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THEORY OF ALPHA DECAY OF NONSPHERICAL NUCLEI

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A consistent theory of α decay of nonspherical nuclei is developed which is based on a suitable generalization of the quasiclassical approximation in quantum mechanics. Formulas are obtained for relative intensities of the fine structure lines of α rays, and also for the α-decay constant, and for the angular distribution of α particles emitted by oriented nuclei. A new method of measuring the quadrupole polarization of oriented nuclei is proposed.

1. INTRODUCTION

DURING the last 30 years the nature of the theoretical interest in the phenomenon of α decay has been greatly altered. In the early papers[1] the principal attention was devoted to the fact that the α decay process is a quantum-mechanical tunnel effect. At the present time the aspect of greatest immediate interest appears to be the use of α decay for the measurement of nuclear deformations. Indeed, the use of other methods of measuring deformations, of which the most important one is Coulomb excitation,[2] has so far turned out to be not very effective in the range A > 220, owing to experimental difficulties.

An essential factor in the following discussion is that the complete wave function for the parent nucleus can be represented in the form

$$\Psi = \sum_{ik} \psi_{ik}(\mathbf{r}, \omega)\, \varphi_i^\alpha \varphi_k, \qquad (1.1)$$

where φ_i^α and φ_k are the wave functions for the stationary states of internal motion of the α particle and the daughter nucleus respectively. The expansion coefficients ψ_{ik} depend on the position vector $\mathbf{r}$ of the α particle relative to the center of mass of the daughter nucleus and on the Eulerian angles ω which characterize the orientation of the nonspherical daughter nucleus with respect to fixed axes. Formula (1.1) holds both inside and outside the daughter nucleus, but the qualitative behavior of the coefficients ψ_{ik} in these two regions is quite different.

In the external region due to the effect of the Coulomb barrier the functions $\psi_{ik}(\mathbf{r}, \omega)$ decay rapidly with increasing r. The decrement of this decay depends strongly on the energy of internal excitation in the state $\varphi_i^\alpha \varphi_k$. As a result, the only components $\psi_{ik}(\mathbf{r}, \omega)$ significant in the asymptotic region $r \to \infty$ are those corresponding to relatively small energies of internal excitation. In this case we have to take into account only the ground state $i = 0$ for the α particle, while the index k denotes several of the lowest levels of internal excitation of the nonspherical daughter nucleus. Thus, in fact, only a small fraction of the components of the total wave function shown in the sum (1.1) manifests itself in α decay. The task of the theory consists in finding these few $\psi_{0k}(\mathbf{r}, \omega)$ in the region external to the nucleus.

It should be emphasized that in this region the previously mentioned components $\psi_{0k}\varphi_0^\alpha\varphi_k$ are not strongly interrelated. After the emission of the α particle the internal state of the daughter nucleus φ_k can change only as a result of being affected by its Coulomb field.* An estimate shows that the probability of such a transition is negligibly small. Therefore we can write for each of the $\psi_{0k}(\mathbf{r}, \omega)$ outside the nucleus its own "single-particle" Schrödinger equation [cf. (2.3)]. It turns out to be fairly complicated, since the electrostatic interaction between the daughter nucleus and the α particle is not a central one, and the wave function ψ_{0k} depends, generally speaking, on all the five variables of the system: the three Cartesian coordinates $\mathbf{r}$ of the α particle, and the two angular variables $\nu = \cos\Theta$ and Φ, which we have previously denoted by the single letter ω. Nevertheless, the generalization we obtained for the quasiclassical method enabled us to solve this equation.† As far as we know, the quasiclassical

*We do not consider the case when such a change occurs as a result of the rotation of the daughter nucleus. In other words, it is assumed that in the spectrum of the nuclear energy levels the rotational bands corresponding to different internal states φ_k do not interact with each other.

†Some of the results of the present work based on this generalization of the quasiclassical approximation have been published in preliminary form without a derivation in the form

approximation has not been previously used for the solution of partial differential equations containing so large a number of independent variables.

