# CONTRIBUTION TO THE THEORY OF ABSORPTION OF ULTRASOUND IN METALS

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The absorption of sound is considered in the low-temperature region, where absorption is due to electrons, and for frequencies sufficiently high so that the acoustic wavelength is small in comparison with the mean free path. The anisotropy of the Fermi surface is taken into account, as are interactions due to the electromagnetic field and to the variation in electron energy as a result of lattice deformations. The acoustic absorption is determined. In the region of wavelengths close to the thickness of the anomalous skin layer at the acoustic frequency, it is shown that a decrease in the ratio of the absorption coefficient to the frequency should be observed with increasing frequency.

**1.** It is well known that in the low-temperature region the absorption of sound in metals is due to the electrons. Moreover, although it is reasonable to discuss the usual picture of absorption as due to an electronic viscosity in the region where the wavelength $\lambda\!\!\!^{-}$ is considerably greater than the electron mean free path $l$, this is not at all true of the opposite case of short waves.

In what follows we consider precisely the latter case. In the limit of sufficiently short waves, the mean free path can be considered infinite. Therefore the processes that occur under such conditions are similar to those in a rarefied electron gas. The damping of sound waves in this case is analogous in nature to the well known damping of plasma waves, first discovered by Landau,[1] and is due to the possibility of the absorption and emission of acoustic waves by electrons moving in phase with the waves. It is plain, therefore, that the consideration of the absorption of sound in the short wavelength region ($\lambda\!\!\!^{-} \ll l$) should lead to corresponding analogous considerations in an electron gas. This is exactly how the decay of longitudinal sound waves was treated in a paper by the author,[2] in which a linear dependence of the absorption upon frequency was obtained. Pippard[3] developed a method which is also applicable to the case $\lambda\!\!\!^{-} \ll l$ and gives a description of the absorption of both longitudinal and transverse sound waves.

The limitations of references 2 and 3 are, first, that they refer to electronic models with spherical Fermi surfaces and, secondly, that they do not take into account the variation of the electron energy under the influence of the sound wave.[4] The next step was therefore to consider a theory of sound absorption using a more realistic model. In spite of the fact that such attempts have been made, it must be admitted that at the present time there is not a single theory of sound absorption which is consistent and free from the deficiencies of references 2 and 3.

In our view, the reason for this state of affairs is that up to now the peculiarities of sound absorption in the region of indefinitely long free paths have not been sufficiently realized. Only in this way, for example, is it possible to explain the attempts to calculate sound absorption in this region with the aid of the heat release formula $Q = T\,dS/dt$, where $S$ is the entropy of the electrons. When the mean free paths of the electrons are infinitely long, it is clear that their entropy is conserved, and therefore if the above formula is used correctly in the calculations, nothing can result.

In what follows we shall put forward a theory of sound absorption which, in the first place, does not make use of the rate of change of electron entropy or of collision integrals (which are equal to zero in our case of infinitely long free paths) in determining the absorption; which, in the second place, is valid for any arbitrary Fermi surface having a center of symmetry; and which, thirdly, takes into account not only the electromagnetic field arising from the passage of the sound wave, but also the variation in electron energy as a result of the lattice deformation. Finally, the idea of a complex modulus of elasticity tensor will be introduced, and in Appendix A the rotation of the plane of polarization of sound will be considered from this point of view.

**2.** Because of the deformation that occurs in a

lattice upon the passage of a sound wave, the energy of the electrons is altered:[4]

$$\varepsilon(\mathbf{p}, \mathbf{r}) = \varepsilon(\mathbf{p}) + \Lambda_{ik}(\mathbf{p})\, \partial u_i/\partial x_k. \tag{1}$$

Here $\epsilon(\mathbf{p})$ is the energy of an electron in the absence of the deformation, $\mathbf{u}$ is the deformation vector, and $\Lambda_{ik}(\mathbf{p})$ is a tensor depending on the quasi-momentum of the electron. Using (1), one can write the following kinetic equation for the electrons:

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \left\{ e\left(\mathbf{E} + \frac{1}{c}\,[\mathbf{v}\times\mathbf{H}]\right) - \Lambda_{ik}\,\frac{\partial^2 u_i}{\partial \mathbf{r}\,\partial x_k}\right\}\frac{\partial f}{\partial \mathbf{p}} = 0. \tag{2}$$

