ANALYSIS OF NUCLEAR INTERACTIONS OF NUCLEONS WITH $E \geq 10^{11}$ ev *IN PHOTOGRAPHIC EMULSIONS*

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The experimental data on collisions between nucleons and emulsion nuclei are compared with the various theories of multiple production of mesons, in which the tunnel model has been employed.

1. METHODS OF ANALYSIS OF THE INTERACTIONS

IN the following discussion, a comparison is made between the experimental data and the various theories of multiple meson production using the tunnel model. By "tunnel" we understand a cylinder of nuclear matter with the base equal to the geometrical cross section of the nucleon. The idea of such a comparison consists of the following: if we take into account[1-3] the experimental data indicating that, at energies $E \geq 10^{11}$ ev, the cross section for meson production coincides sufficiently with the geometrical cross section of the target nuclei[4-7] and varies very little with the energy of the producing particles, then we can calculate the corresponding tunnel length distribution, from the composition of the nuclei of the detector medium. If we take into account that the matter density inside the nucleus is constant, it is easy to calculate the effective mass of the tunnel n in units of the rest mass of the nucleon. In the present experiment, the Ilford G-5 emulsion served as the shower detector. The distribution of the differential probability $\Delta N/N\Delta n$ of observing a tunnel with mass n for this emulsion is shown in Figs. 3 and 4.

On the other hand, generalizing the theory of multiple production for the case of a collision between a nucleon and a nucleus, one can find the variation of an expected multiplicity n_s with the primary energy and the number of nucleons in the tunnel. If we now consider the showers observed in the emulsion, we can estimate the energy independently of n_s for each individual case, using the theoretical relation between them to calculate the corresponding number of nucleons n in the tunnel for each theory. If a sufficiently large number of showers are available, one can construct the distribution of the relative number of showers with respect to the values of n calculated in such a way, and then compare it with the expected distribution for the emulsion. From such a comparison, one can draw conclusions about the applicability of various theories of multiple meson production to nucleon-nucleus collisions within the framework of the assumed model.

In the present paper, data of other laboratories[8-10] were used* in addition to the showers detected in our laboratory.[1] Only the events with $n_s \geq 5$ shower particles, produced by neutral or singly-charged particles, were selected. Secondary showers observed in the emulsion were not taken into consideration. Since the emulsion was exposed in the majority of cases at a high altitude, one can assume that the showers selected in such a way were primarily produced by nucleons. No limitations on the number of grey and black tracks in the stars were imposed. The Lorenz factor γ_c of the nucleon-tunnel center-of-mass system (c.m.s.t.) was found assuming a symmetrical emission of shower particles and the equality of their velocities in the c.m.s.t. to the velocity of the system itself. The latter leads to a systematical overestimate of γ_c.[1,3,11] However, the possible effects of such an overestimate will be discussed below. We then selected showers with $\gamma_c \geq 7$, which corresponds in the laboratory system (l. s.) to energies of $E \geq 10^{11}$ ev. The total number of such showers was equal to 154.

We shall now discuss the various theories.

a) The hydrodynamical theory of Landau was generalized for the nucleon-nucleus collision in the article of Belen'kiĭ and Milekhin[12] where, in par-

*Unpublished data sent to us from the Moscow and Leningrad laboratories were also used.

ticular, a small infraction of the emission symmetry of produced mesons in the c.m.s.t. is shown to exist. Generalizing the relation between the total number of produced particles N and the energy in nucleon-nucleon (NN) collisions, we can write for the nucleon-nucleus collision

$$N = (n+1)\,\gamma_c^{1/2} \text{ for } n \leqslant 3{,}7, \qquad (1)$$

$$N = 1{,}84\,(n - {}^1/_4)^{3/4}\,\gamma_c^{1/2} \text{ for } n > 3{,}7. \qquad (2)$$

Assuming, furthermore, that only π mesons and nucleons are among the shower particles taking part in the collision, we find

$$n_s = 0{,}67\,\gamma_c^{1/2}(n+1) - (n+1)/6 \text{ for } n \leqslant 3{,}7, \qquad (3)$$

$$n_s = 1{,}23\,\gamma_c^{1/2}(n - {}^1/_4)^{3/4} - (n+1)/6 \text{ for } n > 3{,}7. \qquad (4)$$

b) The energy spectrum of the produced mesons as derived from the Heisenberg theory[13] has been widely confirmed experimentally in showers produced on light and heavy nuclei.[7,8,14–16] It will therefore be useful to consider the application of this theory to nucleon-nucleus collisions within the framework of the assumed model. In order to explain the observed multiplicity in NN collisions, Heisenberg considered the inelasticity factor K, which he connected with the impact parameter of nucleons. In spite of the fact that, as a result, it is possible to explain the observed multiplicity, and that, moreover, the experimental data confirm the character of the variation of the average value of K with the energy of colliding particles,[10,11,17] there is also considerable discrepancy between theory and experimental data, e.g., the absence of a logarithmic increase of the meson-production cross section with the energy.[18,19]

