COHERENT ELECTRON RADIATION IN A SYNCHROTRON. III

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Submitted to JETP editor September 30, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 1183-1187 (April, 1960)

The electromagnetic interaction of electrons in a synchrotron is considered with the shielding effect of the chamber walls taken into account for bunches of arbitrary shape. The effect of these forces on the phase motion of the electrons and on the dimensions of a bunch are evaluated.

In earlier papers[1,2] we have considered the coherent radiation forces in a synchrotron and have evaluated the effect of these forces on the phase motion of the electrons. However, in this earlier work the shielding effect of the walls of the synchrotron vacuum chamber was neglected; furthurmore, it was assumed that the bunch moves in an infinite free space. In the present paper we consider the same problems but take the shielding effect into account. The shielding effect of the walls is introduced as an approximation: it is assumed that the bunch moves close to an infinite ideally conducting plane or between two such planes. Under these conditions it is convenient to use the method of images.

In particular, the force which acts on a single electron rotating at a distance b above a shielding plane is equal to the force exerted by a "positron" which rotates in synchronism with the electron at a distance b below the plane. With the method which we have developed earlier[1] it is an easy matter to find the interaction forces in a dipole of this kind. We assume that $p \equiv b/a \ll 1$, where a is the radius of the orbit and expand these forces in powers of p^2. Then, for the tangential force we have

$$f_\tau \approx \frac{2}{3}\frac{e^2}{a^2}\gamma^4(1-2p^2\gamma^4\ldots) \qquad (p \ll 1/\gamma^2), \tag{1}$$

$$f_\tau \approx \frac{e^2}{a^2}\left(\frac{\sqrt{3}}{4}\frac{1}{\gamma^2 p^3} - \frac{\sqrt{3}}{10}\frac{1}{p}\cdots\right) \qquad (p \gg 1/\gamma^2); \tag{2}$$

for the vertical force

$$f_z \approx -\frac{e^2}{\gamma a^2}\frac{1}{(2p)^2} \qquad (p \ll 1/\gamma^2), \tag{3}$$

$$f_z \approx -\frac{e^2}{a^2}\frac{3^{3/4}}{8p^{3/2}} \qquad (p \gg 1/\gamma^2), \tag{4}$$

$$\gamma = (1-\beta^2)^{-1/2}, \quad \beta = v/c.$$

The force given by (1) is balanced by radiation dissipation so that when $p \ll 1/\gamma^2$, i.e., when $b \ll a/\gamma^2$, there is essentially no radiation from the dipole. At first glance this result may appear strange. It would appear that the dipole does not radiate when the distance between the charges, 2b, is small compared with the wavelength at which maximum radiation occurs: $\lambda\!\!\!^{-}_{max} = 2a/3\gamma^3$. Actually, because of the directivity of the radiation this effect comes into play earlier. As is well known, the radiation at the n-th harmonic is concentrated in a cone with opening angle $\alpha \sim n^{-1/3}$. Hence the difference in the path length of waves which emanate from the electron and the positron is $\Delta \sim b\alpha \sim bn^{-1/3}$. For a given value of b, all harmonics with wavelengths $\lambda \gg \lambda_b \equiv b\sqrt{b/a} \equiv ap^{3/2}$ cancel because the charges are of opposite sign. Since $\lambda\!\!\!^{-}_{max} = 2a/3\gamma^3$, all harmonics cancel when $b \ll a/\gamma^2$, i.e., when $p \ll \gamma^{-2}$.

