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ABSORPTION OF ULTRASOUND IN ZINC AT LOW TEMPERATURES

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Magnetic oscillations of the absorption coefficient of sound in zinc for several directions of the wave vector **k** relative to the crystallographic axes of a single crystal are studied under the conditions when $l \gg \lambda$ (l is the electron mean free path and λ is the wavelength of the sound). The magnetic field rotated in a plane perpendicular to the vector **k**. In accordance with the theory, the oscillation period (in 1/H) is found to be constant. It was found to be possible to measure the extreme diameters of Fermi surfaces for a number of crystallographic directions, on the basis of the magnitude of the period. From the anisotropy of the absorption coefficient in a strong field, it was found that the Fermi surface of zinc is open in the direction of the six-fold symmetry axis. A new method is proposed for estimating the electron mean free path. It is shown that for the metal under investigation the mean free path is anisotropic and varies between 0.2 and 0.6 mm.

1. INTRODUCTION

As was shown in a number of researches,[1-5] the ultrasonic absorption coefficient in metals α oscillates upon change of the magnetic field intensity. The theory of this phenomenon has been constructed by V. L. Gurevich[6] for different mutual orientations of the wave vector of the sound **k** and the field **H**. Calculations have been carried out under the assumption of an arbitrary dispersion law for the energy of the electrons, on the basis of a model proposed by Pippard.[7]

The chief consequence of the theory is the constancy of the period of oscillation of the absorption coefficient as a function of the reciprocal of the field intensity. The period of (ΔH^{-1}) is connected with the value of the extreme (relative to the direction of **H**) diameter D_{ext} of the Fermi surface (perpendicular to the vectors **k** and **H**) by the relation

$$D_{\text{ext}} = |p_1 - p_2|_{\text{ext}} = e\lambda/c\Delta H^{-1}. \qquad (1)$$

Thus, measurement of the oscillation period for different directions of the vectors **k** and **H** relative to the crystallographic axes in single crystals of metallic samples makes it possible to obtain valuable information on the characteristics of the Fermi surface.

Two researches on the absorption coefficient of ultrasound in zinc at low temperatures have been reported. In the work of Morse and Bohm,[5] no oscillations of the absorption coefficient were observed in this metal. Our own first measurements,[2] carried out at an ultrasonic frequency of 70 Mcs, have shown that oscillations are observed in zinc, but in a number insufficient for measurement of the period. As is well known, the number and amplitude of the oscillations depend on the ratio of the length of the mean free path of the electron to the sound wavelength, l/λ, and increase with increase of this ratio.

Since the path length is determined by the purity of the metal, it was necessary to increase the frequency of the ultrasonic vibrations for trustworthy measurements. In the present communication, experiments are described which were carried out with a variation of frequency from 60 to 220 Mcs on single crystals of zinc at temperatures 1.65 — 4.2°K in a magnetic field perpendicular to **k**, with intensities up to 7000 oe. The arrangement $\mathbf{k} \perp \mathbf{H}$ was chosen in order to simplify the interpretation of the oscillograms, inasmuch as in this case there are no periodic increases in the absorption coefficient connected with the longitudinal component of the vector **k** in the direction of **H**.[6]

In addition to the study of oscillations, great interest attaches to the investigation of the absorption coefficient of ultrasound in a strong magnetic field, when $l \gg \lambda \gg r$, where r is the radius of the Larmor orbit, $r = cp/eH$. As shown previously,[1,2] the behavior of the absorption coefficient under these conditions is connected in a definite fashion with the asymptotic conductivity tensor. The absorption coefficient in a strong field has been considered theoretically in the papers of Kaner[8] and Gurevich,[9] where it was shown that one can obtain information on the topology of the Fermi surface from a study of the anisotropy of the absorption coefficient in the magnetic field.

2. TECHNIQUE OF OBTAINING SPECIMENS AND METHOD OF MEASUREMENT

Single crystals of zinc, produced by the method of Obreimov-Shubnikov, served as specimens for measurements of the absorption coefficient. The initial metal had a purity of 99.9998 percent and was characterized by the following ratio of resistances at helium and room temperatures: $R_{4.2}/R_{300} = 2 \times 10^{-4}$.