The interaction between the different configurations $\varphi_i^\alpha \varphi_k$ is extremely great inside the nucleus. This can be written as the condition $l_\alpha \ll R_0$, where l_α is the mean free path of the α particle and R_0 is the nuclear radius. Therefore, in addition to the ψ_{0k} in which we are interested, the sum (1.1) contains many other components ψ_{ik}. The normalization condition has the form

$$\sum_{ik} w_\alpha^{ik} = 1; \qquad w_\alpha^{ik} = \int d\omega \int_V |\psi_{ik}(\mathbf{r}, \omega)|^2 dr, \qquad (1.2)$$

where w_α^{ik} are the corresponding "internal probabilities"; the integration is carried out over the volume of the daughter nucleus. The components ψ_{0k} which actually occur in α decay apparently correspond to only a very small fraction of the total internal probability:

$$w_\alpha^{0k} \ll 1. \qquad (1.3)$$

Since the different components $\psi_{ik}\varphi_i^\alpha\varphi_k$ of the total wave function (1.1) are closely interrelated, there exists no single-particle Schrödinger equation for each individual $\psi_{ik}(\mathbf{r}, \omega)$. However, for those few configurations $\varphi_0^\alpha \varphi_k$ which are of the greatest significance for α decay, the form of the function $\psi_{0k}(\mathbf{r}, \omega)$ inside the nucleus can be established.

First of all, we have

$$\psi_{0k}(\mathbf{r}, \omega) = F(\omega), \qquad (1.4)$$

since as a result of the homogeneity and isotropy of nuclear matter and of the condition $l_\alpha \ll R_0$ there exist no special points singled out within the nucleus. The dependence of ψ_{0k} on the Eulerian angles ω is uniquely determined by the values of the total angular momentum of the parent nucleus Ω and of its projection M on a fixed axis, both of which remain constant in time:

$$\Omega^2 \Psi = \Omega(\Omega+1)\Psi, \quad \Omega_z \Psi = M\Psi. \qquad (1.5)$$

The only function that satisfies (1.4) and (1.5) has the form

$$\psi_{0k}(\mathbf{r}, \omega) = \chi \psi_{rot}^{\Omega M;\, K}(\nu, \Phi),$$

where χ is a constant; (1.6)

$$\psi_{rot}^{\Omega M;\, K}(\nu, \Phi) = \sqrt{(2\Omega+1)/4\pi}\, D_{KM}^{\Omega}(0, \nu, \Phi) \qquad (1.7)$$

is the rotational wave function for the nonspherical nucleus, K is the projection of the angular momentum of the daughter nucleus on its symmetry axis in the state of internal motion under investigation, and $D_{KM}^{\Omega}(\alpha, \cos\beta, \gamma)$ is a generalized spherical harmonic.[5] On the nonspherical surfaces of the daughter nucleus, expression (1.6) plays the role of the boundary condition for the Schrödinger equation satisfied by the function $\psi_{0k}(\mathbf{r}, \omega)$ in the external region.

It is of interest to note that condition (1.4) leads to characteristic selection rules for the α decay of nonspherical nuclei. Indeed, the initial state of the parent nucleus is also characterized by a certain value of the component of the angular momentum along the nuclear symmetry axis K_0; the corresponding rotational wave function has the form $\psi_{rot}^{\Omega M; K_0}(\nu, \Phi)$. In processes such as the formation or breakup of an α particle, which are characterized by large energies and small times, the quantum number K is conserved. On comparing with (1.6) and on taking into account the conservation of parity, we obtain

$$K = K_0, \quad \Pi = \Pi_0, \qquad (1.8)$$

where Π is the parity. Precisely such selection rules were established earlier by analysis of the experimental data;[6] the so-called favored α transitions satisfying these selection rules turn out to be the most intense ones when all other conditions remain the same. The unfavored transitions, for which (1.8) is violated, are considerably less intense and we will not consider them.

2. PROBABILITY OF α DECAY

Let us obtain the wave function $\psi_{0k}(\mathbf{r}, \omega)$ in the region exterior with respect to the daughter nucleus, and let us determine the probabilities of excitation of rotational levels of the daughter nucleus corresponding to its internal state φ_k. In carrying out the calculations we shall utilize two coordinate systems: a fixed system x, y, z; ν, Φ and a system tied to the nucleus ξ, η, ζ; ν, Φ, which is rotated with respect to the fixed system through the Eulerian angles Φ, Θ, 0. The corresponding Hamiltonian has the following form

$$H = -(\hbar^2/2m)\nabla^2 + H_{rot} + U,$$

$$H_{rot} = (\hbar^2/2I)[\mathbf{J}^2 - \Omega(\Omega+1)],$$

where m is the reduced mass; $\mathbf{J}^2 = \mathbf{\Omega}(-1)^2 = \mathbf{\Omega}^2 - 2\mathbf{\Omega}\mathbf{l} + \mathbf{l}^2$ is the square of the angular momentum of the daughter nucleus, $\mathbf{l}$ is the orbital

of separate communications,[3] from which it can be seen that the problem has been completely solved analytically. Later articles have appeared[4] in which it is asserted that high-speed electronic computers are necessary to solve the Schrödinger equation describing the α decay of a nonspherical nucleus and that an analytic solution of the problem is impossible. We can not agree with such assertions.

angular momentum of the α particle, and I is the moment of inertia of the daughter nucleus corresponding to the internal state φ_k.