We shall therefore consider an essentially different scheme, in which we postulate the possibility of an inelastic collision of the primary nucleon with a nucleon or a nucleus, without relating it to the impact parameter. The kinematics of such collisions within the framework of the accepted model is as follows (see Fig. 1): before the collision (Fig. 1a), the primary nucleon and the tunnel have equal and opposite momenta in the c.m.s.t. After the collision (Fig. 1b), a strongly excited volume of meson field A is produced in the c.m.s.t., from which the produced mesons are emitted. If the symmetry of emission of these mesons in the c.m.s.t. is conserved, the momenta of the tunnel and of a nucleon after the collision will again be equal. Moreover, the energy ϵ transferred to the meson field is determined as the difference in the total energy of the system nucleon-tunnel before and after the collision. The inelasticity factor K for $\gamma_c^2 \gg 1$ can, in analogy to an NN collision, be determined from the equation*

$$\varepsilon = KM\,(2n\gamma_c - n - 1) \approx 2KMn\gamma_c. \qquad (5)$$

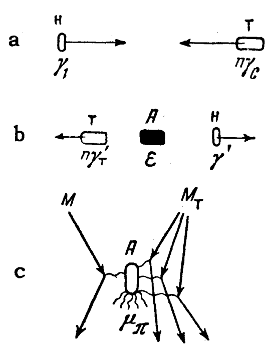

FIG. 1. Scheme of the inelastic nucleon-nucleus collision according to the tunnel model. a – before collision, b – after collision, c – corresponding Feynman diagram.

The scheme under consideration may be represented by the Feynman diagram (Fig. 1c), which describes the whole process as an interaction between a virtual π meson from the cloud of the incident nucleon with virtual π mesons of the nucleons of the tunnel, leading to the production of a single excited volume A. Moreover, the nucleons are not excited, or only weakly excited.[20] It is essential to note that, for large energies ($\gamma_c^2 \gg 1$), consecutive peripheral interactions of the primary nucleon with each of the nucleons of the tunnel ($\pi\pi$ collisions with a production of real mesons), clearly are impossible. This follows from the fact that, in c.m.s.t., the time of existence of a virtual π meson belonging to the cloud of the primary nucleon $\gamma_1 \sim \gamma_c/\mu_\pi$ is much greater than the time $\tau_2 \sim 1/\mu_\pi\gamma_c$ necessary for traversing the distance between the two neighboring nucleons in the nucleus.† The Heisenberg theory is fully applicable to the description of the meson-production mechanism from the excited volume A. An unimportant difference, as compared with the usual theory, consists only in the determination of the maximum energy ϵ_M of produced particles, which, in the given case, is determined from the minimum dimensions of the nucleon-tunnel system.

$$\varepsilon_M \approx \mu_\pi'' n\gamma_c/(1+n^2). \qquad (6)$$

Using the relation (6) and the expression for the energy spectrum of produced mesons of the i-th type,[13] we calculate the number of mesons N_i and the energy ϵ_i carried away by them:‡

*Here and in the following, we used a system of units in which $\hbar = c = 1$.

†The author is thankful to D. S. Chernavskiĭ for a number of useful comments concerning the physical interpretation of the discussed scheme.

‡μ_i is the rest mass of the i-th type of mesons.

$$N_i = A_i \varphi_1(\alpha_i)/\mu_i, \qquad (7)$$

$$\varepsilon_i = A_i \varphi_2(\alpha_i) \qquad (\alpha_i = \mu_i/\varepsilon_M), \qquad (8)$$

$$\varphi_1(\alpha_i) = \frac{2\alpha_i^2+1}{2}\tan^{-1}\frac{\sqrt{1-\alpha_i^2}}{\alpha_i} - \alpha_i\sqrt{1-\alpha_i^2}\tan^{-1}\left(\frac{1-\alpha_i^2}{1+\alpha_i^2}\right)^{1/2} - \frac{\alpha_i}{2}\sqrt{1-\alpha_i^2}, \qquad (9)$$

$$\varphi_2(\alpha_i) = \frac{1}{2}\sqrt{1+\alpha_i^2}\ln\frac{\sqrt{1+\alpha_i^2}+\sqrt{1-\alpha_i^2}}{\sqrt{1+\alpha_i^2}-\sqrt{1-\alpha_i^2}} - \sqrt{1-\alpha_i^2}. \qquad (10)$$

