ROTATIONAL ENERGY AND MOMENTS OF INERTIA OF NON-AXIAL NUCLEI

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It is shown that if the moments of inertia of a non-axially symmetric nucleus depart from their hydrodynamic values, then there is but little change in the dependence of the ratios of the energies of rotational levels on the ratio of the energies of two rotational states with spin 2.

1. Davydov and Filippov[1] and Davydov and Rostovskiĭ[2] have developed a theory of the rotational states of nuclei which do not have axial symmetry. It was shown that the ratios of the energies of the rotational states to the energy of the first excited state with spin 2 were uniquely determined, provided that the same ratio was known for the second level with spin 2. It was also demonstrated that the relative probabilities of electric quadrupole transitions between the rotational levels are also uniquely determined by the same energy ratio.

These results follow from two simplifying assumptions: a) the internal state of the nucleus does not change when it rotates (the adiabatic approximation) and b) the principal moments of inertia of the nucleus can be expressed in terms of only two parameters, A and γ, through the equations

$$I_i = A \sin^2 (\gamma - 2\pi i / 3) \qquad (i = 1, 2, 3). \tag{1}$$

Such a relation between the moments of inertia and γ holds in the hydrodynamic model of the nucleus, and we shall refer to this approximation as the hydrodynamic approximation.

It is natural to wonder how much the results obtained in references 1 and 2 depend on the simplifying assumptions. MacDonald[3*] has used the relation

$$I_i = I_i^H [(I_i^H / I_i^R)^{1/2} + p]^{-2}, \tag{2}$$

where I_i^R are the moments of inertia of the solid body, I_i^H are the moments of inertia that coincide with (1) when $A = 4B\beta^2$, and where p is a new parameter, taken to be 0.1 or 0.2. The relation (2) has the property that $I_i = I_i^R$ as $p \to 0$; for $p \neq 0$ and $\gamma \to 0$, the moment $I_3 \to 0$; for $p \neq 0$ and $\beta \to 0$, all $I_i \to 0$.

*The authors would like to thank N. MacDonald for sending a preprint of his paper prior to publication.

Formula (2) can be considered an empirical one, taking into account the departure of the moments of inertia from their hydrodynamic values. MacDonald considered only levels with spin 2. It will be shown below that in this case it is impossible to say which is the more important, the non-adiabaticity or the deviations of the moments of inertia from their hydrodynamic values.

In this paper we consider, in the adiabatic approximation, the rotational states of non-axial nuclei having arbitrary moments of inertia. We shall show that in the general case the ratios of the rotational energy levels can be expressed through two parameters: ξ – the ratio of the energies of two spin-2 levels, and η – a parameter that depends on the nature of the collective motions which define the rotation of the nucleus. A comparison of our results with experiment shows that the hydrodynamic approximation is good enough for computing the ratios of rotational energy levels. Discrepancies between theory and experiment are due to an interaction between the rotation and the internal state of the nucleus.

2. In the adiabatic approximation, the rotational energy operator for a non-axial even-even nucleus is

$$H = \tfrac{1}{2} \sum_{i=1}^{3} a_i J_i^2,$$

where $a_i = \hbar^2/I_i$; the J_i are the projections of the angular momentum on the principal direction in the nucleus, while the I_i are its principal moments of inertia.

As was pointed out in reference 1, the rotation states of an even-even nucleus are related to the totally-symmetric representation of the D_2 group; only such states will be considered here. It is easy to show that the energy of a rotational state of spin 2 is determined by the equation

$$E^2 - 2(a_1 + a_2 + a_3) E + 3(a_1a_2 + a_1a_3 + a_2a_3) = 0.$$

If $E_1(2)$ and $E_2(2)$ are the roots of this equation, then it follows that

$$\Sigma a_i / E_1(2) = \tfrac{1}{2}(1+\xi), \quad (a_1a_2 + a_1a_3 + a_2a_3)/E_1^2(2) = \tfrac{1}{3}\xi, \tag{3}$$

where

$$\xi = E_2(2)/E_1(2) > 1.$$

The energies of all rotational states will be written in terms of the dimensionless quantity $\epsilon = E/E_1(2)$. Then the energies of rotational states with spin 3 and 5 can be expressed in terms of the experimentally-measureable ratio ξ through the formulas

$$\varepsilon(3) = 1+\xi, \qquad \varepsilon_1(5) = 4+\xi, \qquad \varepsilon_2(5) = 1+4\xi.$$