In the system $\xi, \eta, \zeta; \nu, \Phi$ the shape of the surface of the daughter nucleus is determined by the equation

$$R(\mu) = R_0 + \xi(\mu) = R_0\left\{1 + \sum_n \alpha_n P_n(\mu)\right\}, \qquad (2.1)$$

where $\mu = \cos\vartheta$. The potential energy of the electrostatic interaction U depends only on the variables r and μ; in the approximation linear in α_n we have

$$U(r, \mu) = U_b\left\{\frac{R_0}{r} + \sum_n \frac{3\alpha_n}{2n+1}\left(\frac{R_0}{r}\right)^{n+1} P_n(\mu)\right\}, \qquad (2.2)$$

where $U_b = 2Ze^2/R_0$ is the height of the Coulomb barrier and Ze is the charge of the daughter nucleus.

The form of the operators $\mathbf{\Omega}$ and $\mathbf{l}$ is well known in the system x, y, z; ν, Φ; the transition to the system $\xi, \eta, \zeta; \nu, \Phi$ can also be easily carried out.[7] On taking into account (1.5) we obtain the Schrödinger equation

$$\frac{\partial^2\psi}{\partial r^2} + \frac{2}{r}\frac{\partial\psi}{\partial r} + \left(\frac{m}{I} + \frac{1}{r^2}\right)\left[(1-\mu^2)\frac{\partial^2\psi}{\partial\mu^2} - 2\mu\frac{\partial\psi}{\partial\mu} + \frac{1}{1-\mu^2}\frac{\partial^2\psi}{\partial\varphi^2}\right]$$
$$+ 2\frac{m}{I}\left\{-\sqrt{1-\nu^2}\left(\sqrt{1-\mu^2}\cos\varphi\frac{\partial^2\psi}{\partial\mu\partial\nu} + \frac{\mu}{\sqrt{1-\mu^2}}\sin\varphi\frac{\partial^2\psi}{\partial\varphi\partial\nu}\right) + \frac{iM}{\sqrt{1-\nu^2}}\left(\sqrt{1-\mu^2}\sin\varphi\frac{\partial\psi}{\partial\mu} - \frac{\mu}{\sqrt{1-\mu^2}}\cos\varphi\frac{\partial\psi}{\partial\varphi}\right)\right.$$
$$- \frac{i\nu}{\sqrt{1-\nu^2}}\left[\sqrt{1-\mu^2}\sin\varphi\left(K\frac{\partial\psi}{\partial\mu} - i\frac{\partial^2\psi}{\partial\mu\partial\varphi}\right) - \frac{\mu}{\sqrt{1-\mu^2}}\cos\varphi\left(K\frac{\partial\psi}{\partial\varphi} - i\frac{\partial^2\psi}{\partial\varphi^2}\right)\right]$$
$$\left. - iK\frac{\partial\psi}{\partial\varphi} - \frac{\partial^2\psi}{\partial\varphi^2}\right\} + \frac{2m}{\hbar^2}[E - U(r, \mu)]\psi = 0. \qquad (2.3)$$

Here E is the energy of decay to the rotational level $J = \Omega$; the indices $i = 0$ and k are omitted from the wave function ψ_{0k}.

