operator for the neutron field. Hereafter the equal-time commutator will be omitted; it would give an additive constant in the final expression and would not affect the analytic structure of the matrix element, for example, the locations of the poles and their residues, the branch points, and so on.

We note that

$$T(\eta(z)O(0)) = \theta(-z)[O(0), \eta(z)] + \eta(z)O(0),$$

where $\theta(z) = 1$ for $z_0 > 0$ and $\theta(z) = 0$ for $z_0 < 0$. The second term makes no contribution to the matrix element. Thus we have

$$\langle n|O(0)|p\rangle = -i\bar{u}_n \int d^4z e^{-ip_n z} \theta(-z) \langle 0|[O(0), \eta(z)]|p\rangle. \quad (22)$$

In the coordinate system $p_p = 0$ it is easy to show by the method of Bogolyubov[6] that the function $T(s)$ in Eq. (11) has a pole at $s = m^2$ and a cut that begins at the point $s = 9m^2$. At other points it is analytic, if the following inequality holds:

$$|\operatorname{Im} p_{n0}| > |\operatorname{Im} \sqrt{p_{n0}^2 - M^2}| \qquad (p_{n0} = M - s/2M). \quad (23)$$

Unfortunately, the inequality (23) is satisfied only in the case of imaginary nucleon mass. We assume in what follows that the analyticity of the matrix element in the variable s does not change on analytic continuation with respect to the mass variable.

The residue at the pole $s = m^2$ is easily calculated and is equal to $2^{1/2}G\langle 0|O(0)|\pi\rangle(2q_0)^{1/2}$. Thus we have

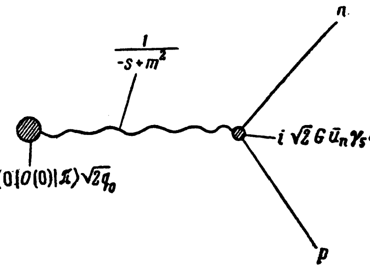
$$T(s) = \frac{\sqrt{2}G}{-s+m^2} \langle 0|O(0)|\pi\rangle \sqrt{2q_0}$$

$$+ T'(s) = -\frac{\sqrt{2}GFm^2}{-s+m^2} + T'(s). \quad (24)$$

The corresponding Feynman diagram for the term with the pole is shown in the drawing.

In Eq. (24), $T'(s)$ is an analytic function with a branch point at $s = 9m^2$. The spectral resolution of the function $T'(s)$ if of the form

$$T'(s) = a_0 + \frac{s}{\pi} \int_{9m^2}^{\infty} \frac{\rho(s')}{s'(s'-s)} ds', \quad (25)$$



where $\rho(s')$ is the spectral function. For small s we can expand $T'(s)$ in a power series in s, which has the radius of convergence $9m^2$.

Setting

$$T'(s) = \sum_n a_n s^n, \quad (26)$$

one can easily show that for large n

$$\lim_{n\to\infty} \left|\frac{a_{n+1}s}{a_n}\right| \leqslant \frac{|s|}{9m^2}. \quad (27)$$

Therefore the series (26) converges rapidly in the region $|s| < m^2$.

If the spectral function does not change sign, the inequality (27) holds also for small n. In this case, for arbitrary $n > 1$ we have

$$|a_{n+1}| = \frac{1}{\pi}\left|\int_{9m^2}^{\infty} \frac{\rho(s')}{s'^{n+2}} ds'\right| \leqslant \frac{1}{9m^2}\frac{1}{\pi}\left|\int_{9m^2}^{\infty} \frac{\rho(s')}{s'^{n+1}} ds'\right| = \frac{|a_n|}{9m^2}.$$

We note that for β decay and μ capture the distance between the points $s = 0$ and $s_\mu = -0.9m_\mu^2$ is much smaller than the radius of convergence $9m^2$. Therefore with good accuracy we can replace $T'(s)$ by a single constant both for β decay and for μ capture (the error is of the order of $0.9m_\mu^2/9m^2 \approx 1/20$).

