PION-PION INTERACTION AND THE ELECTROMAGNETIC STRUCTURE OF THE NUCLEON IN THE STATIC THEORY

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The mean square radius of the pion charge distribution is estimated on the basis of the relationship between this radius and the pion-pion scattering resonance energy derived from the dispersion relations. The mean square radius of nucleon charge is calculated within the framework of the Chew-Low-Wick static theory by taking the electromagnetic structure of the meson into account, and good agreement with experimental data is obtained.

1. INTRODUCTION

THE investigation of the pion-pion interaction aroused great interest in recent years after it became evident that knowledge concerning this interaction is required for further progress in the study of strong interactions.[1] Chew and Mandelstam[2] have studied the pion-pion interaction using a double dispersion representation, and have concluded that the amplitude of pion-pion scattering must exhibit resonance in the state with total angular momentum and isotopic spin both equal to unity. Since the amplitude of pion-pion scattering is still unknown because it is impossible to perform a direct scattering experiment on a system of unstable particles, it is interesting to consider every possible procedure for attempting to determine the basic parameters of this amplitude by analyzing the contributions of the pion-pion interaction to different experimentally observable effects.

Frazer and Fulco[3] have shown how to take into account the contribution of the pion-pion interaction to the isotopic-vector form factor of a nucleon. However, their work is not entirely correct. For example, their solution for the annihilation amplitude of a nucleon-antinucleon pair [Eq. (4)] is wrong, as can be seen by comparing the imaginary part of the solution in Eq. (4) for $s > 4\mu^2c$ with the corresponding imaginary part of the exact solution.[4] Another shortcoming lies in the fact that the expression for the spectral density neglects corrections for secondary pion-nucleon scattering, which plays an essential part.[5]

2. MEAN SQUARE PION CHARGE RADIUS

We shall point out a simple possibility of relating the resonance energy in the pion-pion scattering amplitude to the mean square pion charge radius. We know that the dispersion equation (in subtracted form) for the meson form factor G, after making the reasonable assumption that the largest contribution comes from two-meson intermediate states, has the solution[6]

$$G(s) = \exp\left\{\frac{s}{\pi}\int_{4\mu^2}^{\infty}\frac{\delta(s')\,ds'}{s'(s'-s-i\varepsilon)}\right\}, \tag{1}$$

where $\delta(s)$ is the pion-pion scattering phase in the state with $J = I = 1$ when the total energy in the center-of-mass system is $\sqrt{s}$. (1) leads to a mean square charge radius represented by

$$\langle r^2\rangle_\pi = 6\left.\frac{\partial G(s)}{\partial s}\right|_{s=0} = \frac{6}{\pi}\int_{4\mu^2}^{\infty}\frac{\delta(s')\,ds'}{s'^2}. \tag{2}$$

We know nothing concerning the energy dependence of the phase. Assuming that resonance exists in the state of present interest and that this resonance is quite narrow (i.e., that the scattering amplitude is large only in the resonance region), the behavior of the phase can be approximated by the function

$$\delta(s) = \pi\theta(s - s_0), \tag{3}$$

where s_0 is the square of the resonance energy and $\theta(s)$ is a step function that vanishes for negative values of its argument. We may expect that the total error associated with the approximation (3) will be small even when the resonance is not very narrow. Integration leads to the following relationship between the mean square pion charge radius and the square of the resonance energy:

$$\langle r^2\rangle_\pi s_0 = 6. \tag{4}$$

Using the tentative value $s_0 = 22\mu^2$ for the resonance energy,[7] we obtain

$$(\langle r^2\rangle_\pi)^{1/2} = 0.52\,\mu^{-1} = 0.73\ \text{f}.$$

It is necessary to verify the extent to which this unexpectedly large value of the pion radius is consistent with the experimentally observed nucleon charge distribution.

Riazuddin[8] has calculated the mean square pion charge radius by assuming equality of the experimental and the theoretical mass difference between neutral and charged pions. This work was limited to only a one-meson intermediate state in the expression for the amplitude of a virtual Compton effect for the pion. However, in view of the resonance character of the pion-pion interaction the virtual photoproduction amplitude is not small, so that two-meson intermediate states cannot be neglected. Therefore the result $(\langle r^2\rangle_\pi)^{1/2} = 0.5$ f must be regarded as the lower limit of the pion radius.

