

## THE POLARIZATION OF THE BETA-RAY ELECTRONS FROM ORIENTED RaE

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A formula is obtained for the polarization vector of the electrons emitted in the  $\beta$  decay of RaE nuclei whose spins are oriented in an external field. A region of possible values of the nuclear matrix elements is determined on the basis of the experimental data relating to the shape of the spectrum and the longitudinal polarization of the  $\beta$  particles. These values are used to calculate the polarization of the  $\beta$  particles from oriented RaE. The dependence of the polarization on the assumed radius of the nucleus is considered. It is shown that the transverse polarization of the  $\beta$  particles is very sensitive to possible nonconservation of time parity.

### 1. INTRODUCTION

THE  $\beta$ -decay transition of RaE is a first-forbidden transition ( $1^- \rightarrow 0^+$ ). As is well known, in the overwhelming majority of cases the shape of the  $\beta$ -ray spectrum for first-forbidden transitions with  $\Delta j = 1$  is the same as for allowed transitions. The point here is that the factor that modifies the allowed shape of the spectrum consists of several terms, and those that depend on the energy are usually small in comparison with those that are proportional to the nuclear charge and do not depend on the energy. The spectrum of RaE is an exception to the general rule — its shape differs markedly from that of an allowed spectrum. To explain this anomaly, in earlier papers<sup>1</sup> a special assumption has been made about the values of the nuclear matrix elements of RaE. If these values are such that the large terms in the correction factor cancel each other, then the small terms which depend on the energy will play the dominant role and the shape of the spectrum will be different from the allowed shape. The matrix elements depend on the structure of the nucleus, and such a cancellation can occur only in exceptional cases.

In connection with the unusual shape of the spectrum of RaE, Alikhanov, Eliseev, and Lyubimov<sup>2</sup> have supposed that the longitudinal polarization of the  $\beta$ -ray electrons is also anomalous and different from the value  $v/c$ . A theoretical treatment of this question has been given in a paper by Geshkenbein, Nemirovskaya, and Rudik,<sup>3</sup> who took into account the nonconservation of spatial parity in  $\beta$  decay and also the possible nonconservation of time parity. Alikhanov, Eliseev,

and Lyubimov<sup>2</sup> have determined the longitudinal polarization of the  $\beta$  particles of RaE experimentally and have compared it with the predictions of the theory. This comparison has allowed a rather accurate determination of an upper limit on the possible nonconservation of time parity. Further improvement of this value involves not only experimental difficulties but also the necessity in the interpretation of the experiments of taking into account a number of corrections which are hard to estimate, since they depend on the structure of the nucleus.

We shall show that the difficulties in the interpretation of the experiments can be avoided if we study the transverse polarization of the electrons emitted by oriented RaE nuclei. With conservation of time parity the degree of polarization in the direction  $\mathbf{n} \times \mathbf{n}_0$  does not exceed 2 percent. Here  $\mathbf{n} = \mathbf{p}/|\mathbf{p}|$ ,  $\mathbf{p}$  being the momentum of the  $\beta$  particle;  $\mathbf{n}_0$  is the direction of dominant orientation of the nuclear spins. Within the accuracy of the existing experiments [ $\arg(C_V/C_A) \sim 3^\circ$ , see later discussion], for nonconservation of time parity the degree of polarization reaches 45 percent. In addition we repeat the analysis of the experimental data relating to the spectrum and longitudinal polarization of the  $\beta$  particles, with corrections included that were not dealt with in the previous papers.

