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POSSIBILITY OF APPLICATION OF THE (n, 2n) REACTION IN NUCLEAR SPECTROSCOPY

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The (n, 2n) reaction is treated as a stripping reaction. The angular distribution of the center of mass of two neutrons simultaneously emitted as a result of direct interaction is computed. It is found that it is important to take into account the interaction between the emitted neutrons in the final state. As an example, the angular distributions are calculated for the reactions $Be^9(n, 2n)Be^8$ and $Pb^{208}(n, 2n)Pb^{207}$ corresponding to the 2.9- and 1.63-Mev levels of the final nuclei. It is shown that it should be possible to elucidate the mechanism of the (n, 2n) reaction and determine the characteristics of the nuclear energy levels by investigating the angular dependences in the spectra of neutrons emitted from targets irradiated with neutrons.

1. INTRODUCTION

EVER since Butler[1] showed the possibility of using reactions of the stripping and pick-up type to obtain data on the spectroscopy of the nucleus, there have appeared a large number of papers containing valuable data on the characteristics of nuclear states. The angular distributions of these types of reactions differ in their sharply pronounced structure, which allows one to obtain information on the relation between the spins and parities of the initial and final states of the nuclei that take part in the reaction. Data which are identical from the viewpoint of different formalisms have been obtained for the angular distributions of the products of these reactions.[2] Thus, the stripping and pick-up reactions have occupied an important place in the methods of nuclear spectroscopy.

At the same time, it has been shown that the Coulomb scattering of the products of these reactions lead to appreciable disparities with the results based on theories that do not take into account the effect of the charge of the particles. The Coulomb scattering of the deuteron and proton waves in the (p, d) and (d, p) reactions lead, in the case of light and medium nuclei, to a shift of the angular distribution maximum towards the larger angles, to a broadening and lowering of these maxima, and, in the case of heavy nuclei, to a complete distortion of the shape of Butler's angular distributions. These effects are especially strong at comparatively low incident particle energies.

In the present work, the (n, 2n) reactions are considered as reactions of the pick-up type. It has been shown previously that the energy distribution of the products of reactions in which several particles are emitted, among which are two neutrons, has a narrow peak, which is caused by the interaction of these neutrons in the final singlet state.[3-5] If the strong interaction of neutrons of small relative energy is taken into account, one can speak of the possibility of a pick-up mechanism, i.e., the simultaneous emission, almost in the same direction, of the two interacting neutrons as a result of a direct interaction between the incident neutron and the nucleus. An analogy with the (n, d) and (p, d) reactions occurs in the case of the (n, 2n) reaction, also because the direct interaction in reactions in which two nucleons are emitted is essentially of a surface character.[6]

It is possible that in the distribution of the total momentum of the two emitted neutrons there appear peaks corresponding to the energy levels of the nucleus $A-1$ in the reaction $A(n, 2n)A-1$. As shown later on, for specific examples, the dependence of the area of these peaks on the direction of motion of the center of mass of the two neutrons has the same character both in the stripping and in the pick-up reactions (see also reference 7).

Thus, taking the (n, 2n) reactions, instead of the (n, d) and (p, d) reactions, one may expect to obtain the characteristics of the energy levels of medium and heavy nuclei from the shape of the angular distributions, since the Coulomb effects in the ordinary pick-up reactions have a large value for such nuclei. Neudachin drew the attention of the authors to this fact.

2. CALCULATION OF THE ANGULAR DISTRIBUTIONS

The effective differential cross section of the (n, 2n) reaction, under the assumption of the simultaneous emission of both neutrons with relative energies in the interval between E_{nn} and $E_{nn} + dE_{nn}$ as the result of the direct interaction calculated in the Born approximation, has the same form as in ordinary stripping theory:[8]

$$\frac{d\sigma}{dE_{nn}d\Omega_1 d\Omega_2} = \frac{\mu_{n0}M_n\mu_{2n}}{16\pi^2\hbar^2\mu_n^2}\frac{k_{2n}k_{nn}}{k_{n0}}[B(Q)]^2$$
$$\times V_0^2A^2\left(|E_b| + \frac{\hbar^2}{2\mu_n}z^2\right)^{-2} z^{-2}[j'(za) - g_l(\varkappa a)j(za)]^2\frac{3\theta^2}{a}, \qquad (1)$$