We obtain a particular solution ψ_{Jl} of equation (2.3), which describes at infinity an α particle of angular momentum l moving away from the nucleus and a daughter nucleus in a rotational state with angular momentum J. Having in mind the application of the quasiclassical approximation, we perform the substitution

$$\psi_{Jl}(r, \mu, \varphi, \nu, \Phi) = \sqrt{m/\hbar}\, Y_{Jl}^{\Omega M}(\nu, \Phi, \mu, \varphi)\, e^{\sigma(r,\mu,\varphi,\nu)} \qquad (2.4)$$

and seek σ in the form of the series $\sigma = \sigma_{-1} + \sigma_0 + \sigma_1 + \sigma_2 + \ldots$ (this corresponds in fact to an expansion in powers of the parameter $1/\kappa R_0 \ll 1$, where κ is the wave number of the α particle on the surface of the nucleus). The angular functions $Y_{Jl}^{\Omega M}$ satisfy the equations

$$\Omega^2 Y_{Jl}^{\Omega M} = \Omega(\Omega+1)Y_{Jl}^{\Omega M}, \quad \Omega_z Y_{Jl}^{\Omega M} = MY_{Jl}^{\Omega M},$$
$$\mathbf{l}^2 Y_{Jl}^{\Omega M} = l(l+1)Y_{Jl}^{\Omega M},$$
$$\Omega\mathbf{l} Y_{Jl}^{\Omega M} = \frac{1}{2}[\Omega(\Omega+1) + l(l+1) - J(J+1)]\, Y_{Jl}^{\Omega M} \qquad (2.5)$$

and the normalization condition

$$\int d\omega \int Y_{Jl}^{\Omega M *} Y_{J'l'}^{\Omega' M'} do = \delta_{\Omega\Omega'}\delta_{MM'}\delta_{JJ'}\delta_{ll'}, \qquad (2.6)$$

where

$$d\omega = d\nu d\Phi, \quad do = d\mu d\varphi.$$

We represent each of the quasiclassical functions in the form of a series in powers of the deformations α_n:

$$\sigma_{-1} = \sigma_{-1}^{(0)} + \sigma_{-1}^{(1)} + \sigma_{-1}^{(2)} + \ldots, \quad \sigma_0 = \sigma_0^{(0)} + \sigma_0^{(1)} + \sigma_0^{(2)} + \ldots,$$
$$\sigma_1 = \sigma_1^{(0)} + \sigma_1^{(1)} + \sigma_1^{(2)} + \ldots$$

etc. On taking (2.2) into account, this yields

$$\left(\frac{d\sigma_{-1}^{(0)}}{dr}\right)^2 = -k_{Jl}^2, \quad 2\frac{d\sigma_{-1}^{(0)}}{dr}\frac{\partial\sigma_{-1}^{(1)}}{\partial r} = \varkappa_b^2\sum_n \frac{3\alpha_n}{2n+1}\left(\frac{R_0}{r}\right)^{n+1} P_n(\mu)\ldots,$$
$$2\frac{d\sigma_{-1}^{(0)}}{dr}\frac{d\sigma_0^{(0)}}{dr} + \frac{d^2\sigma_{-1}^{(0)}}{dr^2} + \frac{2}{r}\frac{d\sigma_{-1}^{(0)}}{dr} = 0\ldots, \qquad (2.7)$$

where

$$k_{Jl} = \sqrt{k_\varepsilon^2 - \varkappa_b^2 R_0/r - l(l+1)/r^2}, \quad k_\varepsilon^2 = 2m\varepsilon/\hbar^2,$$
$$\varepsilon \equiv \varepsilon_J = E - (\hbar^2/2I)[J(J+1) - \Omega(\Omega+1)],$$
$$\varkappa_b^2 = 4mZe^2/\hbar^2 R_0.$$

A solution of (2.7) satisfying the boundary condition at infinity has the form

$$\sigma_{-1}^{(0)} = i\int_{a_{Jl}}^{r} k_{Jl}dr, \quad \sigma_0^{(0)} = -\ln(\sqrt{-ik_{Jl}}\, r),$$
$$\sigma_{-1}^{(1)} = \frac{3}{2}i\varkappa_b^2\sum_n \frac{\alpha_n}{2n+1}P_n(\mu)\int_r^\infty \left(\frac{R_0}{r}\right)^{n+1}\frac{dr}{k_{Jl}}. \qquad (2.8)$$

We do not take into account further terms in the double series for the function σ (r, μ, φ, ν) since calculations show that they are negligibly small. In accordance with (2.4), (2.6) and (2.8) the flux of α particles in the state ψ_{Jl} is normalized to unity:

$$\int d\omega \int_{r\to\infty} \frac{\hbar k_\varepsilon}{m}|\psi_{Jl}(r, \mu, \varphi, \nu, \Phi)|^2 r^2 do = 1. \qquad (2.9)$$

At the turning point $r = a_{Jl}$ the wave number k_{Jl} vanishes. We can use Eqs. (2.8) in the region