### 2. THE POLARIZATION OF THE $\beta$ -RAY ELECTRONS

A general expression for the polarization of the  $\beta$  particles emitted by oriented nuclei in first-for-

bidden transitions has been obtained in reference 4. In the special case of a  $1^- \rightarrow 0^+$  transition the polarization is given by the formula

$$\begin{aligned} \zeta = \{ & \mathbf{n} [-a_0 v/c + a_1 \langle \mu \rangle \cos \vartheta \\ & + a_2 \langle \mu^2 \rangle P_2(\cos \vartheta)] + \mathbf{n}_1 a_3 \langle \mu \rangle \sin \vartheta \\ & + \mathbf{n}_2 [a_4 \langle \mu \rangle \sin \vartheta + a_5 \langle \mu^2 \rangle \sin 2\vartheta] \} [W_{\mathbf{nn}_0}]^{-1}, \end{aligned} \quad (1)$$

$$\begin{aligned} W_{\mathbf{nn}_0} = & 1 - b_0 \langle \mu \rangle \cos \vartheta - b_1 \langle \mu^2 \rangle P_2(\cos \vartheta), \quad (2) \\ \mathbf{n}_1 \parallel & [\mathbf{n} \times \mathbf{n}_0], \quad \mathbf{n}_2 \parallel [\mathbf{n}_1 \times \mathbf{n}], \quad \cos \vartheta = \mathbf{nn}_0, \end{aligned}$$

$$\langle \mu \rangle = \sum_{\mu=-1}^1 \mu w(\mu), \quad \langle \mu^2 \rangle = \sum_{\mu=-1}^1 (3\mu^2 - 2) w(\mu). \quad (3)$$

$\mu$  is the projection of the spin of the nucleus on the axis  $\mathbf{n}_0$ ;  $w(\mu)$  is the probability of a given value of  $\mu$ ;  $W_{\mathbf{nn}_0}$  is the angular distribution of the  $\beta$  particles emitted by the oriented RaE. The explicit form of the coefficients  $a_i$  and  $b_i$  is presented in Appendix I.\* They depend on the energy of the  $\beta$  particle, on the  $\beta$ -decay interaction constants, and on two ratios of nuclear matrix elements  $x$  and  $y$  defined by

$$\begin{aligned} x = i\varepsilon_V \int \mathbf{r} / \varepsilon_A \int [\boldsymbol{\sigma} \times \mathbf{r}], \quad y = \varepsilon_V \int \boldsymbol{\alpha} / \varepsilon_A \int [\boldsymbol{\sigma} \times \mathbf{r}], \\ \varepsilon_V^2 = |C_V|^2 + |C'_V|^2, \quad \varepsilon_A^2 = |C_A|^2 + |C'_A|^2, \end{aligned} \quad (4)$$

$x$  and  $y$  are real quantities;  $C_A$ ,  $C'_A$ ,  $C_V$ , and  $C'_V$  are the  $\beta$ -decay interaction constants. In the theory of the two-component neutrino  $C_V = C'_V$ ,  $C_A = C'_A$ . We shall deal mainly with the case of the VA interaction, which is evidently confirmed by experiment, and shall show only at the end how to go over to the ST interaction.  $\int \mathbf{r}$ ,  $\int \boldsymbol{\alpha}$ , and  $\int \boldsymbol{\sigma} \times \mathbf{r}$  are nuclear matrix elements of the  $\beta$ -decay transition.

The spectrum of RaE is of the form

$$N(E) dE = (G^2 / 2\pi^3) F(Z, E) pE (E_0 - E)^2 C(E) dE, \quad (5)$$

where  $G$  is the Fermi interaction constant,  $F(Z, E)$  is the Fermi function, and  $C(E)$  is the correction factor for the spectrum.

In the diagram the dashed curves show the region of spread of the experimental points  $C(E)$ . The main inaccuracy is that associated with the determination of the limiting energy of the spectrum:<sup>6</sup>  $3.26 \text{ mc}^2 \lesssim E_0 \lesssim 3.32 \text{ mc}^2$ . The solid curves are the theoretical values for certain possible values of  $x$  and  $y$ .