The energies of rotational states with other values of the spin depend not only on the parameter ξ but also on another parameter η,

$$\eta = a_1a_2a_3/E_1^3(2). \tag{4}$$

For example, the energies of levels with spin 4 and 6 are given by

$$\varepsilon^3 - 5(1+\xi)\varepsilon^2 + 4[\xi^2 + \tfrac{19}{3}\xi + 1]\varepsilon - 40[\tfrac{1}{3}\xi(1+\xi) + 7\eta] = 0,$$

$$\varepsilon^4 - 14(1+\xi)\varepsilon^3 + 49[1 + 4\xi + \xi^2]\varepsilon^2 - [36(\xi^3+1) + 578\xi(1+\xi) + 3888\eta]\varepsilon + [252\xi(1+\xi^2) + 889\xi^2 + 13608(1+\xi)\eta] = 0.$$

If the moments of inertia are determined by formula (1), then

$$\eta = \eta_{\text{hydr}} = \xi^2/18(1+\xi),$$

and the energies of all rotational states depend only on ξ, which in this case is greater than or equal to 2. In general, however, there is a second parameter η, whose values lie in a certain interval determined by ξ.

To find the limits of the variation of η with ξ, we note that according to (3) and (4) the quantities $a_i/E_1(2)$ $(i = 1, 2, 3)$ are the roots of the cubic equation

$$x^3 - \tfrac{1}{2}(1+\xi)x^2 + \tfrac{1}{3}\xi x - \eta = 0.$$

The condition that the roots of this equation be real and positive implies that η must lie in the following intervals, whose end points depend on ξ:

$$\xi^2(3-\xi) \leqslant 54\eta \leqslant 3\xi - 1 \qquad (1 < \xi \leqslant 3),$$
$$0 \leqslant 54\eta \leqslant 3\xi - 1 \qquad (\xi \geqslant 3). \tag{5}$$

Figure 1 shows the ratios $\epsilon_1(4)$ and $\epsilon_2(4)$ as functions of ξ for various values of η satisfying

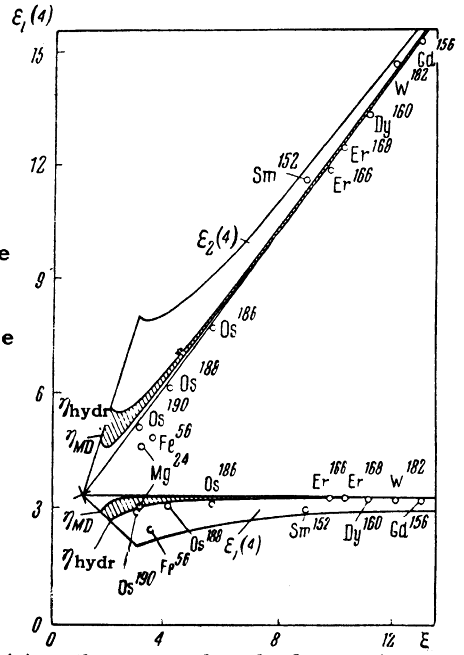

FIG. 1. Possible values of the ratios $\varepsilon_1(4)$ and $\varepsilon_2(4)$ for various values of the parameters ξ and η.

the inequalities (5). The cross hatched area is bounded by two curves, $\eta_{\text{hydr.}}$ (corresponding to the hydrodynamic approximation) and η_{MD} (corresponding to moments of inertia determined by (2) with $\beta = p = 0.2$). Figure 2 shows $\epsilon_1(6)$ as a function of ξ and η, for the range defined by (5).

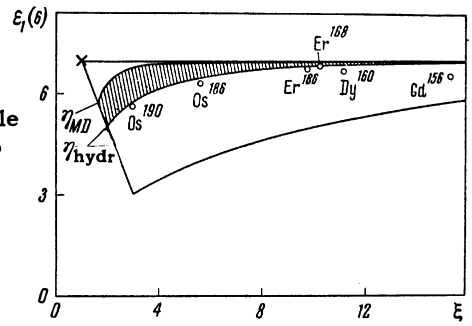

FIG. 2. Possible values for the ratio $\varepsilon_1(6)$ for various values of the parameters ξ and η.

It should be noted, naturally, that the values of η in (5) correspond to all possible ratios of the principal moments of inertia, including some that are utterly unrealistic. For example, in Figs. 1 and 2, the point marked with a cross corresponds to the rotation of the nucleus as a rigid sphere ($I_1 = I_2 = I_3$; $\xi = 1$); in such cases η is zero or close to it.