$r < a_{Jl}$, if we make the substitution $k_{Jl} \rightarrow i\kappa_{Jl}$. We represent the true wave function ψ_{0k} in the form of the superposition:

$$\psi = \sum_{Jl} b_{Jl}\psi_{Jl}. \tag{2.10}$$

At the nonspherical surface S of the daughter nucleus, i.e., for $r = R(\mu)$, we have in the required approximation

$$\sigma_{-1}^{(0)}(S) = \sigma_{-1}^{(0)}(R_0)\big|_{\substack{l=0\\ \Delta E=0}} - \varkappa_{\Omega_0}(R_0)\,\xi(\mu) + \frac{1}{2}\gamma_E \Delta E + \frac{1}{2}\gamma_l l(l+1),$$

$$\sigma_{-1}^{(1)}(S) = \sigma_{-1}^{(1)}(R_0)\big|_{\substack{l=0\\ \Delta E=0}}, \quad \sigma_0^{(0)}(S) = \sigma_0^{(0)}(R_0)\big|_{\substack{l=0\\ \Delta E=0}}. \tag{2.11}$$

Here we have carried out an expansion in terms of the dimensionless small parameters α_n, $l(l+1)/\kappa^2 R_0^2$ and $\Delta E/E$, where $\Delta E \equiv \Delta E_J = E - \epsilon_J = (\hbar^2/2I) \times [J(J+1) - \Omega(\Omega+1)]$ is the energy of excitation of the daughter nucleus measured from the level $J = \Omega$.

It can be easily shown that the expansion coefficients are given by

$$\gamma_E = \frac{1}{E}\left(\frac{\varkappa_b^2 R_0}{k}\tan^{-1}\frac{\varkappa}{k} + \varkappa R_0\right), \quad \gamma_l = \frac{2\varkappa}{\varkappa_b^2 R_0}, \tag{2.12}$$

where $k \equiv k_E = \sqrt{2mE}/\hbar$, $\kappa \equiv \kappa_{\Omega_0}(R_0) = \sqrt{\kappa_b^2 - k^2}$. We substitute (2.11) into (2.4), (2.10), and the boundary condition (1.6). Taking (2.8), (2.1), and (2.6) into account we obtain, after straightforward transformations,

$$b_{Jl} = \sqrt{\frac{\hbar\varkappa R_0^2 \chi^2}{m}}\,\Gamma \exp\left(-\frac{\gamma_E}{2}\Delta E - \frac{\gamma_l}{2} l(l+1)\right) \times \int d\omega \psi_{rot}^{\Omega M;K} \int Y_{Jl}^{\Omega M*} \exp\left\{\sum_n \beta_n P_n(\mu)\right\} do, \tag{2.13}$$

$$\beta_n = \left\{\varkappa R_0 - \frac{3\varkappa_b^2}{2(2n+1)}\left[\int_{R_0}^{a_{\Omega 0}} \left(\frac{R_0}{r}\right)^{n+1} \frac{dr}{\varkappa_{\Omega 0}} + i\int_{a_{\Omega 0}}^{\infty} \left(\frac{R_0}{r}\right)^{n+1} \frac{dr}{k_{\Omega 0}}\right]\right\}\alpha_n, \tag{2.14}$$

$$\Gamma = \exp\left\{-2\left(\frac{\varkappa_b^2 R_0}{k}\tan^{-1}\frac{\varkappa}{k} - \varkappa R_0\right)\right\}. \tag{2.15}$$

Here Γ is the penetrability factor for the Coulomb barrier in the case of a spherical nucleus of radius R_0.

In the laboratory system x, y, z; ν, Φ we denote the angular function Y by $\tilde{Y}_{Jl}^{\Omega M}(\nu, \Phi, \mu, \varphi)$. Then evidently we have

$$\tilde{Y}_{Jl}^{\Omega M} = \sum_{M'm'} C_{JM'lm'}^{\Omega M} \psi_{rot}^{JM',K}(\nu, \Phi)\, Y_{lm'}(\mu, \varphi), \tag{2.16}$$

where $C_{..}^{..}$ are the Clebsch-Gordan coefficients and Y_{lm} are the normalized spherical harmonics.[5]