We now consider the interaction forces in a bunch. The coherent radiation force which acts on an individual electron in an unshielded bunch is given approximately by the following expression[1]

$$f_{\tau\,\mathrm{coh}}(\psi) \approx -\frac{2}{3^{1/3}}\frac{Ne^2}{a^2}\frac{d}{d\psi}\int_0^\infty \frac{\varphi(\psi - x)}{x^{1/3}}\,dx, \tag{5}$$

$$1 \gg \vartheta_0 \gg 1/\gamma^3, \qquad \vartheta_0^{4/3} \gg (\sigma_0/a)^2. \tag{6}$$

Here ϑ_0 is the effective angular dimension of the bunch; σ_0 is the effective cross sectional radius of the bunch; N is the number of electrons in a bunch; ψ is the phase (azimuth) of the electron being considered; $\varphi(\psi)$ is the phase distribution of the electrons in the bunch. In the derivation of Eq. (5) we consider only the interaction of the electron in question with the part of the bunch which is behind it. The interaction with charges in front of the electron can be neglected. This procedure is valid because the interaction forces are not symmetrical forward and backward since the radiation is highly directive. The maximum value of the force (5) is of order $Ne^2/a\vartheta_0^{4/3}$.

Using the same method as that employed in the derivation of Eq. (5), we find that the coherent radiation force acting on an individual electron in a bunch which rotates over a single shielding plane is given approximately by

$$f_{\tau\,\mathrm{coh}}(\psi, p) \approx \frac{2}{3^{2/3}} \frac{Ne^2}{a^2} p^2 \frac{d^2}{d\psi^2} \int_0^\infty \frac{\varphi(\psi - x)}{x^{2/3}} dx. \tag{7}$$

In the derivation of Eq. (7) it is assumed that

$$1 \gg \vartheta_0 \gg 1/\gamma^3, \qquad 1 \gg p \gg 1/\gamma^2, \qquad \vartheta_0^{4/3} \gg p^2, \qquad \sigma_0/a \ll p. \tag{8}$$

It is further assumed that the phase distribution of electrons in the bunch $\varphi(\psi)$ is a smooth function. The smoothness criterion is given below. As in the derivation of Eqs. (5) and (7), account is taken only of the interaction between the electron being considered and the charges behind it. Hence, Eqs. (7) and (5) cannot be used behind the bunch.

The condition $1 \gg p \gg 1/\gamma^2$ can be written in the form $a \gg \lambda_b \gg \lambdabar_{max}$ which implies that the short wave radiation of each individual electron must be unshielded. The condition $\vartheta_0^{4/3} \gg p^2$ means that the dimension of the bunch $a\vartheta_0$ must be large compared with $\lambda_b \equiv ap^{3/2}$. The coherent radiation of a small unshielded bunch is concentrated in the region of wavelengths which are of the order of the dimensions of the bunch; hence, the condition $a\vartheta_0 \gg \lambda_b$ means that the shielding is strong, i.e., that a large part of the coherent radiation is shielded. The ratio $p^2/\vartheta_0^{4/3}$ characterizes the fraction of unshielded coherent radiation.

In Fig. 1 we show the dependence of the force (7) on azimuth ψ for a bunch of Gaussian shape. It is assumed that the bunch moves to the right. The maximum value of this force is of order $Ne^2p^2/a^2\vartheta_0^{8/3}$.

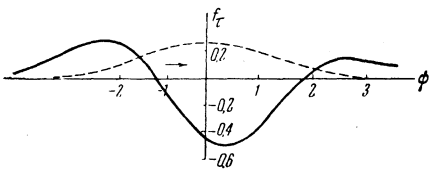

FIG. 1. The coherent force f_τ (in units of $Ne^2p^2/a^2\vartheta_0^{8/3}$, acting on a single electron of a Gaussian shielded bunch as a function of azimuth ψ (in units of ϑ_0). The dashed curve shows the phase distribution of particles in the bunch.

The power expended by the forces (5) and (7) in acting on the bunch is equal to the power of the coherent radiation (with opposite sign) of the unshielded bunches respectively. Integrating (5) and (7) over a bunch we find that these powers are of order $N^2e^2c/a^2\vartheta_0^{4/3}$ and $N^2e^2cp^2/a^2\vartheta_0^{8/3}$ respectively. In the particular case in which the bunch is Gaussian, using Eq. (7) we obtain an expression for the power which coincides exactly with that obtained by Schiff;[3] Schiff's result is obtained by a phase analysis and summation of the intensities of the harmonics in the spectrum.