For a more accurate determination of the orientation of the axes in the crystal, the latter was grown in the form of a sphere of diameter 15 mm. The direction of the axes was determined from the reflection of the crystallographic planes on a double-curve goniometer, after which plane parallel plates 4 — 6 mm thick were cut off by an electroerosion method.

An X-cut quartz disc of diameter 5—6 mm with a fundamental resonant frequency of 20 Mcs was attached to each plane face of the specimen. Good acoustic contact with the specimen was obtained over a wide range of temperatures by employing as an adhesive a low-temperature vacuum putty. One of the quartz plates mentioned served as the source of ultrasonic vibration, and the second as a receiver. The direction of propagation of sound coincided with the direction normal to the plane surface of the specimen.

To obtain ultrasonic frequencies above 20 Mcs, the quartz plates were excited in the odd harmonics: third, fifth, ninth and eleventh (which correspond to frequencies of 60, 100, 180 and 200 Mcs). The generator driving the quartz worked in a pulse system with pulse length of 0.5—0.6 microsecond.

The pulse was passed through the specimen, amplified in the receiving circuit, and detected by a peak detector. A voltage proportional to the amplitude of the input signal was applied to a recorder which plotted it as a function of the magnetic field. Since the absorption coefficient is proportional to the logarithm of the intensity of the signal, one could determine the change of the coefficient in the magnetic field by this recording.

The necessity of the comparatively short-length pulses produced by the generator arose because of the fact that the time of flight of the ultrasonic signal from the generating crystal to the receiver through the specimen amounted to ~1 microsecond, while the normal operation of the receiver was possible only in the absence of the powerful initial pulse of the generator, which produced great parasitic signals.

The small thickness of the specimen was connected with the difficulty of "sounding" through it: with increase in the number of the harmonic in which the quartz generator operated, the power of the radiated sound decreased and the sensitivity of the detecting crystal also decreased. Furthermore, with increase in frequency, the coefficient of sound absorption in the metal increased. All this led to the result that, in thick specimens, the signal is lost in the noise of the detector circuit.

The construction of the low-temperature equipment allowed the possibility of rotating the specimen relative to the direction of the magnetic field while maintaining the perpendicularity of the vectors **k** and **H**. The oscillation curves were recorded for a change in the direction of the field through each 3°.

3. RESULTS OF MEASUREMENTS

The periods of the oscillation of the ultrasonic absorption coefficient were measured for different orientations of the magnetic field, which rotated in a plane perpendicular to the sound wave vector.

Figure 1 illustrates one of the oscillation curves, recorded on the recorder for a frequency of the ultrasound of 220 Mcs. The wave vector was directed along $[10\bar{1}0]$ (the axes OX, OY and OU passed through the centers of the prism faces), and the **H** vector along the $[11\bar{2}0]$ direction.

Figure 2 shows the dependence of the number of oscillations (plotted along the ordinate) on the reciprocal of the magnetic field for a number of directions of the vector **H** in the $(10\bar{1}0)$ plane. As

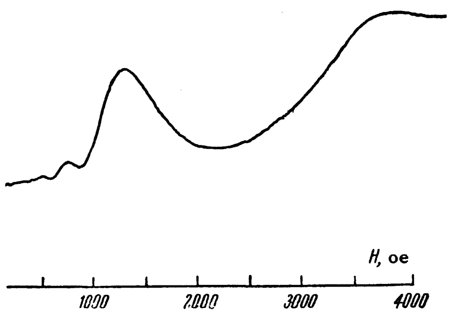

FIG. 1. Record of the amplitude of an ultrasonic pulse transmitted through the specimen as a function of the magnetic field. T = 1.65° K, ν = 220 Mcs.