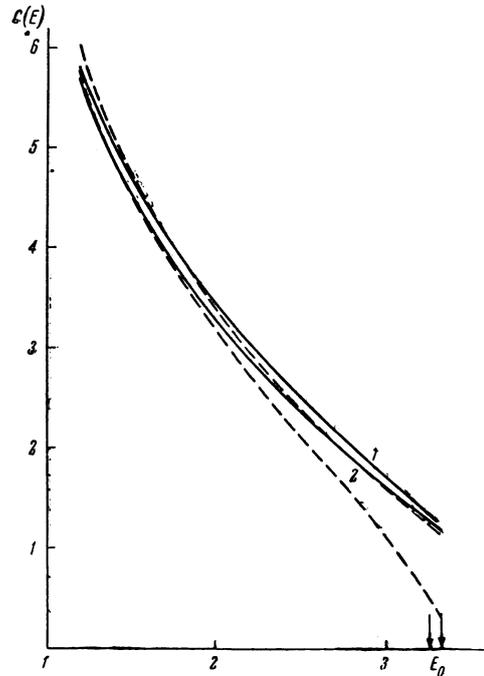
The formulas of Appendices I and II have been used to calculate the coefficients  $a_i$  and  $b_i$  that appear in Eq. (1) for the values of  $x$  and  $y$  fitted in this way to the shape of the spectrum; the val-

ues of the coefficients so obtained are shown in Appendix III, Table I. Agreement cannot be obtained with values of  $x$  and  $y$  that differ much from those given in the tables.

The finite size of the nucleus was taken into account in the calculation of the coefficients  $a_i$ ,  $b_i$ . A uniform volume distribution of charge in the nucleus was assumed for two possible values of the nuclear radius,  $r_0 = 1.2$  and  $r_0 = 1.5$ , where  $R = r_0 A^{1/3} \times 10^{-13} \text{ cm}$  and  $A = 210$  for RaE.\* The limiting energy  $E_0$  of the spectrum was taken to be  $3.26 \text{ mc}^2$ .  $a_0$  is the coefficient of the departure of the longitudinal polarization of the  $\beta$  particles of un-oriented RaE from the value  $v/c$ . Table II of Appendix III shows the experimental values of  $a_0$  available at present from the data of various authors.

### 3. DISCUSSION OF THE RESULTS

An examination of the curves of  $C(E)$  and the values for  $a_0$  shows that the shape of the spec-



The dashed lines bound the region of the experimental values of the form factor  $C(E)$ . The solid curves are calculated for the following possible values of the parameters  $x$  and  $y$ : 1— $x = 1$ ,  $y = 30.5$ ,  $r_0 = 1.2$ ; 2— $x = 1$ ,  $y = 25.1$ ,  $r_0 = 1.5$ .

\*In the approximation  $(\alpha Z)^2 \ll 1$  a surface distribution of charge in the nucleus is equivalent to a volume distribution with the radius of the nucleus increased by 20 percent. This approximation is not valid for RaE ( $Z = 83$ ), but nevertheless by increasing the radius of the nucleus and choosing new values of  $x$  and  $y$  that fit the shape of the spectrum we get a result corresponding to a distribution of charge in the nucleus that is close to a surface distribution with the original value of the radius of the nucleus.

\*The coefficients  $b_i$  have been calculated independently of us by Geshkenbein, Nemirovskaya, and Rudik.<sup>5</sup>

trum and the size of the longitudinal polarization as observed experimentally can be best described with values of  $x$  in the range 0.67–1 and corresponding values of  $y$  that depend on the radius of the nucleus. These values do not depart too far from those that have been obtained by other authors.