Following Heisenberg,[13] we put $A_i = g_i A$, where g_i is the number of possible charge states of the i-th type of mesons (taking strangeness into account), and A is a constant. In the case of multiple production of π and K mesons, $g_\pi = 3$ and $g_K = 4$. The ratio of the number of $K^\pm$ mesons to the total number of charged K and π mesons can be obtained, assuming charge symmetry, by means of Eqs. (7) and (9):

$$\frac{N_K^\pm}{(N_K^\pm + N_\pi^\pm)} = \frac{3\mu_\pi g_K \varphi_1(\alpha_K)}{3\mu_\pi g_K \varphi_1(\alpha_K) + 4\mu_K g_\pi \varphi_1(\alpha_\pi)}. \qquad (11)$$

In Fig. 2, this ratio is shown as a function of γ_c for NN collisions (curve 1), and for the collision between a nucleon and a tunnel containing seven nucleons (curve 2). The contribution of heavier particles is small and has been neglected. The ratio $N_K^\pm/(N_K^\pm + N_\pi^\pm)$ for high energies ($\gamma_c > 30$) tends towards the value 0.2. This is in satisfactory agreement with experimental estimates[10,16,21] of the fraction of heavy charged particles in showers.

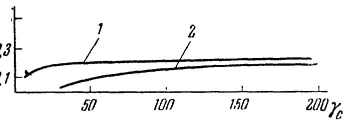

FIG. 2. Ratio between the number of charged K mesons to the total number of charged mesons according to the Heisenberg theory. Curve 1 – for a NN collision, curve 2 – for a collision between a nucleon and a nucleus consisting of seven nucleons.

In order to obtain the variation of multiplicity with the energy, we shall use the equations (5), (8), (9), and (10), from which we shall determine the constant A. Substituting its value into (7), and assuming that $n_s = N_\pi^\pm + N_K^\pm$, we obtain

$$n_s = \frac{KM(2n\gamma_c - n - 1)}{g_\pi\varphi_2(\alpha_\pi) + g_K\varphi_2(\alpha_K)}\left[\frac{2}{3}\frac{g_\pi}{\mu_\pi}\varphi_1(\alpha_\pi) + \frac{1}{2}\frac{g}{\mu_K}\varphi_1(\alpha_K)\right]. \qquad (12)$$

An additional unknown parameter in Eq. (12) is the quantity K, which may be different in each shower. In order to give an estimate of this quantity to a roughest approximation, let us assume that it is constant for all showers with energy higher than 100 Bev. The order of magnitude of the quantity K can be estimated from the condition that the distribution of showers with respect to the tunnel length corresponds to the distribution of the relative differential probability of collision with the tunnel for the detector medium.

The value of n for each shower has been calculated from the hydrodynamical theory [(3) and (4)] and the Heisenberg theory (12), for various K = 0.1, 0.2, 0.3, and 0.5. The histograms of the distribution of the relative number of showers with respect to the values of n calculated in such a way are shown in Figs. 3 and 4.

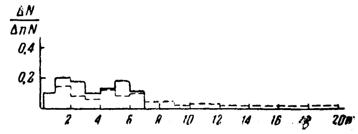

FIG. 3. Distribution of the relative differential density of the number of showers $\Delta N/N\Delta n$ with respect to n according to the Landau theory (dotted line). Density of the probability of distribution of tunnels with respect to n for the Ilford G-5 emulsion is shown by the solid line.

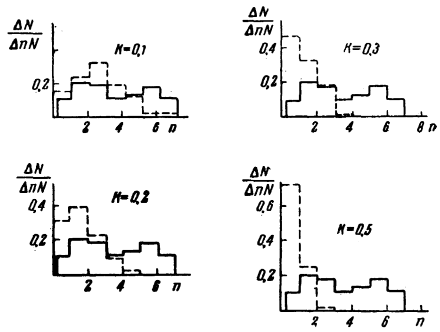

FIG. 4. Distributions of the relative differential density of the number of showers $\Delta N/N\Delta n$ with respect to n (dashed line) obtained from Heisenberg's theory for inelasticity factor K = 0.5, 0.3, 0.2, and 0.1 respectively. The solid line represents the probable distribution of tunnels with respect to n for the Ilford G-5 emulsion.