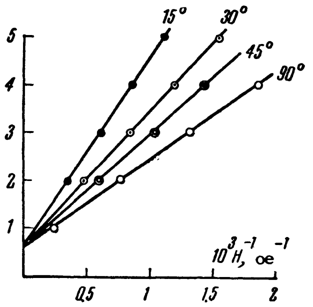

FIG. 2. Dependence of the number of oscillations (determined by the maximum of the absorption) on the inverse value of the magnetic field for a number of directions of **H** in the $(10\bar{1}0)$ plane.

is seen from the drawing, the experimental points, which correspond to maximum absorption, fall on a straight line, which testifies to the constancy of the period in the reciprocal field.

In the work of Gurevich,[6] the oscillating part of the absorption coefficient is described by the equation

$$\Delta\alpha = \alpha' \sin[(ck/eH)\,|\,p_1 - p_2\,| \pm \pi/4]. \qquad (2)$$

From the extrapolation of the curves of Fig. 2 to H → ∞, one can obtain a value of the constant phase which, according to theory, is equal to $-\pi/4$; the negative sign of the phase shows that the corresponding diameter of the surface responsible for the oscillations is a maximum.[6]

The measured periods of oscillation of ΔH^{-1} were used to calculate the extreme diameters of the Fermi surface according to Eq. (1). The angular dependencies of the extreme diameter of the Fermi surface perpendicular to the vector **H** are shown in Fig. 3, a, b, c, for rotation of the magnetic field in the planes $(11\bar{2}0)$, (0001), and $(11\bar{1}0)$ The less reliable values obtained from oscillation curves for a small number of oscillations are indicated by dashed lines. The accuracy of the measurement of the period of oscillation in the inverse field, according to which the diameter is calculated, was within 5—10 percent.

In zinc, at liquid helium temperatures, there is a significant anisotropy of the absorption coefficient in the absence of the magnetic field, and when the wave vector **k** is directed along the [0001] axis the absorption increases so much that it has not yet been possible to investigate the crystal along this axis at 220 Mcs.

The periods of oscillation in the (0001) and $(11\bar{2}0)$ planes were measured at 100 and 180 Mcs, respectively, at a temperature of 1.65°K. Simultaneously with the measurement of the period of oscillations, the dependence of the absorption coefficient on the direction of the magnetic field was studied at a frequency of 60 Mcs. The vector **k** in these measurements was also perpendicular to the direction of the field, the value of which was 7000 oe.

The diagrams of the change of the quantities $\alpha(H) - \alpha_0$, where $\alpha(H)$ is the absorption coefficient in the field and α_0 in the absence of the field are represented in Fig. 4a, b and c for directions of the wave vector **k**: a—along [0001], b—along $[10\bar{1}0]$ and c—along $[11\bar{2}0]$. The angle $\phi = 0$ corresponds to the direction of the magnetic field in planes perpendicular to the vector **k**: for the case a—along $[11\bar{2}0]$, b—along [0001] and c—along [0001].

As is seen from the drawing, for the direction of the vector **k** along the [0001] axis, the value of the absorption coefficient lies entirely outside the circle $\alpha = \alpha_0$. For other directions of the line, $\alpha(H)$ is located inside the circle mentioned, having a strongly anisotropic character.

4. ESTIMATE OF THE FREE PATH OF THE ELECTRON

In accord with the theory,[6] to each oscillation of the absorption coefficient of sound in the magnetic field there corresponds a change in the diameter of the electronic orbit by a value of the