Here collisions have been neglected and it has also been assumed that, in addition to the Lorentz forces, there is a force acting on the electrons due to the dependence of the electron energy on the lattice deformation. Furthermore, formula (1) yields for the velocity $\mathbf{v}$

$$\mathbf{v} \equiv \frac{\partial \varepsilon(\mathbf{p}, \mathbf{r})}{\partial \mathbf{p}} = \frac{\partial \varepsilon}{\partial \mathbf{p}} + \frac{\partial \Lambda_{ik}}{\partial \mathbf{p}}\,\frac{\partial u_i}{\partial x_k}. \tag{3}$$

In order to determine the electromagnetic field we need to know the charge density and current density of the electrons. These are found in the usual form, with the aid of the solutions of (2) and the velocity formula (3):

$$\rho_e = e\int d\mathbf{p} f, \qquad \mathbf{j}_e = e\int d\mathbf{p}\, f\mathbf{v}.$$

We describe the lattice vibrations by the equation

$$\rho_m \ddot{u}_i = \lambda^{(0)}_{iklt}\,\frac{\partial^2 u_j}{\partial x_k \partial x_l} + \rho_p E_i + \frac{1}{c}\,[\mathbf{j}_l\times\mathbf{H}]_i + \frac{\partial}{\partial x_k}\int \Lambda_{ik} f\, d\mathbf{p}. \tag{4}$$

Here $\rho_m$ is the density of the material in the lattice, $\lambda^{(0)}_{ijkl}$ is the modulus of elasticity tensor, and $\rho_l$ and $\mathbf{j}_l = \rho_l\dot{\mathbf{u}}$ are the charge density and current density of the lattice. The final term on the right-hand side of (4) represents the vibration-electron interaction, which appears in (2).

It should be mentioned that the consideration of the electric field, which in itself leads to an interaction between the electrons and the lattice, and also the consideration of the interaction due to the variation of electron energy in the sound-wave field (2), lead to the appearance of an additional elasticity. In other words, the interactions under consideration lead to a renormalization of the elastic-modulus tensor, and consequently to a renormalization of the speed of sound. Thus, in fact, in Eq. (4), $\lambda^{(0)}_{ijkl}$ must be interpreted as the renormalized elasticity tensor. Note that the appearance of similar renormalizations in the modulus, even when electric interactions alone are considered, is evident from references 2 and 5, in which no elasticity is taken into account except what results from the electrical forces acting between the electrons and the ions. In a model that takes into account an interaction of the type (1), such as Fröhlich's model,[6-8] the occurrence of renormalization in the speed of sound is also well known.

The electromagnetic field is described by Maxwell's equations. For wavelengths large in comparison with the radius of the longitudinal screening field, the equation

$$\rho_e + \rho_l = 0 \tag{5}$$

can be used in place of the equation $\operatorname{div}\mathbf{E} = 4\pi(\rho_e + \rho_l)$. Accordingly, if we neglect the displacement current, we obtain from Maxwell's system of equations

$$\operatorname{curl}\operatorname{curl}\mathbf{E} = -\frac{4\pi}{c^2}\frac{\partial}{\partial t}(\mathbf{j}_e + \mathbf{j}_l). \tag{6}$$

Equations (2), (4), (5), and (6) constitute a system which permits us to describe the sound absorption in which we are interested.

**3.** The subsequent treatment must be devoted to the elimination of all quantities except the lattice-vibration variables from the above system of equations. For this purpose Eq. (4) can be written as

$$\ddot{u}_i\rho_m = \partial\sigma_{ij}/\partial x_j, \tag{4'}$$

where

$$\sigma_{ij}(\mathbf{r}, t) = \int_{-\infty}^{t} dt' \int d\mathbf{r}\, \hat{\lambda}_{ij\,lm}(t - t', \mathbf{r} - \mathbf{r}')\,\frac{\partial u_l(\mathbf{r}', t)}{\partial x'_m}. \tag{7}$$

The relationship (7) can be considered as a generalization of the usual connection between the stress tensor and the strain to the case of dispersion of the elastic modulus. In treating the propagation and absorption of sound it is convenient to use the quantity

$$\lambda_{ij\,lm}(\omega, \mathbf{k}) = \int d\mathbf{r}\, e^{-i\mathbf{kr}}\int_0^\infty dt e^{i\omega t}\hat{\lambda}_{ij\,lm}(t, \mathbf{r}) = \lambda'_{ij\,lm} + i\lambda''_{ij\,lm}, \tag{8}$$

which we shall call the complex modulus of elasticity tensor. In the problem of sound absorption in metals, which is treated in this paper, the dispersion of the real part $\lambda'$ proves to be extremely small, and will not be considered. On the other hand, the dispersion of the imaginary part $\lambda''$, both in time and in space, is of considerable importance [see Eq. (16)].