The possible amount of nonconservation of time parity obtained by Geshkenbein and others<sup>3</sup> agrees with the results of our work. A change of the assumed nuclear radius has a marked effect on the quantities  $x$  and  $y$ , but we can always take values of  $x$  and  $y$  that satisfy all the experimental data without resorting to the hypothesis of a considerable nonconservation of time parity. The inaccuracy associated with the determination of  $E_0$  affects the values of  $a_i$  and  $b_i$  mainly near the limit of the spectrum. For example, when we replace  $E_0 = 3.26$  by  $E_0 = 3.32$  the value of  $a_0$  at  $E = 3.0$  decreases by 9 percent for  $x = 0.2$  and by 14 percent for  $x = 1.2$ . At  $E = 2.0$ , however, the maximum change of  $a_0$  for permissible values of  $x$  does not exceed 3 percent.

We see from the tables that the change of the longitudinal polarization is of the same order of magnitude as the transverse polarization in the direction  $[\mathbf{n} \times \mathbf{n}_0] \times \mathbf{n}$ . For conservation of time parity the transverse polarization in the direction  $\mathbf{n} \times \mathbf{n}_0$  is very small. For example, for  $E = 1.2$  the coefficient  $a_3$  is equal to 2 percent.

It is a remarkable fact, however, that  $a_3$  depends sharply on a possible nonconservation of time parity. If we write  $C_A = aC_V e^{i\theta}$ , then if the angle  $\theta$  that gives the phase shift between the A and V interactions is  $3^\circ$  (which is within the limits of error of the existing experiments), the coefficient  $a_3$  reaches a value of 45 percent. Therefore we can recommend the study of the transverse polarization of the  $\beta$  particles from oriented RaE as one of the most sensitive ways of testing the law of conservation of time parity.

The polarization formula (1) is valid for nuclei oriented in any way (for example, by the method of Gorter and Rose). For nuclei aligned by the method of Bleaney or Pound there remain only the terms proportional to the quadrupole polarization of the nuclei. In this case the amount of longitudinal polarization will differ from that of unoriented RaE by not more than 15 percent. There can be a transverse polarization only in the direction  $[\mathbf{n} \times \mathbf{n}_0] \times \mathbf{n}$ . In the special case  $x = 0.5$  the coefficients  $a_2$ ,  $a_5$ , and  $b_1$  are equal to zero. Thus in this case with aligned nuclei there can be only a longitudinal polarization of the  $\beta$  particles, of an amount that agrees exactly with the polarization of the  $\beta$  particles from unoriented RaE.

We can make an estimate of the corrections from the third degree of forbiddenness. Using an estimate for the matrix elements of the third order in forbiddenness that is clearly too high

$$\varepsilon_V \int \alpha r^2 / \varepsilon_A \int [\sigma \times \mathbf{r}] \lesssim R^2 \varepsilon_V \int \alpha / \varepsilon_A \int [\sigma \times \mathbf{r}] \approx \left(\frac{1}{50}\right)^2 y \quad (6)$$

and keeping the largest term in the expression for the correction, we get

$$a'_i = \frac{a_i C(E) + 2 |G(E, R)| \sqrt{a_i C(E)}}{C(E) + 2 |G(E, R)| \sqrt{C(E)}}, \quad (7)$$

where the  $a'_i$  are the corrected values and the  $a_i$  the original values of the coefficients in Eq. (1), and

$$G(E, R) = \frac{2q \alpha Z}{15 R} \frac{\varepsilon_V}{\varepsilon_A} \int \alpha r^2 / \int [\sigma \times \mathbf{r}]. \quad (8)$$

The  $b'_i$  and  $b_i$  are related in a similar way.

It follows from Eqs. (7) and (8) that the third-forbidden terms make their largest contribution to the soft part of the  $\beta$ -ray spectrum. Their total contribution is small. At  $E = 1.2$  the coefficient  $a_0$  is decreased by 2 percent.

The writers are grateful to K. A. Ter-Martirosyan, who informed them about earlier papers<sup>2,3</sup> before their appearance in print and called attention to the problem of RaE.

