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ALLOWANCE FOR THE MEDIUM IN RADIATION CORRECTIONS TO COULOMB SCATTERING

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Radiation corrections to electron scattering are treated with allowance for the "density effect." When condition (5) holds, the corrections due to the medium become important and are given by formula (4).

As already pointed out by the author,[1] the polarization effect in bremsstrahlung must affect the calculation of radiation corrections to Coulomb scattering. The reason for this effect can be easily understood from the following considerations. Experimentally the sum of two cross sections is always measured — the elastic Coulomb cross section, and the bremsstrahlung cross section for quanta of energy lower than some specified energy $h\omega_{min}$ determined by the resolving power of the experimental apparatus.

Taking this circumstance into account enables us to avoid in the theoretical calculations the difficulties associated with the infrared divergence, which renders invalid the calculation of a purely elastic scattering cross section. However, the bremsstrahlung formulas for relativistic particles undergo considerable changes, due both to multiple scattering and to the polarization of the medium. It is evident that these changes in the bremsstrahlung cross section will also affect the calculation of radiation corrections to Coulomb scattering, since, as just noted, we actually always have to calculate the sum of two cross sections. Simple addition of the new bremsstrahlung cross section and of the purely elastic scattering cross section (with the radiation corrections evaluated for the case of vacuum) would be inconsistent.

Obviously, it is necessary to take into account the effect of the medium also in evaluating the radiation corrections themselves. Let us investigate the effect of the polarization of the medium. In the case of elastic scattering which is of interest to us there are no photons both at the beginning and at the end of the process, so that the whole effect of the polarization of the medium reduces to a change in the photon propagator.

In contrast to the vacuum case, a photon moving in a medium undergoes repeated absorptions and emissions. To obtain the altered propagator we have to sum a number of Feynman diagrams in which each succeeding diagram differs from the preceding one by an additional absorption and emission of the photon with no change of frequency. In this way we obtain a geometric progression whose sum yields the desired propagator. The results of calculations using such an altered propagator must then be averaged over the states of all the atoms. This procedure of calculation can be given a simple graphic interpretation by noting that such a method of calculation is in a certain sense equivalent to the transition from the microscopic to the macroscopic Maxwell's equations.[2] To obtain the rules for writing down the matrix elements, taking the effect of the medium into account, it is convenient to utilize the following formal method.* We write the macroscopic Maxwell's equations for the potentials in four-dimensional invariant form (we shall write them in the Mandel'shtam-Tamm form which is completely equivalent to the usual one)

$$\varepsilon^{ik}\varepsilon^{lm}\partial^2 A_k/\partial x^l \partial x^m = J^i, \qquad \varepsilon^{lm}\partial A_l/\partial x^m = 0. \quad (1)$$

$\epsilon^{ik} = \epsilon^{ki}$ is the dielectric permittivity and magnetic permeability tensor introduced by Mandel'shtam and Tamm, with non-zero components $\epsilon^{00} = \epsilon\sqrt{\mu}$ and $\epsilon^{\alpha\alpha} = -1/\sqrt{\mu}$ in the rest system of the medium ($\alpha = 1, 2, 3$); $\epsilon^{ik} = 0$ for $i \neq k$. The transition to the vacuum case consists of replacing the tensor ϵ^{ik} by the metric tensor g^{ik} ($g^{00} = 1$;

*Ryazanov[3] has utilized similar rules for writing down the matrix elements on the basis of quantization of the macroscopic Maxwell's equations. However, as has been brought out in a discussion with K. M. Polievktov-Nikoladze, E. L. Feĭnberg and M. I. Ryazanov, quantum electrodynamics in a medium must be used with a certain amount of caution, and in each specific case must be justified on the basis of the microscopic quantum equations.

$g^{\alpha\alpha} = -1$ ($\alpha = 1, 2, 3$); $g^{ik} = 0$ for $i \neq k$).