The transition to the rotating system ξ, η, ζ; ν, Φ is accomplished with the aid of the generalized spherical harmonics D^l. On taking (1.7) into account we easily obtain

$$Y_{Jl}^{\Omega M} = \sqrt{\frac{2J+1}{2\Omega+1}} \sum_m C_{JKlm}^{\Omega, K+m} \psi_{rot}^{\Omega M; K+m}(\nu, \Phi)\, Y_{lm}(\mu, \varphi). \tag{2.17}$$

We substitute (2.17) into (2.13) and integrate over ω, and also over the angle φ. In accordance with (1.2) and (1.6), the constant χ is related by the equation $w_\alpha = \frac{4}{3}\pi R_0^3 \chi^2$ to the internal probability of formation of an α particle in the nucleus w_α. As can be seen from (2.10), in the case of the normalization (2.9) the probability per unit time of decay to the rotational level of angular momentum J accompanied by the emission of an α particle of angular momentum l, is equal to $W'_{Jl} = |b_{Jl}|^2$. As a result, after interchanging the indices of the Clebsch-Gordan coefficients, we finally obtain*

$$b_{Jl} = \sqrt{\frac{3\hbar\varkappa w_\alpha}{mR_0}\Gamma}\,(-1)^{J-\Omega} C_{\Omega K l0}^{JK} \sqrt{B_l}\, e^{-\gamma_E \Delta E/2}\, \tilde{X}_l,$$

$$W_{Jl} = \frac{3\hbar\varkappa w_\alpha}{mR_0}\Gamma\,(C_{\Omega K l0}^{JK})^2 B_l\, e^{-\gamma_E \Delta E}\, |\tilde{X}_l|^2, \tag{2.18}$$

where

$$B_l = (2l+1)\, e^{-\gamma_l l(l+1)}, \tag{2.19}$$

$$\tilde{X}_l = \int_0^1 \exp\left\{\sum_n \beta_n P_n(\mu)\right\} P_l(\mu)\, d\mu. \tag{2.20}$$

Here we have utilized the fact that the shape of the daughter nucleus possesses mirror symmetry: no odd values of n appear in the right hand side of (2.1).

With the aid of (2.18) we obtain the useful relation

$$W_{J'l}/W_{Jl} \equiv w_{J'l}/w_{Jl} = (C_{\Omega K l0}^{J'K}/C_{\Omega K l0}^{JK})^2 \exp(-\gamma_E \Delta E_{J'J}), \tag{2.21}$$

where $\Delta E_{J'J} = (\hbar^2/2I)[J'(J'+1) - J(J+1)]$ is the separation between the rotational levels of angular momenta J′ and J.

The total probability of excitation of a rotational level with angular momentum J is obtained by summing the second of formulas (2.18) with respect to l. We normalize the relative probability w_J by the condition $w_\Omega = 1$. As a result of this we obtain

*Independently of the present author and practically simultaneously with him, Fröman[8] has obtained a formula similar to (2.18), based on semi-intuitive considerations.

$$w_J = \exp(-\gamma_E \Delta E) \sum_{l=|J-\Omega|}^{J+\Omega}{}' B_l (C^{JK}_{\Omega K l0})^2 \times |\widetilde{X}_l|^2 \Big/ \sum_{l=0}^{2\Omega}{}' B_l (C^{\Omega K}_{\Omega K l0})^2 |\widetilde{X}_l|^2, \quad (2.22)$$

where the prime on the summation sign indicates that the summation is taken only over even values of l.

For many purposes it is sufficient to consider only the quadrupole deformation $\xi = R_0\alpha_2 P_2(\mu)$ which exceeds considerably the other terms in the sum (2.1). In this case the substitution $\widetilde{X}_l \to X_l$ should be performed, where

$$X_l(\beta) = \int_0^1 e^{\beta P_2(\mu)} P_l(\mu)\, d\mu,$$

$$\beta \equiv \beta_2 = \left[\frac{4}{5}\varkappa R_0 (1 - k^2/2\varkappa_b^2) - i2k^3 R_0/5\varkappa_b^2\right]\alpha_2. \quad (2.23)$$

On summing the second of relations (2.18) over all J and l, we obtain the α-decay constant.[9] In the special case of even-even nuclei we have $\Omega = K = 0$; substitution into (2.22) and (2.18) leads to further simplifications.[3,9]