## APPENDIX I

Analytical form of the functions  $a_i$ ,  $b_i$  for the VA interaction:

$$\begin{aligned} a_0 C(E) pE^{-1} = & x^2 [\beta_{VV} (M'_0 + 2L'_1 - \frac{2}{3} q N'_0) \\ & + \frac{1}{3} q^2 L'_0 - \frac{2}{3} q \beta'_{VV} N''_0] + y^2 \beta_{VV} L'_0 + \beta_{AA} (M'_0 + \frac{1}{2} L'_1 \\ & + \frac{2}{3} q N'_0 + \frac{1}{6} q^2 L'_0) + \frac{2}{3} q \beta'_{AA} N''_0 \\ & + 2y [\beta_{VA} (N'_0 + \frac{1}{3} q L'_0) - \beta'_{VA} N''_0] \\ & + 2x [\beta_{VA} (M'_0 - L'_1) + \frac{2}{3} q \beta'_{VA} N''_0] \\ & + 2xy [\beta_{VV} (N'_0 - \frac{1}{3} q L'_0) + \beta'_{VV} N''_0], \\ b_0 C(E) = & x^2 \{ \beta_{VV} [M'_0 - 2M'_1 + L'_1 - \frac{2}{3} q (N'_0 - N'_1)] \\ & - 2\beta'_{VV} [M'_2 + \frac{1}{3} q (N''_0 - N'_2)] \} + \beta_{AA} [M'_0 + M'_1 + \frac{1}{4} L'_1 \\ & + \frac{1}{3} q (2N'_0 + N'_1) + \frac{1}{12} q^2 L'_0] \\ & + \beta'_{AA} [M'_2 + \frac{1}{3} q (2N''_0 + N'_2)] \\ & + y^2 \beta_{VV} L'_0 + y [\beta_{VA} (2N'_0 + N'_1 + \frac{2}{3} q L'_0) - \beta'_{VA} (2N''_0 + N'_2)] \\ & + x [\beta_{VA} (2M'_0 - M'_1 - L'_1 - q N'_1 - \frac{1}{3} q^2 L'_0) \\ & - \beta'_{VA} (3M'_2 - \frac{4}{3} q N''_0 + \frac{1}{3} q N'_2)] \\ & + 2xy [\beta_{VV} (N'_0 - N'_1 - \frac{1}{3} q L'_0) + \beta'_{VV} (N''_0 - N'_2)], \end{aligned}$$

$$a_2 C(E) = x^2 [\beta_{VV} (2M'_1 + L'_1 - \frac{2}{3} qN'_1) + \beta'_{VV} (2M'_2 - \frac{2}{3} qN'_2)] \\ + \beta_{AA} (\frac{1}{4} L'_1 - M'_1 - \frac{1}{3} qN'_1) - \beta'_{AA} (M'_2 + \frac{1}{3} qN'_2) \\ - y (\beta_{VA} N'_1 + \beta'_{VA} N'_2) + x [\beta_{VA} (M'_1 - L'_1 + qN'_1) \\ + \beta'_{VA} (3M'_2 + \frac{1}{3} qN'_2)] + 2xy (\beta_{VV} N'_1 + \beta'_{VV} N'_2).$$

We get  $C(E)$ ,  $b_1$ , and  $a_1$  from  $a_0 C(E) pE^{-1}$ ,  $a_2$ , and  $b_0$ , respectively, if in the latter expressions we make the replacements

$$(M'_i, L'_i, N'_i) \rightarrow (M_i, L_i, N_i), \beta_{ii'} \rightarrow \alpha_{ii'}, \beta'_{ii'} \rightarrow \alpha'_{ii'}, N''_0 \rightarrow 0.$$

We get  $a_4$  from  $a_1$  by the replacements

$$M_0 \rightarrow \bar{M}_0, L_0 \rightarrow \bar{L}_0, N_0 \rightarrow \bar{N}_0, L_1 \rightarrow -2\bar{L}_1, \\ M_1 \rightarrow -\frac{1}{2} \bar{M}_1, N_1 \rightarrow -\frac{1}{2} \bar{N}_1, \\ M_2 \rightarrow -\frac{1}{2} \bar{M}_2 \text{ and } N_2 \rightarrow -\frac{1}{2} \bar{N}_2.$$