where

$$\mu_{n0} = M_nM_i/(M_i+M_n), \qquad \mu_n = M_nM_f/(M_f+M_n),$$
$$\mu_{2n} = 2M_nM_f/(M_f+2M_n), \qquad \frac{\hbar^2k_{nn}^2}{M_n} = E - \frac{\hbar^2k_{2n}^2}{2\mu_{2n}},$$
$$\mathbf{z} = \mathbf{k}_{2n} - \left(\frac{M_f}{M_f+M_n}\right)\mathbf{k}_{n0}, \qquad \mathbf{Q} = \frac{\mathbf{k}_{2n}}{2} - \mathbf{k}_{n0};$$

M_i and M_f are the masses of the initial and final nuclei; $\mathbf{k}_{n0}$, $\mathbf{k}_{2n}$, and $\mathbf{k}_{nn}$ are the wave vectors of the incident neutron, of the center of mass of the two emitted neutrons, and of their relative motion; E and E_b are the energy in the c.m. system after collision and the binding energy of a neutron in the initial nucleus; $d\Omega_1$ and $d\Omega_2$ are elements of the solid angle for the vectors $\mathbf{k}_{2n}$ and $\mathbf{k}_{nn}$; θ^2 is a dimensionless normalized width; l is the orbital angular momentum of the picked-up neutron; a is the radius of interaction;

$$g_l(\varkappa a) = \frac{1}{rh_l^{(1)}(i\varkappa r)}\frac{d}{dr}(rh_l^{(1)}(i\varkappa r))\Big|_{r=a}$$

is the logarithmic derivative at the boundary of the nucleus expressed in terms of spherical Hankel functions of the first kind; $\kappa = 2\mu_n|E_b|)^{1/2}/\hbar$; the function $j_l(za)$ is the same as in reference 9. The interaction in the final state of the two neutrons emitted as a result of the reaction is included in the matrix element in the integral

$$\int V_{nn}e^{i\mathbf{Q}\rho}\varphi_{2n}(\rho)\,d\rho.$$

Here ρ is the vector between the neutrons and the final state, V_{nn} is the interaction potential of the neutrons in the final state expressed in the form of a potential well of depth V_0 and radius $\rho_0 = 2.65$ fermi. The radial part of the wave function of the two neutrons has the form[7]

$$\varphi_{2n}^{(1)} = A\sin k\rho/\rho \qquad (\rho < \rho_0),$$
$$\varphi_{2n}^{(2)} = \frac{i\sqrt{\pi}}{k_{nn}\rho}[e^{-ik_{nn}\rho} - e^{ik_{nn}\rho}] + \frac{\sqrt{\pi}}{\alpha_s - ik_{nn}}\frac{e^{ik_{nn}\rho}}{\rho} \quad (\rho > \rho_0). \qquad (2)$$

The coefficient A and the modulus of the wave vector k are found by matching the logarithmic derivatives of the functions $\varphi_{2n}^{(1)}$ and $\varphi_{2n}^{(2)}$ at $\rho = \rho_0$. These values depend only on the choice of the form of the interaction between the neutrons, the scattering length, and the magnitude of the relative energy of the two neutrons; α_s is the generalized scattering length;

$$1/\alpha_s = 1/\alpha_{s0} - k_{nn}^2\rho_0/2, \qquad \alpha_{s0} = \sqrt{\mu_n\varepsilon}/\hbar;$$

$\epsilon = 70$ kev is the interaction energy of the neutrons.

In formula (1) the integral given above takes the form

$$B = \int_0^{\rho_0}\frac{\sin Q\rho}{Q}\sin k\rho\,d\rho.$$

The energy distribution described by expression (1) has a narrow maximum caused by the interaction of the two neutrons in the final singlet state in the region $E_{nn} = 0$ to 160 kev. The magnitude of k weakly depends on the relative energy of the two neutrons and is equal to 11.04×10^{12} cm^{-1}.