3. ELECTROMAGNETIC STRUCTURE OF THE NUCLEON

For the purpose of estimating the role of pion-pion interaction in the electric charge structure of nucleons, the vector part of the mean square nucleon charge radius was calculated in the static theory of Chew, Low, and Wick.[9] We know that with a suitable cutoff constant this theory yields the same basic results as local-field theory with dispersion relations.[5,10]

Although the dispersion-relation method provides a more consistent approximation, and takes, for example, relativistic corrections into account, it requires that we know the pion-nucleon scattering amplitude for non-physical momentum transfers $s_3 > 4\mu^2$. Because of the strong pion-pion interaction, this amplitude is not analytic with respect to s_3 for $s_3 > 4\mu^2$, and cannot be continued outside of the physical region by expansion in Legendre polynomials. The problem of analytic continuation does not arise in the static theory, which from this point of view yields more reliable results than the dispersion relations.

Earlier calculations using both the static and the dispersion theory yield comparatively acceptable results in the approximation where the pion-nucleon scattering amplitude is represented by a one-nucleon term.[10-12] However, this agreement with experiment must be regarded as accidental; it was shown in reference 5 that consideration of secondary pion-nucleon scattering seriously changes the results and leads to a very small proton charge radius that conflicts with the experimental results obtained by Hofstadter's group. The same authors pointed out the necessity of taking into account the pion electromagnetic structure, which they estimated by means of perturbation theory.

In our case the pion-pion resonance interaction is partially accounted for phenomenologically through the introduction of an electromagnetic form factor for the pion. The pion-nucleon scattering amplitude is effectively corrected by means of a suitable cutoff parameter. $v(k) = L^2/(L^2 + k^2)$ was used as the cutoff function; the cutoff parameter $L = 6.5\mu$ was chosen to give the correct value for the vector part of the anomalous nucleon magnetic moment. Secondary scattering terms were subject to the same assumptions as in reference 13, where small phase shifts of the pion-nucleon scattering amplitude were calculated.

A similar calculation was performed for the mean charge of the meson cloud, which determines the degree Q of virtual dissociation of the nucleon into a nucleon and charged pion. We obtained $Q = 23\%$, which results from a semi-phenomenological analysis of experimental data on nucleon structure when a physical nucleon is regarded as a system consisting of a nucleonic core and a virtual pion cloud.

For the vector part of the mean square of the nucleon charge radius we obtained

$$\langle r^2\rangle_V = 0.047\ \mu^{-2} + 0.063\ \mu^{-2} + 0.039\ \mu^{-2} = 0.149\ \mu^{-2};$$

the first term accounts for the charge distribution of the cloud of virtual point pions; the second term, which is $Q\langle r^2\rangle_\pi$, is the contribution of the meson electromagnetic structure. The last term, which is $(3 - 6Q)\,[\partial^2 v(k)/\partial k^2]_{k=0}$, takes into account the charge distribution of the meson core as well as the contributions of virtual high-energy pions and the possible contributions of other heavier particles.

It is impossible at present to calculate the scalar part of the nucleon form factor, since this part results from a photon-nucleon interaction through a three-meson intermediate state. Amplitudes corresponding to diagrams with three external meson lines, knowledge of which is required for such calculations, have thus far not been investigated. We may assume, however, that the vector and scalar parts of the mean square nucleon charge radius are equal, as is indicated by the experimentally observed vanishing of the neutron electric charge. The rms radius of the proton charge then becomes

$$(\langle r^2\rangle_p)^{1/2} = 0.77\ \text{f}$$

in good agreement with experimental data.

4. CONCLUSION

We have thus shown that the hypothesis of a pion-pion resonance interaction makes it possible for the static theory to describe experimental data regarding the mean square nucleon charge radius if resonance occurs for $s_0 = 22\mu^2$.[7] Our results indicate, furthermore, that corrections for secondary pion-nucleon scattering, which had previously been considered small,[10-12,14] actually played an important part, as has been confirmed in reference 5. It should be noted that the point pion cloud, which has thus far been regarded as most important, actually plays only a small part. On the other hand, the electromagnetic structure of the pion is very important. The static theory may be corrected through the use of dispersion relations, but this evidently requires a previous solution of Mandelstam's equations.

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