At present it is considered that the moments of inertia in the nucleus are intermediate between their hydrodynamic values and those obtaining for a rigid body. Hence the actual ratios of the moments of inertia correspond apparently to values of η for which the energy ratios $\epsilon_i(J)$ are displaced from their hydrodynamic values toward the cross hatched area. Taking this into account, we conclude that the energy ratios $\epsilon_i(J)$ depend

Experimental results for the ratios ϵ

Nucleus	$E_1(2)$ (kev)	ξ	$\varepsilon(3)$	$\varepsilon_1(4)$	$\varepsilon_2(4)$	$\varepsilon_1(6)$
Os^{190}[4,6]	186.7	2.99	4.04	2.94	5.12	5.61
Mg^{24}[4]	1368	3.09	3.82	3.01	4.61	—
Fe^{56}[7]	845	3.49	4.54	2.47	4.85	—
Os^{188}[5]	155	4.09	5.10	3.08	6.17	—
Os^{186}[5]	137.2	5.60	6.63	3.16	7.73	6.33
Sm^{152}[5]	122.3	8.92	10.14	3.01	11.68	—
Er^{166}[8]	80.7	9.76	10.67	3.29	11.87	6.76
Er^{168}[8]	79.9	10.29	11.22	3.31	12.47	6.86
Dy^{160}[9]	87.0	11.16	12.11	3.27	13.35	6.70
Gd^{156}[4,5]	89.0	13.01	14.0	3.24	15.34	6.56
W^{182}[5]	100.9	12.11	13.20	3.26	14.68	—

only weakly on η, at least for values of η which actually occur in nuclei. This is especially true for $\xi \geq 4$. Almost all of the experimental values for $\epsilon_1(4)$ and $\epsilon_1(6)$ now known to us (see the table) lie below the cross hatched area in Figs. 1 and 2, which represents the theoretically allowed region in the adiabatic approximation. The experimental values $\epsilon_2(4)$ are less than the theoretically predicted ones for all values of η satisfying the inequalities (5). This shows that the agreement between theory and experiment cannot be improved by modifying the moments of inertia so that they lie somewhere between the hydrodynamic ones and those corresponding to a rigid body. Furthermore, it follows from Fig. 1 that if we formally pick those ratios between the three principal moments of inertia which minimize the disagreement between theory and experiment for the ratios $\epsilon_1(4)$, then we shall have automatically worsened the agreement for $\epsilon_2(4)$. We thus conclude that the difference between theory and experiment is due to the use of the adiabatic approximation in the calculations.

Our results lead us to hope that corrections for the interaction between the rotation and the internal state of a nucleus can be made using the hydrodynamic-model dependence of the moments of inertia on the parameter γ, which describes the departure of the nucleus from axial symmetry. The values of the relative energies $\epsilon_1(4)$, $\epsilon_2(4)$ and $\epsilon_1(6)$ are much less sensitive to departures of the parameter η from its hydrodynamic value than they are to deviations from the adiabatic condition.

[1] A. S. Davydov and G. F. Filippov, JETP **35**, 440 (1958), Soviet Phys. JETP **8**, 303 (1959); Nucl. Phys. **8**, 237 (1958).

[2] A. S. Davydov and V. S. Rostovskiĭ, JETP **36**, 1788 (1959), Soviet Phys. JETP **9**, 1275 (1959); Nucl. Phys. **12**, 58 (1959).

[3] N. MacDonald, Nucl. Phys. **14**, 70 (1959).

[4] B. S. Dzhelepov and L. K. Peker, Схемы распада радиоактивных ядер (Decay Schemes for Radioactive Nuclei), U.S.S.R. Acad. Sci., 1958.

[5] B. S. Dzhelepov and L. K. Peker, Возбужденные состояния радиоактивных ядер (Excited States of Radioactive Nuclei), preprint RP-218, Joint Inst. Nuc. Res.

[6] Nielsen, Poulsen, Sheline, and Jensen, Nucl. Phys. **10**, 475 (1959).

[7] S. Cook, Nucl. Phys. **7**, 480 (1958).

[8] Jacob, Mihelich, Harmatz, and Handley, Bull. Am. Phys. Soc. **3**, 358 (1958).

[9] E. P. Grigor'ev and M. P. Avotina, Izv. Akad. Nauk SSSR, Ser. Fiz. **24**, 324 (1960), Columbia Tech. Transl., in press.

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