With the aid of (4') it is easy to show that the energy lost by the sound wave per unit time is given by

$$Q = {}^1/_4 i\omega\,(\lambda_{ij\,lm} - \lambda^*_{lm\,ij})\,k_j k_m u_i^* u_l. \qquad (9)$$

According to Onsager's symmetry relations, we have also

$$k_j k_m \lambda_{ij\,lm}(\mathbf{H}_0, \omega, \mathbf{k}) = k_j k_m \lambda_{lm\,ij}(-\mathbf{H}_0, \omega, -\mathbf{k}) \qquad (10)$$

($\mathbf{H}_0$ — constant magnetic field).

In what follows we shall determine the complex modulus of elasticity tensor within the framework of the model outlined in Sec. 2.

**4.** We take the decrement of the acoustic damping to be small compared with the frequency. In particular, this will enable us to assume a time dependence of the form $\exp(-i\omega t)$ in all equations except (4). If we then assume that there is no constant field, and take the coordinate dependence to be of the form $\exp(-i\mathbf{k}\cdot\mathbf{r})$, we obtain with the aid of (2) the following expression for the nonequilibrium addition to the electron distribution function:

$$\delta f = \frac{\partial f_0}{\partial \varepsilon}\left(e\frac{\partial \varepsilon}{\partial \mathbf{p}}\mathbf{E} + \Lambda_{jl}k_l u_j \mathbf{k}\frac{\partial \varepsilon}{\partial \mathbf{p}}\right)\left(\mathrm{P}\,\frac{i}{\mathbf{k}\,\partial\varepsilon/\partial\mathbf{p} - \omega} - \pi\delta\left[\mathbf{k}\frac{\partial \varepsilon}{\partial \mathbf{p}} - \omega\right]\right). \qquad (11)$$

The occurrence of the delta function $\delta_+(\omega - \mathbf{k}\partial\epsilon/\partial\mathbf{p})$ corresponds to adiabatically turning on the interaction an infinite time ago.*

We have at our disposal a parameter of smallness given by the ratio of the speed of sound to the electron velocity at the Fermi surface. If we limit the expansion in terms of this parameter to the most important terms, after substituting (11) into (5) and (6), we obtain equations for the transverse field $\mathbf{E}^\perp$ and the longitudinal field $\mathbf{E}\cdot\mathbf{k}/\mathrm{k}$:

$$E_\alpha^\perp = ie\omega B^{-1}_{\alpha\beta}\left\{u_j k_l \left\langle \frac{v^\perp_{0\beta}}{\mathbf{k}v_0}\left(\Lambda_{jl} - \frac{\langle\Lambda_{jl}\rangle}{\langle 1\rangle} - \frac{N\delta_{jl}}{\langle 1\rangle}\right)\right\rangle + N u^\perp_\beta\right\}, \qquad (12)$$

$$\frac{e\mathbf{k}\mathbf{E}}{k^2} = -\frac{u_j k_l}{\langle 1\rangle}\langle\Lambda_{jl} + N\delta_{jl}\rangle - \frac{e\,\mathbf{E}^\perp}{\langle 1\rangle}\left\langle\frac{\mathbf{v}_0^\perp}{\mathbf{k}v_0}\right\rangle - \frac{i\pi\omega}{\langle 1\rangle}\left\langle \delta(\mathbf{k}v_0)\left(\Lambda_{jl} - \frac{\langle\Lambda_{jl}\rangle}{\langle 1\rangle} - \frac{N\delta_{jl}}{\langle 1\rangle}\right)\right\rangle u_j k_l. \qquad (13)$$

Here $\mathbf{v}_0$ is the electron velocity, N is the number of electrons per unit volume, $\langle\ \rangle$ denotes the mean value calculated according to the formula† $\langle A\rangle = \int d\mathbf{p}\, A\, \partial f_0/\partial\epsilon$, and, finally, the tensor $B_{\alpha\beta}$ is of the form

$$B_{\alpha\beta} = -\pi e^2 \langle v^\perp_{0\alpha} v^\perp_{0\beta}\delta(\mathbf{k}v_0)\rangle - i\,(c^2k^2/4\pi\omega)\,\delta_{\alpha\beta}. \qquad (14)$$