We get  $a_5$  from  $\frac{1}{2} a_2$  by the replacements

$$M'_1 \rightarrow \frac{3}{2} \bar{M}_1, N'_1 \rightarrow \frac{3}{2} \bar{N}_1, N'_2 \rightarrow \frac{3}{2} \bar{N}_2, \\ \bar{M}'_2 \rightarrow \frac{3}{2} \bar{M}'_2 \text{ and } L'_1 \rightarrow 0.$$

We get  $a_3$  from  $b_0$  by the replacements

$$M'_1 \rightarrow -\frac{1}{2} \bar{M}'_1, L'_1 \rightarrow 2\bar{L}'_1, N'_1 \rightarrow -\frac{1}{2} \bar{N}'_1, M'_2 \rightarrow -\frac{1}{2} \bar{M}'_2, \\ N'_2 \rightarrow -\frac{1}{2} \bar{N}'_2, \beta_{ii'} \rightarrow \beta'_{ii'} \text{ and } \beta'_{ii'} \rightarrow -\beta_{ii'}.$$

$$\alpha_{ii'} = \text{Re}(C_i C_{i'}^* + C_i^* C_{i'}) / \varepsilon_i \varepsilon_{i'}, \\ \alpha'_{ii'} = \text{Im}(C_i C_{i'}^* + C_i^* C_{i'}) / \varepsilon_i \varepsilon_{i'}, \\ \beta_{ii'} = \text{Re}(C_i C_{i'}^* - C_i^* C_{i'}) / \varepsilon_i \varepsilon_{i'}, \\ \beta'_{ii'} = \text{Im}(C_i C_{i'}^* - C_i^* C_{i'}) / \varepsilon_i \varepsilon_{i'}, \\ \varepsilon_i^2 = C_i^2 + C_i'^2, \alpha_{ii} = 1, \alpha'_{ii} = \beta'_{ii} = 0.$$

For the case of the ST interaction we must make the following replacements in the formulas presented above: in  $\beta_{ii'}$  and  $\beta'_{ii'}$  replace indices  $A \rightarrow T$ ; in the terms in  $x^2$ ,  $x$ , and  $1$  replace  $V \rightarrow S$ ; in the terms in  $y^2$ ,  $xy$ , and  $y$  replace  $V \rightarrow T$ ; also replace  $a_2 \rightarrow -a_2$ ,  $b_0 \rightarrow -b_0$ ,  $q \rightarrow -q$ , and let

$$x = i\varepsilon_S \int \beta r / \varepsilon_T \int \beta [\sigma \times r], \quad y = \int \beta \alpha / \int \beta [\sigma \times r].$$

## APPENDIX II

Values of the functions  $M_i$ ,  $L_i$ , etc., as functions of the energy of the  $\beta$  particle and the radius of the nucleus, as found for a volume charge distribution in the nucleus:

$$M_0 = \frac{2\pi}{9E\rho F} \left\{ \left[ 0.95(W-1)^2 - \frac{3\alpha Z}{5R}(W-1) + \left( \frac{3\alpha Z}{10R} \right)^2 \right] a_{1/2-1/2}^2 \right. \\ \left. + \left[ 0.95(W+1)^2 - \frac{3\alpha Z}{5R}(W+1) + \left( \frac{3\alpha Z}{10R} \right)^2 \right] a_{1/2+1/2}^2 \right\}, \\ L_0 = \frac{2\pi \cdot 0.88}{E\rho F} (a_{1/2-1/2}^2 + a_{1/2+1/2}^2),$$