2. DISCUSSION OF RESULTS

1. The histogram of shower distribution with respect to the tunnel length obtained from the hydrodynamical theory of Landau (Fig. 3) generally follows the distribution expected for emulsions. However, a considerable shift towards greater tunnel lengths is observed. The fraction of showers to which one has to ascribe a fictitious tunnel length $n > 7$ in order to explain the observed multiplicity amounts to 30%. This is also reflected in the ratio of the number of showers N_1 produced only on heavy nuclei ($n > 3.7$) to the number of showers N_2 produced both on heavy and light nuclei (n

≤ 3.7). For the emulsion, this ratio is equal to 0.9, while, according to the hydrodynamical theory of Landau, it equals 1.8. The multiplicity of showers to which, according to the theory, it was necessary to ascribe a fictitious tunnel length ($n > 7$) is great ($n_s > 15$), and the taking into account of the fluctuations in the angular distribution of shower particles will therefore not lead to considerable corrections in γ_c. Since the values of γ_c used[3,11] were systematically overestimated, one can conclude that the disagreement between the theory and the actual conditions cannot be ascribed to experimental errors.*

2. The modified Heisenberg theory leads, within the framework of the accepted model, to a sharp discrepancy between the tunnel-length distribution of the showers and the expected distribution for emulsion for values $K \geq 0.5$ (Fig. 4). For values $K = 0.1$ and 0.2, the ratio N_1/N_2 is equal to 0.3 and 0.1. The value $N_1/N_2 = 0.9$ could have been obtained for $K < 0.1$. However, such a value of inelasticity cannot be accepted, since it lies outside the limits of estimates carried out directly for showers with a known energy distribution of secondary particles.[1,7,11,17] It should, however, be noted that a systematical overestimate of γ_c leads to a worse agreement between the histograms shown in Fig. 4 (γ_c is found[3] to be overestimated by roughly a factor of 1.5). Taking this into account, one can expect an agreement between the present model and the experiment for values of the inelasticity factor substantially smaller than unity ($K \approx 0.1 - 0.2$). The value of K corresponding to the Feynman diagram (Fig. 1c) is approximately equal to $\mu_\pi/M = 0.15$.[20] For the analysis of high-energy showers ($E \geq 10^{11}$ ev) with a known energy distribution of secondary particles and a small number of grey and black tracks ($N_h \leq 3$, i.e., mainly NN interactions), the mean value of the inelasticity factor K was found to equal $0.2 - 0.3$ (references 7, 11, 17), which is not very different from the estimate obtained above. Analogous measurements on showers produced in interactions of nucleons with heavy nuclei ($N_h > 7$) are of great interest.

For two showers with $N_h > 10$ (of type 20 + 12 p and 18 + 11 p), it was found possible in our laboratory to measure the energies of shower particles† (reference 15).

For high energy ($\gamma_c^2 \gg 1$) we can write*

$$Kn \approx \frac{1.5\,\mu_\pi}{2M\gamma_c} \sum_{i=1}^{n_s} E_i'. \tag{13}$$

In columns 1 and 2 of the table, the type of the shower and the corresponding mean value of the transverse momentum of shower particles are given. In the columns 3, 4, and 5, the values of γ_c, Kn and the shower energy in the l.s. E_0 are given. If we assume that, in these showers, a collision of a primary nucleon occurred with a tunnel of at least average length ($n \approx 3.5$), then the value K is found to be substantially smaller than unity ($K \approx 0.1 - 0.2$). For $K \approx 1$, the quantity Kn should amount to several units, which does not correspond to the table. These data are a very tentative indication that the inelasticity coefficient should be considerably less than unity in showers produced in the collision between a nucleon and a heavy nucleus. The final solution of this problem depends on a marked increase in the number of similar showers observed.