Following Feynman's arguments[4] we arrive at the conclusion that the change that must be introduced in order to take into account the effect of the medium reduces to replacing the photon propagator by $D = -ihc/(2\pi)^4 \epsilon^{lm} k_l k_m$. The ends of the internal photon line will now correspond to the matrices $ie\gamma^i$ and $ie\epsilon^{-1}_{\mu i}\gamma^\mu$. Let us make use of this formal method for evaluating the electron self-energy. From physical considerations it follows that an electron in a medium will polarize the material. Moreover, the use of the ordinary dielectric constant will lead to the damping of the initial electron wave function due to energy losses. If we represent the electron wave function in the form $\exp(iEt/h - \gamma t/2)$, then we obtain for the quantity $i(E - E_v)/h - \gamma/2$ (where E_v is the electron self-energy in vacuum) the expression (for $v \ll c$):

$$\gamma = -\frac{e^2}{2\pi^2 hv} \operatorname{Im} \int_0^{k_{max}} \frac{dk}{k} \int_0^{kv} \frac{d\omega}{\varepsilon(\omega)},$$

$$\Delta E = -\frac{e^2}{4\pi^2 v} \int_0^{k_{max}} \frac{dk}{k} \int_{-kv}^{+kv} d\omega \left(\frac{1}{\varepsilon(\omega)} - 1\right). \quad (2)$$

From the quantity γ we can easily obtain the energy losses of a nonrelativistic particle by multiplying the integrand by $h\omega$. The quantity ΔE is the polarization energy of the medium (cf. reference 5). It is also necessary to note that the use of the usual value of $\epsilon(\omega)$ is justified for distances larger than interatomic distances. Therefore γ determines the energy losses for "distant" collisions. It is natural that in order that the method of summation of diagrams should be equivalent to the method using macroscopic equations we must make the same approximation in the former method of calculation.

We return to the problem of the radiation corrections to scattering by a Coulomb field. By following the usual calculation procedure we must, in order to avoid the infrared catastrophe, also consider the bremsstrahlung cross section for soft quanta of frequency ω. The latter formula can be easily obtained if we evaluate the emission of radiation for a given deflection of the electron classically. The scattering probability is given by the Coulomb scattering cross section $d\sigma_C$. The formula has the form

$$d\sigma = d\sigma_C \frac{2\alpha}{\pi\sqrt{\varepsilon}} \{2\Phi \coth 2\Phi - 1\} \frac{d\omega}{\omega},$$

$$\sinh\Phi = \frac{v}{c}\sqrt{\varepsilon} \frac{\sin(\theta/2)}{\sqrt{1 - \varepsilon v^2/c^2}}; \quad (3)$$

θ denotes the scattering angle of the electron. In evaluating the elastic scattering cross section up to terms of order e^6 we must take the absolute value of the square of the sum of the matrix elements of second and fourth order with respect to e. It should be noted that in evaluating the second-order matrix element (simple Coulomb scattering) we can neglect the effect due to the neighboring atoms, since the scattering occurs in the region inside the atom. Since the correction terms contain the fourth-order matrix element linearly, we can use in the matrix elements the photon propagator averaged over the states of the system, which for small momenta (large distances) coincides with the Green's function for Eq. (1). The infrared divergence in the corrections cancels the integral in (3) over ω from $\omega = 0$ to $\omega = \omega_{min}$. We assume that $\omega_{min} \gg \omega_{atom}$ and utilize the limiting value $\epsilon(\omega) = 1 - 4\pi NZe^2/m\omega^2$. The change in the radiation corrections in the medium will be associated with formula (3) in which, in spite of the fact that $\epsilon \approx 1$, it is necessary to take the medium into account when $1 - v^2/c^2 \approx 4\pi NZe^2/m\omega^2$. Finally, an additional term will appear in the radiation corrections to Coulomb scattering which is associated with the effect under consideration. The formula for the scattering cross section in second order perturbation theory has the form

$$d\sigma = d\sigma_v(\omega_{min}) - d\sigma_C \frac{4\alpha}{\pi} \ln^2 \frac{m^{3/2}c^2\omega_{min}}{E\sqrt{4\pi N Ze^2}}, \quad (4)$$

where $d\sigma_v$ is the ordinary differential scattering cross section including radiation corrections in the case of vacuum. Formula (4) is valid for scattering angles

$$\theta > \sqrt{1 - v^2/c^2 + 4\pi NZe^2/m\omega^2}$$

and under the condition

$$\omega_{min} \ll \sqrt{4\pi NZe^2/m}\, E/mc^2. \quad (5)$$

In the opposite case the effect of the medium is unimportant. It is possible to neglect the effect of multiple scattering on the calculation under discussion.

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[2] V. B. Berestetskiĭ, JETP **8**, 148 (1938).

[3] M. I. Ryazanov, JETP **32**, 1244 (1957), Soviet Phys. JETP **5**, 1013 (1957).

[4] R. P. Feynman, Phys. Rev. **76**, 769 (1949).

[5] J. Lindhard, Niels Bohr and the Development of Physics, edited by W. Pauli, (Russ. Transl.), 1958, p. 244.

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