The Coulomb part of the total tangential force which acts on a single electron in a bunch differs from the corresponding expression for Coulomb force in an unshielded bunch[1] only in the logarithmic term which contains the shielding parameter $p \equiv b/a$; this component is approximately

$$f_{\tau\,\mathrm{Coul}}(\psi, p) \approx -\frac{2}{\gamma^2} \frac{Ne^2}{a^2} \ln\left(2e\frac{b}{\sigma}\right) \varphi'(\psi), \tag{9}$$

where σ is the cross sectional radius of the bunch. Hence, shielding is less important for the Coulomb force than for the coherent radiation force.

Although shielding acts mainly to reduce the coherent force, the Coulomb force can be smaller than the coherent force when $\gamma \gg 1$. From a comparison of the orders of magnitude of the quantities in (7) and (9) it follows that the Coulomb force can be neglected (as compared with the coherent force) if

$$(p\gamma)^2 \gg \vartheta_0^{2/3} \ln(b/\sigma). \tag{10}$$

Equation (7) applies only when a bunch is smooth; it cannot be used if a bunch has highly irregular sections or sharp ends. Hence we consider the case of a rectangular bunch separately:

$$\varphi(\psi) = \begin{matrix} 1/4\vartheta_0, & |\psi| < 2\vartheta_0; \\ 0, & |\psi| > 2\vartheta_0. \end{matrix}$$

We will not derive general expressions for the forces and radiation power because these expressions are not particularly illuminating, but shall only give certain results. The coherent forces for a rectangular bunch over a single shielding plane for the case $p = 0.1$ and $\vartheta_0 = \pi/8$ are shown in Fig. 2. It is assumed that the bunch moves to the right. It is apparent that the regions close to the ends of the bunch are the most important.

The force which acts on an inner portion of order $b\sqrt{b/a} \equiv ap^{3/2}$ close to the front of the bunch is given approximately by

$$f_\tau \approx -\frac{Ne^2}{a^2} \frac{1}{4\vartheta_0} \frac{3}{2(\sqrt{3}p)^{1/2}}. \tag{11}$$

At a distance $a\eta$ from the end, inside the bunch, the force is

$$f_\tau \approx -\frac{Ne^2}{a^2} \frac{1}{4\vartheta_0} \frac{2}{|6\eta|^{1/3}} \qquad (0 < \eta < p^{3/2}). \tag{12}$$

The same situation obtains for a bunch between two shielding plates; the power expended by these forces in acting on the end portions of a bunch of

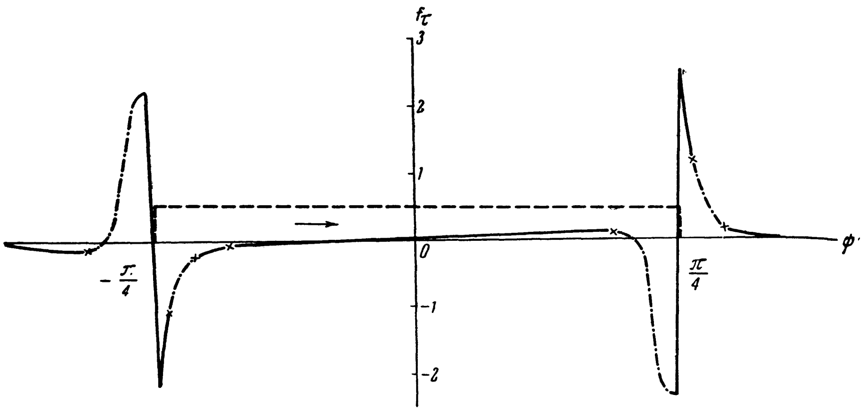

FIG. 2. The coherent force f_τ (in units of Ne^2/a^2) acting on an individual electron in a rectangular bunch as a function of azimuth ψ for the particular case $\vartheta_0 = \pi/8$, $p = 0.1$. The dashed line shows the phase distribution of particles in the bunch.