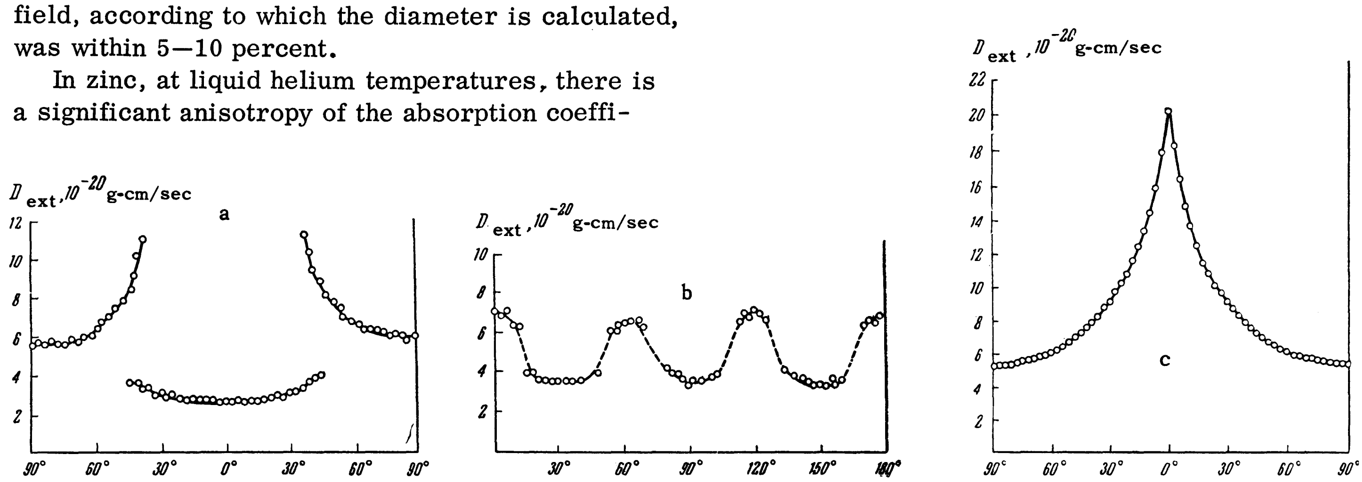

FIG. 3. Angular dependence of the extreme diameter of the Fermi surface. Diameter is perpendicular to **H** and lies in the plane: a – $(11\bar{2}0)$, b – (0001), c – $(10\bar{1}0)$. The angle $\varphi = 0°$ corresponds to the direction of the field for a – along $[11\bar{2}0]$, for b and c – along [0001].

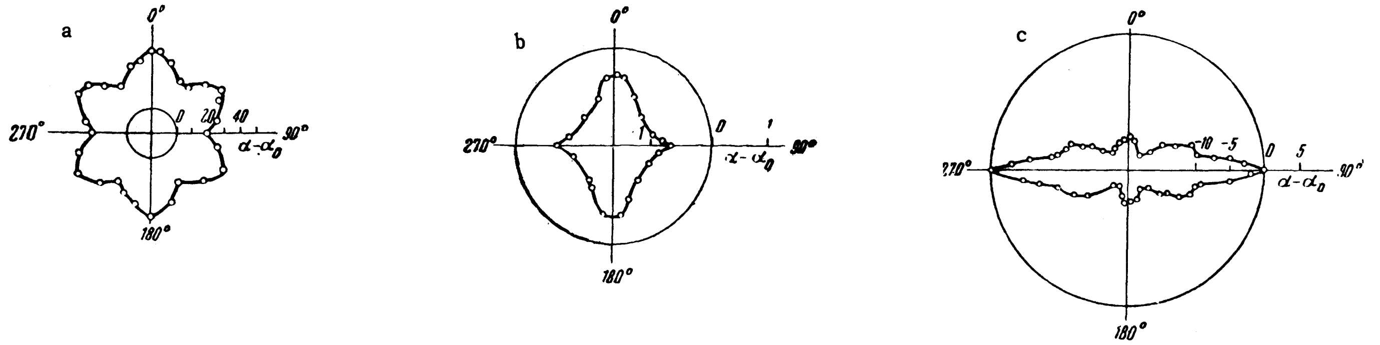

FIG. 4. Dependence of the difference of the absorption coefficients in a field of 7000 oe and without the field (in db/cm) on the direction of the magnetic field. a – k directed along [0001], b – k directed along [10$\bar{1}$0], c – k along [11$\bar{2}$0]. T = 4.2°K, ν = 60 Mcs.

sound wavelength. If we assume that the form of the trajectory of the electron differs little from a circle, one can estimate the length of its free path l by taking into account the number of oscillations and determining from them the maximum diameter of orbit in a weak field:

$$l = \pi\lambda (n + 1). \tag{3}$$

The factor n + 1 is obtained by virtue of the fact that the oscillations from the side of the strong field can begin for a minimum diameter of the electronic orbit, equal to λ at least.