Thus we get the final result given by the formulas (14) and (15).

Let us now go on to the consideration of the quantity α. If the matrix element of the equal-time commutator is not zero, then in general one must use a dispersion relation with a subtraction. In this case the quantity α is proportional to the subtraction constant a_0, which can be very large.

If, on the other hand, the matrix element of the commutator is zero, than there is a dispersion relation without a subtraction. Then it is reasonable to suppose that the contribution of the nearest singularity predominates and the quantity α is small ($\alpha \approx 0.2 \ll 1$).

Thus it is reasonable to assume that $\varphi(s)$ is slowly varying for any theory in which there is a dispersion relation without subtraction.

Let us consider the ordinary pseudoscalar theory, with the Lagrangian

$$L = -\bar{N}(\hat{\partial} + M_0 - iG_0(\tau\pi)\gamma_5) \times N - m_0^2\pi^2/2 - (\partial_\alpha\pi)^2/2 - \lambda_0\pi^4. \quad (28)$$

By means of the gauge transformation

$$N \to (1 + i(\tau v)\gamma_5)N, \qquad \pi \to \pi + v(4M_0 + 2M)/3G_0$$

we get by the standard method, explained in reference 3,

$$\mathbf{P}_\alpha = \bar{N}\tau\gamma_\alpha\gamma_5 N - i\partial_\alpha\pi(4M_0 + 2M)/3G_0, \quad (29)$$

$$\mathbf{O}(x) = i\partial_\alpha\mathbf{P}_\alpha = 2G_0\bar{N}N\pi + i\tfrac{2}{3}(M_0 - M)\bar{N}\tau\gamma_5 N + (m_0^2\pi + 4\lambda_0\pi^2\pi)(4M_0 + 2M)/3G_0. \quad (30)$$

We shall show in this case that the equal-time commutator for the operator O makes no contribution to the matrix element $\langle n|O(0)|p\rangle$. Let us examine the matrix element of the commutator

$$\mathbf{I} = \langle 0|2G_0N\pi + i\tfrac{2}{3}(M_0 - M)\tau\gamma_5 N|N\rangle.$$

From symmetry properties we have

$$\mathbf{I} = iA\tau\gamma_5 u_N. \quad (31)$$

Multiplying Eq. (31) on the left by the matrix $\tau\gamma_5$, we get

$$i3Au_N = -2i\langle 0|\eta(0)|N\rangle,$$

where

$$\eta(0) = iG_0(\tau\pi)\gamma_5 N + (M - M_0)N$$

is the current of the nucleon field. It is known that the matrix element $\langle 0|\eta(0)|N\rangle$ is equal to zero, and therefore $\mathbf{I} = 0$.

Thus we have shown that in the ordinary pseudoscalar theory there exists the pseudovector current (29), which satisfies all the necessary requirements.

If the pseudovector current is of the ordinary form

$$\mathbf{P}_\alpha = \bar{N}\tau\gamma_\alpha\gamma_5 N,$$

then the matrix element of the commutator is not zero, and in general there is no dispersion relation without subtraction. Even in this case there is hope that the G.T. result is valid. This question will be discussed in the Appendix.

4. LEPTON DECAYS OF HYPERONS AND K MESONS

The experimental limit for the probabilities of lepton decays of Λ and Σ hyperons is an order of magnitude smaller than the theoretical value calculated on the hypothesis that the effective coupling constants in hyperon decays are equal to those in β decays.[7] Many authors have expressed the opinion that the universality of the weak interactions evidently does not extend to strange-particle decays. Nevertheless, it is reasonable to assume the existence of a limited universality [a lepton current in the form (2)[8]].