1	2	3	4	5
Type of shower	$\bar{p}_\perp/\mu_\pi$	γ_c	Kn	E_0, Bev
$20+12(p)$	0.9	5—6	0.8—0.6	$\gtrsim 100$
$18+11(p)$	1.1	8—14	0.3—0.2	>100

An interesting consequence of the discussed model of multiple meson production in nucleon-nucleus collisions is the possibility of explaining the appearance of a large number of grey and black tracks in high-energy showers. According to the usual hydrodynamical theory, the appearance of such tracks cannot be explained without additional assumptions, since such a large momentum is transferred to the tunnel as a whole (for $K \approx 1$) that it manages to leave the nucleus before disintegrating into separate particles, and the excitation energy equal to the variation of the surface energy of the nucleus is insufficient to explain the observed number of grey and black tracks.

In the case where $K \ll 1$, the nucleons of the tunnel conserve high velocity in their original direction after the collision with a primary nucleon. In the l.s., their velocity β_l relative to the nucleus is small, and they can therefore initiate intranuclear cascades. Such cascades can, in prin-

*Transformation to the system of equal velocities does not lead to a greater similarity between the histograms shown in Fig. 3.[2]

†The energy of all particles was measured in the shower 20 + 12 p. In the shower 18 + 11 p, the energy of three shower particles emitted at the greatest angle could not be measured because of the small length of the track in the emulsion layer. The energy of these particles was found by assuming that their transverse momentum $p_\perp$ is equal to the value $\overline{p_\perp}$ of all remaining particles.

*The energy of shower particles in the c.m.s.t. E_i' is measured in units of rest mass of the π meson.

ciple, be calculated according to the scheme of Goldberger.[22] Since the initial velocity of nucleons of the tunnel is, in the l.s., mainly in the same direction as the primary particle, one should expect the appearance of anisotropy in the angular distribution of recoil nucleons with the maximum in the same direction.

A simple kinematic calculation shows that, if we take the time of flight τ of the primary nucleon (without interaction) through a distance equal to the tunnel length ($\tau \sim n/\mu_\pi$) as the time scale in the l.s., then the fraction of nucleons $\Delta n/n$ in the tunnel remaining during that time inside the nucleus amounts to $\Delta n/n \approx (1 - \beta_l)$. For a high energy ($\gamma_c^2 \gg 1$), $\beta_l \approx K$ and $\Delta n/n \approx (1 - K)$. The mean energy of these nucleons in the l.s. is $\gamma_l \approx M(1 - K^2)^{-1/2}$. For $K \lesssim 0.5$, the kinetic energy of the tunnel nucleons is ~ 1000 Mev, which is wholly sufficient for the ejection of recoil nucleons from the nucleus. If we assume that the mean transverse momentum of the tunnel nucleons $p_\perp \approx M$, we can estimate the opening angle ϑ of the cone in which they are collimated as $\tan\vartheta \approx \sqrt{1 - K^2}/K$. The formulas presented are only estimates. However, it already follows from them that, for $K \approx 1$, the development of the internuclear cascades is impossible, since the fraction of nucleons remaining in the nucleus and also the angle of their collimation are very small. This practically coincides with the ideas of the hydrodynamical theory.

c) The statistical Fermi theory[23] is not applicable in the energy range $E \geq 10^{11}$ ev, since, by means of it, it is impossible to explain the anisotropy in the angular distribution of the produced mesons[24] without using the law of conservation of moment of momentum. In that case, however, the theory becomes self-contradictory even when using it for NN collisions,[25] and all the more so for the tunnel model.

CONCLUSIONS

1. An analysis of different theories of multiple meson productions carried out on the basis of the tunnel model shows that none of the discussed theories in the energy range $E \geq 10^{11}$ ev leads to a good agreement between the tunnel-length distribution of showers and the distribution expected for photographic emulsion.

2. The comparison shows that the hydrodynamical theory of Landau, and the theory of Heisenberg extended for the case of nucleon-nucleus collisions, lead to diametrically opposite effects. Using the first theory, one obtains too great a number of collisions with long tunnels ($N_1/N_2 = 1.8$), while the second theory results in an excess of collisions with short tunnels ($N_1/N_2 = 0.3$) as compared with the number expected for emulsion ($N_1/N_2 = 0.9$). This difference is mainly determined by the value of the inelasticity factor. The hydrodynamical theory is applicable under the assumption that only head-on collisions ($K \sim 1$) occur, while, for the field theory, one has to expect an agreement with the experiment assuming that only peripheral interactions of a primary nucleon with nucleons of the nucleus (leading to the production of a small number of mesons, $K \sim 0.1$) occur. At the present time, the ideas on the multiple meson production in nucleon-nucleus collisions with a small energy transfer to the particles produced ($K \ll 1$) should be regarded as a model whose experimental confirmation requires that many more showers produced on heavy nuclei ($N_h > 10$) be analyzed in detail. Within the framework of this model, it is possible to explain, at least qualitatively, the appearance of a great number of grey and black tracks in corresponding stars.