width of order λ_b is

$$W_{\text{coh}} \approx \frac{N^2e^2c}{a^2}\frac{\sqrt{3}\,p}{(4\vartheta_0)^2}. \qquad (13)$$

This result is in agreement with that obtained by Schwinger (cited by Nodvick and Saxon);[4] Schwinger's expression is obtained by summation of the wave-zone radiation over the entire spectrum. The Schwinger method, however, cannot be used to compute the effect of individual portions of the bunch in the general expression for the radiation power. Nodvick and Saxon[4] assume that the Schwinger expression (13) can be applied to a bunch of arbitrary shape. Actually, however, this procedure is not valid because the radiation (13) comes from the ends of the bunch; the radiation is in fact due to the existence of sharp ends in the bunch.

A bunch may be assumed smooth if there is no marked variation in charge density in a distance large compared with $\lambda_b \equiv b\sqrt{b/a}$. Under these conditions Eq. (7) can be used. If this condition is not satisfied the expressions for the forces in a rectangular bunch can be used as an approximation.

In practice, because of the coherent forces it is probable that there are considerable charge-density gradients near the front of a bunch. However, because of the same forces high density gradients cannot exist for long at the rear of the bunch; any sharp density variation is spread out so that the bunch always has a smeared-out tail. In what follows it will be assumed that the bunch is smooth and that Eq. (7) can be used.

We now estimate the effect of the coherent forces (5) and (7) on the phase motion of electrons in a "cumulative" system. This effect is intensified to the extent that the angular dimension of a bunch is reduced because of the usual incoherent radiation damping of the phase oscillations. Hence the coherent forces limit the dimension to which a bunch can be compressed without an external agency. The minimum angular dimension of a bunch due to a force such as (5) when there is no shielding is of the following order of magnitude:[2]

$$\vartheta_0 \sim (2\pi Ne/aV)^{3/7}, \qquad (14)$$

where V is the peak value of the radio-frequency voltage. Equation (14) is easily obtained from the condition that close to the rear of the bunch the phasing electric force is comparable with the coherent force which tends to disturb phase stability.

Similarly, when shielding is taken into account, i.e., when the forces in (7) are considered, this dimension is of the order of

$$\vartheta_0 \sim (2\pi Nep^2/aV)^{3/11}. \qquad (15)$$

The estimates in (14) and (15) apply for the same conditions as Eqs. (5) and (7) respectively.

Substituting in Eq. (14) $a = 50$ cm, $V = 10$ kev, and assuming $N = 10^{13}$, we find $\vartheta_0 \sim 3.5$. This result means that because of the coherent radiation force a bunch will not be small in the absence of a shielding wall; however, in this case (14) does not apply. Substituting the same parameters in (15) and assuming that $p = 0.05$, we find $\vartheta_0 \sim 0.4$.

The vertical force due to the shielding planes is directed toward the planes and leads to an instability. In the case of a closed current ring this force is given approximately by

$$f_z \approx \frac{Ne^2}{a^2}\frac{z}{a}\left\{\frac{1}{\gamma^2}\frac{\pi}{8p^2} + \beta^2 \ln\frac{1}{p}\right\}, \qquad (16)$$

where z is the displacement of the bunch from the median plane. It is assumed that $|z| \ll b$. Using reasonable values we find that the force in (16) is considerably smaller than the magnetic focusing force and need not be considered in practice.

[1] L. V. Iogansen and M. S. Rabinovich, JETP **37**, 118 (1959), Soviet Phys. JETP 10, **83** (1960).

[2] L. V. Iogansen, JETP **37**, 299 (1959), Soviet Phys. JETP **10**, 211 (1960).

[3] L. Schiff, Rev. Sci. Instr. **17**, 6 (1946).

[4] J. S. Nodvick and D. S. Saxon, Phys. Rev. **96**, 180 (1954).

Translated by H. Lashinsky
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