Another method of estimating the path length consists in the following. If we measure the component ($p_\perp$) of the momentum of the electron perpendicular to the vectors **k** and **H**, and also the minimum magnetic field for which oscillations cease, then we can estimate the path length from the relation

$$p_\perp / r_{max} = eH_{min} / c.$$

Since the quantity $p_\perp = D_{ext}/2$ is measured from the period of oscillations of ΔH^{-1}, it becomes possible to use this relation for determination of l.

The path lengths determined by the two methods for different directions of k in the crystal are shown in the table.

Direction of **k**	Path l, mm		Temperature, °K
	From no. of oscillations	From $p_\perp$	
[0001]	0.5	0.6	1.65
[10$\bar{1}$0]	0.22	0.2	4.2
[11$\bar{2}$0]	0.27	0.24	1.65

5. DISCUSSION OF RESULTS

The values obtained for the extreme diameters of the Fermi surface testify to the sharp departure of the law of energy dispersion of the electrons from the quadratic.

It is not yet possible to establish the complete form of the Fermi surface from the momentum values obtained. The difficulties are connected with the fact that it is not known to what value of p_z correponds the particular value of p_y obtained in the experiment (the z axis is directed along the field, the x axis along the wave vector). If we assume that the oscillations are connected with the central cross section of the surface, then the values of the diameters for certain directions do not "join" one another. This points to the result that the oscillations are possibly connected not only with the central but with other cross sections, that is, the Fermi surface for zinc has a complicated configuration. On the other hand, investigation of the oscillations of the absorption along the six-fold axis was carried out at comparatively low frequency, but this did not permit a sufficiently accurate measurement of the extreme value of the momentum. It is possible that carrying out investigations at much higher frequencies would permit the construction of the Fermi surface.

Analysis of the diagrams of rotation of the absorption coefficient, carried out on the basis of theory,[8,9] shows that there are directions in zinc of open trajectories of the electrons. The experiments are best explained if one assumes that the Fermi surface for zinc contains a "corrugated cylinder" in it, with axis in the direction [0001]. In fact, since the fundamental contribution to the coefficient α is made by electrons moving perpendicular to the wave vector of the sound over a range of the wavelength λ, while the quantity of such electrons in the case in which **k** is directed along the axis of the corrugated cylinder increases in a strong field (for $\mathbf{k} \perp \mathbf{H}$), then the absorption coefficient in the field must increase, and for an arbitrary direction of **H** must be larger than α_0.

The same situation occurs if the vector **k** is directed along the axis [0001]. Small maxima along the binary axes indicate the presence of closed electronic trajectories in the case of direction of the field along these axes. Rotation diagrams for **k** along the axes $[10\bar{1}0]$ and $[11\bar{2}0]$ show that the Fermi surface is closed along these directions.

It is interesting to note that the position of the maxima of the sound absorption coefficient on the rotation diagram in planes $(10\bar{1}0)$ and $(11\bar{2}0)$ coincides with the position of the minima of the resistance $\Delta R/R$ on similar diagrams in galvanomagnetic measurements.[10,11]

An estimate of the density of electrons made from the mean value of the momentum shows that the fundamental groups of current carriers are responsible for the oscillations of the absorption coefficient. Thus, the mean value of the momentum in the plane (1010), $\bar{p} = 6 \times 10^{-20}$ g-cm/sec, which gives an estimate of 0.6×10^{22} electrons per cm^3 for the density.

Thus measurements of the diameters in the principal crystallographic directions are not sufficient for construction of the complete Fermi surface in zinc: it is also necessary to obtain a more detailed diagram of rotation, to increase the purity of the metal in this case, and to carry out investigation at higher frequencies.

In conclusion, we consider it our pleasant duty to thank É. A. Kaner and M. I. Kaganov for discussion of the results and valuable advice, and also V. I. Bogatov for the steady supply of liquid helium.

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