3. ANGULAR DISTRIBUTION OF α PARTICLES

The angular distribution of the α particles emitted by oriented nuclei depends on the quantum number M. In accordance with (2.10), (2.4), (2.8), and (2.16), we have in the laboratory system x, y, z; ν, Φ as $r \to \infty$

$$\psi_M = \sqrt{\frac{m}{\hbar}} \sum_{JlM'm'} b_{Jl} \exp(\sigma_{-1}^{(0)} + \sigma_0^{(0)}) \times C^{\Omega M}_{JM'lm'} \psi_{rot}^{JM';K}(\nu, \Phi)\, Y_{lm'}(\mu, \varphi). \quad (3.1)$$

In the general case we have $\psi = \sum c_M \psi_M$. The spatial density $\int |\psi|^2 d\omega$ of the α particles breaks up into a sum of independent components corresponding to different values of J. To go over to the current density j we must multiply each of these components by the corresponding value of the velocity $\hbar k_\epsilon/m$. The expression for the differential intensity has the form $jr^2 do$. The final averaging over the spin orientations of the parent nuclei in the initial state is achieved by the replacement $c_{M'} c^*_{M''} \to \rho_{M'M''}$, where ρ is the spin density matrix of the parent nuclei. After some straightforward transformations we obtain

$$dW = \sqrt{4\pi(2\Omega+1)} \sum_{M'M''} \rho_{M'M''} \sum_L C^{\Omega M'}_{\Omega M'' L, M'-M''} Y_{L, M'-M''}(\mu, \varphi) \times \sum_{Jl'l''} (-1)^{J-\Omega} \sqrt{(2l'+1)(2l''+1)}\, b^*_{Jl''} b_{Jl'} \times \exp\{ig[l''(l''+1) - l'(l'+1)]\} C^{L0}_{l'0l''0} W(\Omega l' \Omega l''; JL) \frac{do}{4\pi}, \quad (3.2)$$

where $g = k/\kappa_b^2 R_0$, W(abcd; ef) are the Racah coefficients.

At not too low temperatures, when the nuclear orientation is not very pronounced, the main contribution to the anisotropy in the angular distribution is made by the term $L = 2$. If the orienting field possesses some symmetry axis, then the angular distribution becomes axially symmetric.* We restrict ourselves henceforth to the case $\Omega = K \neq 1/2$. Then from (2.18) it follows† that $b_{Jl} = (-1)^{J-K}\sqrt{W_{Jl}}$. In fact, the great majority of the emitted α particles is due to transitions to the levels $J = K$ and $J = K + 1$, with $l = 0$ and 2, while the admixture of other values of the orbital angular momentum is very small. On taking (2.21) into account and on normalizing the total probability of α decay to unity, we easily obtain

$$dw = \{1 + A_2 P_2(\mu)\} \frac{do}{4\pi},$$

$$A_2 = \sqrt{\frac{K(2K-1)}{(K+1)(2K+3)}} \frac{f_2}{1 + w_{K+1}} \times \left\{ 2\sqrt{5}\left[\frac{(K+2)(2K-1)}{3(2K+3)\exp(-\gamma_E \Delta E)} w_{K+1} \times \left[1 - \frac{(K+2)(2K-1)}{3(2K+3)\exp(-\gamma_E\Delta E)} w_{K+1}\right]\right]^{1/2} + \frac{5}{7} w_{K+1} \frac{\sqrt{2K-1}}{\sqrt{K(K+1)(2K+3)}} \times \left[(K+6) + \frac{(K+2)(2K-3)(2K+5)}{3(2K+3)\exp(-\gamma_E\Delta E)}\right]\right\}, \quad (3.3)$$

where ΔE is the separation between the levels $J = K + 1$ and $J = K$; f_2 is the quadrupole polarization of the parent nuclei.[10] Relations (3.3) can be utilized to measure the quadrupole polarization of nonspherical α-active nuclei by means of the angular distribution of the α particles emitted by them.

Thus, the concepts of the collective model,[11] taken together with the assumption that the mean free path of the α particle in the nucleus is small, enable us to present a complete theory of the α decay of nonspherical nuclei.

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*For this it is sufficient that the field should have a symmetry axis of order >2.

†Indeed, even though according to (2.23) the quantities β and X_l are complex, their imaginary part is actually negligibly small (in an analogous manner, the imaginary exponent of the exponential $\exp\{ig[l''(l''+1) - l'(l'+1)]\}$ is small for these values of the orbital angular momentum which play an important role, and we shall everywhere replace it by unity). Comparison with experiment shows that $\alpha_2 > 0$. Therefore we have $\beta, X_l > 0$.

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