In what follows we assume that the K meson is pseudoscalar and the V and A interactions exist for the lepton decays of strange particles. In this case the Hamiltonian for the weak decays of strange particles is of the form (1). Following the example given in Sec. 2 for the pseudoscalar theory with pseudoscalar coupling, we can construct the pseudovector current in such a form that a dispersion relation without subtraction holds for the matrix element $\langle N|\partial_\alpha P_\alpha|Y\rangle$.

Generally speaking, the matrix element for hyperon decay consists of three terms:

$$\langle N|P_\alpha(0)|Y\rangle = \bar{u}_N\{g_{AY}\gamma_\alpha\gamma_5 + i\xi_Y[(\hat{p}_N - \hat{p}_Y)\times\gamma_\alpha - \gamma_\alpha(\hat{p}_N - \hat{p}_Y)]\gamma_5 + if_Y(p_Y - p_N)_\alpha\gamma_5\}u_Y, \quad (32)$$

from which we have

$$\langle N|O(0)|Y\rangle = i\langle N|\partial_\alpha P_\alpha|Y\rangle = i[(M_N + M_Y)g_{AY} + f_Y s]\bar{u}_N\gamma_5 u_Y, \quad (33)$$

where $s = -(p_Y - p_N)^2$. Repeating one after another the arguments presented in Secs. 2 and 3, we easily get the following equation:

$$[(M_N + M_Y)g_{AY} + f_Y s] = -G_{KY}F_K m_K^2/(-s + m_K^2) + T_Y(s), \quad (34)$$

where G_{KY} is the renormalized coupling constant for the KYN interaction, and F_K is a constant parameter associated with the decay of K mesons. We have further

$$\langle 0|P_\alpha(0)|K\rangle = -q_\alpha F_K/\sqrt{2q_0}. \quad (35)$$

We can determine F_K from data on the lifetime for the decay $K \to \mu + \nu$. In Eq. (34) $T_Y(s)$ is a function that is analytic in the region

$$|s| < (m_K + 2m)^2. \quad (36)$$

Let us denote by T_N the kinetic energy of the nucleon recoil in the rest system of the hyperon. Expressing s in terms of T_N, we get

$$s = (M_Y - M_N)^2 - 2M_Y T_N. \quad (37)$$

In the present case the values of s that correspond to β and μ decays are very close together, as compared with the distance between the s given by Eq. (37) and $s = (m_K + 2m)^2$. Therefore with good accuracy we can replace $T_Y(s)$ by a constant a_Y.

Thus we have

$$[(M_N + M_Y) g_{AY} + f_Y s] = - G_{KY} F_K m_K^2 / (-s + m_K^2) + a_Y. \quad (38)$$

The relation (38) can be used to test the universality of the pseudovector current in lepton decays of strange particles.

Applying the dispersion theory of Goldberger and Treiman, we find for the function f_Y:

$$f_Y = - G_{KY} F_K / (-s + m_K^2) + T'_Y(s), \quad (39)$$

where $T'_Y(s)$ is a function analytic in the region (36), which with good accuracy can be replaced by a constant a'_Y.

Substituting Eq. (39) in Eq. (38), we get

$$(M_N + M_Y) g_{AY} = - G_{KY} F_K + a_Y - s a'_Y. \quad (40)$$

The relation (40) is a generalization of the formula of Goldberger and Treiman for the decay of strange particles.

The experimental data on the lifetimes of K and π mesons show that $F_K^2 \ll F_\pi^2$. Therefore it can be seen from a comparison of Eqs. (40) and (16) that to accuracy $a_Y - s a'_Y$

$$g_{AY}^2 \leqslant g_{A\beta}^2, \quad (41)$$

even for the case in which G_{KY} and G are of the same order of magnitude. This fact was first pointed out by Sakita.[9]

We emphasize that our method can also be easily extended to the case of a scalar K meson and to other types of weak interactions (for example, S + P).