In conclusion, the author wishes to express his deep gratitude to Prof. Zh. S. Takibaev for proposing the subject, and for his constant attention to the present research.

[1] Zh. S. Takibaev, Тр. ИЯФ АН КазССР Proceedings of the Nuclear Physics Institute, Academy of Sciences Kazakh S.S.R. **1**, 114 (1958).

[2] San'ko, Takibaev, and Shakhova, JETP **35**, 574 (1958), Soviet Phys. JETP **8**, 396 (1959).

[3] É. G. Boos and Zh. S. Takibaev, op. cit. ref. 1 S.S.R. **3**, 46 (1960).

[4] N. A. Dobrotin, Космические лучи (Cosmic Rays) Gostekhizdat, 1954, p. 122.

[5] Grigorov, Murzin, and Rappoport, JETP **36**, 1068 (1959), Soviet Phys. JETP **9**, 759 (1960).

[6] A. A. Alekseeva and N. L. Grigorov, JETP **35**, 599 (1958), Soviet Phys. JETP **8**, 416 (1959).

[7] S. Powell, Proceedings of the Second International Conference on the Peaceful Applications of Atomic Energy, Geneva 1958; Proceedings of the 9th Conference on High-Energy Physics, Kiev 1959.

[8] Fenyves, Gombosi, and Syranyi, Nuovo cimento **11**, 21 (1959).

[9] C.B.A. McCusker and F. C. Roesler, Nuovo cimento **5**, 1141 (1957); Gurevich, Mishakova, Nikol'skiĭ, and Surkova, JETP **34**, 265 (1958), Soviet Phys. JETP **7**, 185 (1958); G. Bertolino, Nuovo cimento **3**, 141 (1956).

[10] Edwards, Losty, Perkins, Pinkau, and Reynolds, Phil Mag. **3**, 237 (1958).

[11] É. G. Boos and Zh. S. Takibaev, Tr. Международной конференции по космическим лучам (Proceedings of the International Conference on Cosmic Rays), Vol I, Moscow 1960.

[12] S. Z. Belen'kiĭ and G. A. Milekhin, JETP **29**, 20 (1955), Soviet Phys. JETP **2**, 14 (1956); S. Z. Belen'kiĭ and L. D. Landau, Usp. Fiz. Nauk **56**, 309 (1955); G. A. Milekhin, JETP **35**, 978 (1958), Soviet Phys. JETP **8**, 682 (1959).

[13] W. Heisenberg, Z. Naturforsch, **133**, 65 (1952); Kosmische Strahlung, Berlin 1953, p. 149.

[14] M. Schein, op. cit. ref. 11, Vol I, Moscow 1960.

[15] Vinitskiĭ, Takibaev, Golyak, and Chasnikov, op. cit. ref. 11, Vol. I, Moscow 1960.

[16] S. Hasegawa, Nuovo cimento **14**, 909 (1959).

[17] W. Fretter and J. Hansen, op. cit. ref. 11, Vol. I, Moscow 1960.

[18] E. L. Feĭnberg and D. S. Chernavskiĭ, Dokl. Akad. Nauk SSSR **91**, 511 (1953).

[19] I. L. Rozental' and D. S. Chernavskiĭ, Usp. Fiz. Nauk **52**, 185 (1954).

[20] D. S. Chernavskiĭ, Postepy Fizyki 9, 653 (1958).

[21] E. Lohrmann and M. W. Teucher, Phys. Rev. **112**, 587 (1958).

[22] M. L. Goldberger, Phys. Rev. **74**, 1269 (1948); Metropolis, Bivins, Strom, Miller, Friedlander, and Turkevich, Phys. Rev. **110**, 185 (1958); Phys. Rev. **110**, 204 (1958).

[23] E. Fermi, Nuclear Processes at High Energies, Russ. Transl. in Usp. Fiz. Nauk **46**, 71 (1952).

[24] Bartke, Ciok, Gierula, Holynski, Miesowicz, and Saniewska, Nuovo cimento **15**, 18 (1960); G. Bozoki and E. Gombosi, Nuclear Phys. **9**, 400 (1959).

[25] E. L. Feĭnberg and D. S. Chernavskiĭ, Dokl. Akad. Nauk SSSR **81**, 795 (1951).

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