$$L_1 = \frac{2\pi \cdot 0.93}{E\rho F} (a_{3/2-1/2}^2 + a_{3/2+1/2}^2),$$

$$N_0 = -\frac{2\pi}{3E\rho F} \left\{ \left[ 0.92(W-1) - \frac{3\alpha Z}{10R} \right] a_{1/2-1/2}^2 \right. \\ \left. + \left[ 0.92(W+1) - \frac{3\alpha Z}{10R} \right] a_{1/2+1/2}^2 \right\}$$

$$M'_0 = \frac{4\pi}{9E\rho F} \left[ 0.95x^2 - \frac{3\alpha Z}{5R}W + \left( \frac{3\alpha Z}{10R} \right)^2 \right] a_{1/2+1/2} a_{1/2-1/2} \cos \delta_0,$$

$$L'_0 = \frac{4\pi \cdot 0.88}{E\rho F} a_{1/2+1/2} a_{1/2-1/2} \cos \delta_0, \quad L'_1 = \frac{4\pi \cdot 0.93}{E\rho F} a_{3/2+1/2} a_{3/2-1/2} \cos \delta_1,$$

$$N'_0 = -\frac{4\pi}{3E\rho F} (0.92W - \frac{3\alpha Z}{10R}) a_{1/2+1/2} a_{1/2-1/2} \cos \delta_0,$$

$$N''_0 = -\frac{3.68\pi}{3E\rho F} a_{1/2+1/2} a_{1/2-1/2} \sin \delta_0,$$

$$M_1 = \frac{2\pi}{3E\rho F} \left\{ \left[ 0.94(W-1) - \frac{3\alpha Z}{10R} \right] a_{1/2-1/2} a_{3/2+1/2} \cos \delta_2^+ \right. \\ \left. + \left[ 0.94(W+1) - \frac{3\alpha Z}{10R} \right] a_{1/2+1/2} a_{3/2-1/2} \cos \delta_2^- \right\},$$

$$N_1 = -\frac{1.8\pi}{E\rho F} (a_{1/2-1/2} a_{3/2+1/2} \cos \delta_2^+ + a_{1/2+1/2} a_{3/2-1/2} \cos \delta_2^-).$$

We get  $M'_1$  and  $N'_1$  from  $M_1$  and  $N_1$ , respectively, if we replace  $a_{3/2,1/2}$  by  $a_{3/2,-1/2}$  and conversely, and take  $\delta_2^\pm$  instead of  $\delta_2^\pm$ .  $M_2$ ,  $N_2$ ,  $M'_2$ , and  $N'_2$  differ from  $M_1$ ,  $N_1$ ,  $M'_1$ , and  $N'_1$ , respectively, by replacement of  $\cos \delta_1^\pm$  by  $\sin \delta_1^\pm$ .

We get  $\bar{M}_0$ ,  $\bar{L}_0$ ,  $\bar{N}_0$  from  $M_0$ ,  $L_0$ ,  $N_0$  by the replacement  $a_{1/2+1/2}^2 \rightarrow -a_{1/2+1/2}^2$ , and we get  $\bar{M}_1$ ,  $\bar{N}_1$  from  $M_1$ ,  $N_1$  by replacing  $\cos \delta_2^-$  by  $-\cos \delta_2^-$ .

$$\delta_0 = \delta_{1/2-1/2} - \delta_{1/2+1/2}, \quad \delta_1 = \delta_{3/2+1/2} - \delta_{3/2-1/2}, \quad \delta_2^\pm = \delta_{1/2\mp 1/2} - \delta_{3/2\pm 1/2},$$

$$\delta_3^\pm = \delta_{1/2\mp 1/2} - \delta_{3/2\mp 1/2}, \quad W = E + \frac{3}{2} \frac{\alpha Z}{R}, \quad x^2 = W^2 - 1,$$

$$p^2 = E^2 - 1, \quad q = E_0 - E.$$

$a_j \lambda$  and  $\delta_j \lambda$  are tabulated in the work of Sliv and Volchok<sup>7</sup> for  $r_0 = 1.2$  and  $r_0 = 1.5$ .  $F$  is the Fermi function, which is tabulated in the work of Dzhelepov and Zyryanova.<sup>8</sup> If we neglect the change of the electron wave function inside the nucleus,  $M_0$ ,  $L_0$ ,  $L_1$ , and  $N_0$  coincide with the functions determined by Konopinski and Uhlenbeck.<sup>9</sup> Corrected for the finite size of the nucleus, but for different values of  $Z$ , these functions have been tabulated in reference 8.