In the case in which the relative parity of K and YN is positive, we have to deal with the divergence of a vector current

$$i\partial_\alpha V_\alpha = O(x). \quad (42)$$

The matrix element $\langle N | V_\alpha(0) | Y \rangle$ is of the form

$$\langle N | V_\alpha(0) | Y \rangle = \bar{u}_N \{ g_{VY} \gamma_\alpha + i C_Y [(\hat{p}_N - \hat{p}_Y) \gamma_\alpha - \gamma_\alpha (\hat{p}_N - \hat{p}_Y)] + i d_Y (p_Y - p_N)_\alpha \} u_Y. \quad (43)$$

From this we have

$$\langle N | O(0) | Y \rangle = i [(M_N - M_Y) g_{VY} + d_Y s] \bar{u}_N u_Y. \quad (44)$$

It is easy to repeat the remaining arguments, and the final formula will be of the form

$$(M_N - M_Y) g_{VY} + d_Y s = - G_{KY} F_K m_K^2 / (-s + m_K^2) + a_Y. \quad (45)$$

Applying the dispersion theory for the function d_Y, we find

$$d_Y = - G_{KY} F_K / (-s + m_K^2) + a'_Y. \quad (46)$$

Substituting (46) in (45), we get

$$(M_N - M_Y) g_{VY} = - G_{KY} F_K + a_Y - a'_Y s. \quad (47)$$

Comparing (47) and (16), one sees that to accuracy $a_Y - s a'_Y$

$$\left(\frac{g_{VY}}{g_{A\beta}}\right)^2 \approx \left(\frac{2M_N}{M_N - M_Y} \frac{F_K G_{KY}}{F_\pi G_\pi}\right)^2 \approx 5C \left(\frac{G_{KY}}{G_\pi}\right)^2,$$

where C is of the order of unity. Therefore in the case of the scalar K meson the small probability of lepton decay of hyperons could be explained only by having the coupling constant G_{KY} for the KYN interaction be smaller than the pion-nucleon constant G_π.

We note that Λ and Σ can have different relative parities. Let us consider this case. For simplicity we call the K particle a scalar, if the relative parity of K and Λ N is positive, and a pseudoscalar if it is negative. In the case of the pseudoscalar K meson, Eq. (40) holds for the decay of Λ particles, and Eq. (47) holds for the decay of Σ particles, if we write $\langle N | P_\alpha | \Sigma \rangle$ in the form (43). In the case of the scalar K meson, conversely, Eq. (47) holds for the decay of Λ particles and Eq. (40) for Σ particles, if we write $\langle N | V_\alpha | \Sigma \rangle$ in the form (32).

We note that the relations (38) and (45) can be used for the determination of the renormalized coupling constants G_{KY}, if precise experiments are made on the decays of strange particles.

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APPENDIX

In the usual theory the pseudovector current has the form

$$P_\alpha = \bar{N} \tau \gamma_\alpha \gamma_5 N, \quad (A.1)$$

for which the divergence has been calculated in reference 3 and is given by

$$\partial_\alpha P_\alpha = 2M_0 \bar{N} \tau \gamma_5 N - 2iG_0 \bar{N} N \pi. \quad (A.2)$$

In order to calculate the matrix element $\langle 0 | \partial_\alpha P_\alpha | \pi \rangle$, we write Eq. (A.2) in the form

$$\partial_\alpha P_\alpha = - i \frac{2M_0}{G_0} j + i \frac{2M_0}{G_0} [(m_0^2 - m^2) \pi - 4\lambda_0 \pi^2 \pi] - 2iG_0 \bar{N} N \pi, \quad (A.3)$$

where j is the meson-field current. Using the fact that the matrix element $\langle 0 | \mathbf{j}(0) | \pi \rangle$ is equal to zero, we get

$$\langle 0 | \partial_\alpha P_\alpha (0) | \pi \rangle = i2M_0 G_0^{-1} \delta m^2 \sqrt{Z_3} / \sqrt{2q_0} - 8M_0 \lambda_0 G_0^{-1} \langle 0 | \pi^2 \pi | \pi \rangle - 2iG_0 \langle 0 | \bar{N} N \pi | \pi \rangle, \quad (A.4)$$

where Z_3 is the renormalization constant for the pion wave function.