We list the values of A^2 for different values of relative energy of the two neutrons E_{nn}:

E_{nn}, Mev	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16
$A^2\times10^{22}$, cm²	298	237	159	139	114	97	84	60

The angular distribution relative to the motion of the center of mass of the two interacting neutrons for the angle ϑ in the c.m. system of the two neutrons and the final nucleus is obtained by integrating the energy distribution over the narrow region E_{nn} from 0 to 160 kev for each ϑ. By comparing the absolute values of the cross sections for transition to different levels of the final nucleus, we can determine the relative values of the normalized widths.

3. REACTION Be^9(n, 2n)Be^8. EXAMPLE OF CALCULATION

As an example of the use of this method of calculation, the angular distributions have been plotted for the center of mass of the two emitted neutrons from the (n, 2n) reaction on Be^9, at an incident neutron energy of 14 Mev, for the 2.9-Mev excited state of the Be^8 nucleus.

In Fig. 1a are shown the angular distributions for an orbital angular momentum of the picked-up

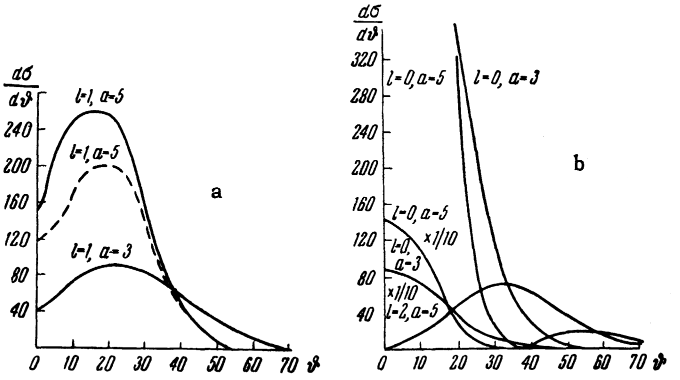

FIG. 1. Angular distribution of the center of mass of two interacting neutrons from the reaction $Be^9(n, 2n)Be^8$ for the 2.9 Be^8 level at an incident neutron energy of 14 Mev. a) $l = 1$, dotted curve was calculated according to Butler's theory; b) $l = 0$ and 2.

neutron equal to unity. This case agrees with experiment, since the neutron is probably picked up from the p shell of Be^9. It is seen from the figure that the angular distributions have a marked dependence on the radius of interaction a. Since the final state of the two neutrons is unbound, the maximum of the angular distribution is somewhat different from the maximum on Butler's curve (dotted curve in Fig. 1a) for the emission of a bound state (dineutron). The angular distributions for $l = 0$ and $l = 2$ (curve in Fig. 1b) are considerably different from those shown in Fig. 1a. Comparison of the curves of Figs. 1a and 1b shows that the angular momentum of the picked-up neutron can be determined from the experimental angular distributions by comparison with their calculated angular distributions, just as is done when the usual Butler curves are used.

4. REACTION $Pb^{208}(n, 2n)Pb^{207}$. EXAMPLE OF CALCULATION

The interaction of two neutrons in the final state distorts somewhat the shape of the usual Butler curve (Fig. 1a), especially at large angles. Therefore the angular dependence for the reaction $Pb^{208}(n, 2n)Pb^{207}$, with an excitation energy of 1.63 Mev for the Pb^{207}, was calculated with only those terms of our formula similar to Butler's formula taken into account. The radius of interaction was obtained from the formula $1.4 \times 10^{-13} A^{1/3}$ cm. In this case the orbital angular momentum $l = 6$ is uniquely determined, since the levels of Pb^{208} and Pb^{207} have been studied in detail, and their spins are equal to 0^+ and $13/2^+$. The levels neighboring on the investigated level of Pb^{207}, namely, 0.89 and 2.34 Mev, are sufficiently far apart, and the angular distribution for them is determined by orbital angular momenta of different parity. The calculated curve is shown in Fig. 2.