## APPENDIX III

The values of  $a_i$  and  $b_i$  in Table I are calculated for  $r_0 = 1.2$ . A change of  $r_0$  can be compensated by a suitable change of  $y$ . The values of  $y$  obtained from the data on the shape of the  $\beta$ -ray spectrum for  $r_0 = 1.2$  and  $r_0 = 1.5$  are given in Table III. It was found that the quantities  $a_i$  and  $b_i$  calculated with these values of  $y$  were practically identical for the two values of  $r_0$ , being a

TABLE I. Calculated values of the coefficients  $a_i$  and  $b_i$

$E$	1,2	1,5	2,0	3,0	1,2	1,5	2,0	3,0
	$a_0$				$b_0$			
0.2	0.84	0.85	0.86	0.89	0.41	0.44	0.68	0.61
0.5	0.80	0.82	0.81	0.84	0.38	0.39	0.64	0.70
0.67	0.78	0.80	0.79	0.81	0.35	0.35	0.63	0.74
1.0	0.74	0.76	0.74	0.75	0.30	0.28	0.59	0.75
1.2	0.72	0.74	0.70	0.71	0.28	0.18	0.54	0.71
	$a_1$				$a_4$			
0.2	0.93	0.82	0.89	0.85	0.37	0.58	0.65	0.93
0.5	0.89	0.78	0.89	0.98	0.17	0.50	0.65	1.01
0.67	0.86	0.74	0.93	1.05	0.02	0.60	0.66	1.06
1.0	0.76	0.69	0.89	1.07	-0.40	0.63	0.70	1.08
1.2	0.70	0.55	0.87	1.08	-0.58	0.42	0.64	1.11
	$b_1$				$a_2$			
0.2	0.010	0.026	0.057	0.148	0.016	0.035	0.069	0.167
1.0	-0.010	-0.029	-0.062	-0.124	-0.027	-0.047	-0.089	-0.175
1.2	-0.011	-0.035	-0.071	-0.103	-0.028	-0.062	-0.112	-0.181
	$a_5$							
0.2	0.016	0.031	0.056	0.122				
1.0	-0.012	-0.032	-0.084	-0.220				

TABLE II. Experimental values of  $a_0$

$E$	$a_0$	$E$	$a_0$
1.23	$0.69 \pm 0.04$ [10]	1.57	$0.66 \pm 0.06$ [10]
1.24	$0.733 \pm 0.06$ [2]	1.76	$0.725 \pm 0.06$ [2]
1.30	$0.75 \pm 0.04$ [10]	2.28	$0.69 \pm 0.10$ [11]
1.41	$0.75 \pm 0.04$ [10]		

TABLE III

$x$	$y$	
	$r_0 = 1,2$	$r_0 = 1,5$
0.2	19.2	16.0
0.5	23.5	19.5
0.67	25.9	21.4
1.0	30.5	25.1
1.2	33.2	27.3

TABLE IV

$E$	1,2	1,5	2,0	3,0
0.2	0.84	0.85	0.85	0.83
0.5	0.80	0.81	0.81	0.75
0.67	0.78	0.79	0.78	0.69
1.0	0.74	0.74	0.72	0.60
1.2	0.71	0.71	0.68	0.55

bit different only at the limit of the spectrum.

As an example, values of  $a_0$  for  $r_0 = 1.5$  are shown in Table IV.

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