We now go on to the consideration of $\langle n | \partial_\alpha P_\alpha | p \rangle$. We rewrite Eq. (A.4) in the form

$$\partial_\alpha \mathbf{P}_\alpha = -i \frac{4M_0 + 2M}{3G_0} \mathbf{j} + \mathbf{O}(x), \quad (A.5)$$

where the operator O is that of Eq. (30).

In the present case we can write a dispersion relation without subtraction for the matrix element $\langle n | O(0) | p \rangle$, with the term with the pole defined by Eq. (12). Thus we have

$$i \langle n | \partial_\alpha P_\alpha (0) | p \rangle = \tfrac{1}{3}(4M_0 + 2M) G G_0^{-1} d(s) F(s) \sqrt{Z_3} i \bar{u}_n \gamma_5 u_p + \langle n | O(0) | p \rangle, \quad (A.6)$$

where d(s) and F(s) are the respective form factors for the π-meson propagation function and the vertex part. If the first term in Eq. (A.6) is small in comparison with the term with the pole, then the G.T. result would hold also for the usual theory.

We assume that the first term in Eq. (A.5) predominates. Comparing Eqs. (A.5) and (A.6), we get

$$F_7 \approx \frac{2M_0}{G_0} \frac{\delta m^2}{m^2} \sqrt{Z_3}. \quad (A.7)$$

By means of Eq. (A.7) the first term in Eq. (A.6) can be expressed in the form

$$i \frac{2M_0 + M}{3M_0} G \frac{Fm^2}{\delta m^2} \bar{u}_n \gamma_5 u_p d(s) F(s). \quad (A.8)$$

Since in perturbation theory the quantity δm^2 diverges quadratically, it is very probable that

$$\frac{2M_0 + M}{3M_0} \frac{m^2}{\delta m^2} \ll 1. \quad (A.9)$$

In this case the first term in Eq. (A.6) is actually small in comparison with the term that has the pole. Therefore it seems to us that the G.T. result is also valid for the usual theory.

It is interesting to note one more example, in which the pseudovector current has the form

$$i\mathbf{P}_\alpha (x) = \partial_\alpha \pi (x). \quad (A.10)$$

It is easy to show directly from Eq. (A.10) that the matrix element $\langle n | P_\alpha (0) | p \rangle$ for β decay is equal to zero.

In this case the divergence of the pseudovector current is

$$i\partial_\alpha \mathbf{P}_\alpha = m_0^2 \pi - iG_0 \bar{N} \tau \gamma_5 N - 4\lambda_0 \pi^2 \pi = m^2 \pi (x) - \mathbf{j}(x) = \mathbf{O}(x) - \mathbf{j}(x). \quad (A.11)$$

From this we get

$$\langle 0 | O(0) | \pi \rangle = i \langle 0 | \partial_\alpha P_\alpha (0) | \pi \rangle = m^2 \sqrt{Z_3} / \sqrt{2q_0}, \quad (A.12)$$

$$i \langle n | \partial_\alpha P_\alpha | p \rangle = \langle n | O(0) | p \rangle - i \sqrt{2} G d(s) F(s) \sqrt{Z_3} \bar{u}_n \gamma_5 u_p. \quad (A.13)$$

The term with the pole is of the form

$$\sqrt{2} G \sqrt{Z_3} m^2 / (-s + m^2). \quad (A.14)$$

Comparing the expression (A.14) with the second term in Eq. (A.13), we verify that they are of the same order and cancel each other.

From this example it can be seen that for those theories in which a dispersion relation without subtraction does not hold for the matrix element $\langle n | \partial_\alpha P_\alpha (0) | p \rangle$ the G.T. result is in general not a good approximation.

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