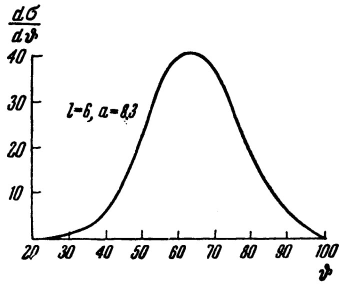

FIG. 2. Angular distribution of the center of mass of two interacting neutrons from the reaction $Pb^{208}(n, 2n)Pb^{207}$ for the 1.63-Mev level at an incident neutron energy of 14 Mev (in arbitrary units).

The maximum of the distribution lies at an angle large enough for experimental measurements to be conveniently carried out. The value of the effective cross section at the maximum is approximately one order lower than the value of the cross section at the maximum of the angular distribution for $l = 1$.

5. POSSIBILITY OF EXPERIMENTAL VERIFICATION AND CONCLUSIONS

Special investigations to reveal the marked peculiarities in the angular distributions of neutrons from the (n, 2n) reaction were not carried out. It is almost impossible to obtain directly by experiment the distributions of the total momentum vector of the two neutrons, except for individual cases, for example, the reaction $Be^9(n, 2n)Be^8$. However, the energy distributions of the emitted neutrons can be investigated.

The peak of these distributions, which is due to the interaction of the two neutrons at small relative energies, should occur when E_n is equal to half the maximum energy of a neutron from the (n, 2n) reaction for an infinitely heavy nucleus, and, when the energy is $M_f/2M_i$ times the maximum for a finite nucleus. It should be mentioned that there is a possibility of erroneously determining the energy states of the nuclei from the peaks in the energy distributions of inelastically scattered neutrons, since these peaks may be caused by the (n, 2n) process with simultaneous emission of two neutrons in the same direction, and not by the (n, n′) process.

The spectra of inelastically scattered neutrons, taken at different angles with respect to the direction of the incident neutrons, contain neutrons produced as a result of the investigated process in the (n, 2n) reaction, statistical processes in the (n, 2n) reaction, the cascade process in the (n, 2n) reaction, the (n, n′) process, and, finally, the accompanying reactions, for example, (n, pn). It is possible that the great majority of neutrons in the spectra owe their origin to the enumerated competitive mechanisms. Independently of this, one can determine the effect of the simultaneous emission of two neutrons in the same direction, since in this case the neutrons will be concentrated in a narrow region (of the order of hundreds of kev) of the spectrum corresponding to a definite state of the final nucleus.

The experimental problem consists of investigating the angular dependence for the narrow energy region of the neutron spectrum corresponding to the investigated state of the nucleus $A-1$ for the reaction $A(n, 2n)A-1$. In this case, one may expect a typical angular dependence similar to that shown in the figures, which allows one to determine the characteristics of the investigated levels or groups of neighboring levels, if the experimental method employed does not have the required resolving power.

The investigation of the spectra of inelastically scattered neutrons was not sufficiently complete to find any experimental confirmation of the above discussion. Rosen and Stewart[10] have shown that the probability of direct interactions in the (n, 2n) reaction on some medium and heavy nuclei at an incident neutron energy of 14 Mev was 10 — 15%. The angular distributions of the emitted neutrons in the energy interval 4 — 12 Mev, as shown in that work, resemble the angular distributions in the (n, d) and (p, d) reactions. In the work of O'Neill[11] the spectrum of neutrons inelastically scattered on lead was measured for an incident neutron energy of 14.8 Mev. The maximum obtained at an energy of $E_n = 2$ Mev may be due to the interaction of neutrons from the (n, 2n) reaction.

The experimental difficulties of the problem we are treating are offset by the possibility of elucidating the mechanism of the (n, 2n) reaction and determining the characteristics of the states of medium and heavy nuclei, and also of determining more accurately the states of the light nuclei. Of special interest is the determination of the characteristics of active nuclei which cannot be investigated by other methods, and also the investigation of secondary excited levels of even-even nuclei in order to consider the question of the splitting of these levels into a number of states with different angular momenta.

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