The relationships (11), (12), and (13) enable us to represent Eq. (4) in the form (4′). When this is done,

$$k_m k_l \lambda'_{im\,jl} = k_m k_l\left\{\lambda^{(0)}_{im\,jl} + \langle\Lambda_{im}\Lambda_{jl}\rangle - \frac{1}{\langle 1\rangle}(\langle\Lambda_{im}\rangle + N\delta_{im})(\langle\Lambda_{jl}\rangle + N\delta_{jl})\right\}, \qquad (15)$$

$$k_m k_l \lambda''_{im\,jl} = \pi\omega\langle\delta(\mathbf{k}v_0)\,L_i L_j\rangle - \omega e^2\left(\left\langle\frac{v^\perp_{0\alpha}}{\mathbf{k}v_0}L_i\right\rangle + \delta_{i\alpha}N\right)\mathrm{Im}(iB^{-1}_{\alpha\beta})\left(\left\langle\frac{v^\perp_{0\beta}}{\mathbf{k}v_0}L_j\right\rangle + \delta_{j\beta}N\right). \qquad (16)$$

The summation with respect to $\alpha$ and $\beta$ on the right-hand side of (16) is carried out over the components that are perpendicular to the direction of the wave vector $\mathbf{k}$. In addition we introduce the notation

$$L_i = k_m\left(\Lambda_{im} - \frac{\langle\Lambda_{im}\rangle}{\langle 1\rangle} - \frac{N\delta_{im}}{\langle 1\rangle}\right).$$

It follows from (15) that the introduction of the interaction between the sound wave and the electrons has led to a renormalization of the elastic modulus. Furthermore, owing to the symmetry of equations (15) and (16) with respect to the indices i and j, the absorption is determined by the expression (16), according to formula (9).

**5.** We first compare the results achieved so far with some already known.

In the simplest case, where $\Lambda_{ij} = 0$ and $\lambda^{(0)}_{ijkl} = 0$, taking only the electrical forces into account, we have for an isotropic Fermi surface (this corresponds to the results of reference 2)

$$\lambda'_{im\,jl} = \delta_{im}\delta_{jl}p_0 v_0 N/3, \qquad \omega^2 = p_0 v_0 k^2 N/3\rho_m,$$

$$q^t = (\pi/2)\,(\omega^2/v_0 k).$$

Here $p_0$ and $v_0$ are the momentum and velocity of an electron at the Fermi surface, and $q^t$ is the ratio of the energy lost by the wave per unit time to the energy of the wave; the frequency $\omega$ is given by $|\,\delta_{il}\rho_m\omega^2 - k_j k_r \lambda'_{ij\,lr}\,| = 0$, as usual. We also note that in the isotropic case, when $\lambda^{(0)} \neq 0$ and $\Lambda_{ij} = 0$, the renormalization occurs only for the hydrostatic-compression modulus. Hence the velocity of transverse sound is not renormalized.

The case is quite different if the electric field is completely neglected, but the changes in energy due to lattice deformations are considered. In this case we have

$$k_j k_m(\lambda' - \lambda^{(0)})_{ij\,lm} = k_j k_m\langle\Lambda_{im}\Lambda_{jl}\rangle,$$

$$k_m k_l \lambda''_{im\,jl} = \pi\omega\langle\delta(\mathbf{k}v_0)\,\Lambda_{im}\Lambda_{jl}\rangle\, k_m k_l.$$

For the case of the Fröhlich model, considered above, $\Lambda_{ij} = C\delta_{ij}$, and we obtain

*In all that follows, $1/\mathbf{k}\cdot\mathbf{v}$ will be understood to mean its principal value.

†Here the process of integration is understood to include also summation over the zones.

$$\delta\lambda'_{imjl}k_mk_l = -3k^2\delta_{ij}\,(C^2/p_0v_0)\,N,$$

$$\delta\omega^2 = -3\,(C^2/p_0v_0)\,(N/\rho_m)\,k^2,$$

$$q^t = (3\pi/2)\,(C^2/p_0v_0)\,(N/\rho_m)\,(k/v_0),$$

which corresponds to the results of references 6 – 8.

Let us now consider the frequency dependence of the absorbed energy. To this end, we must first consider the behavior of the tensor $B_{\alpha\beta}$. The real part of the tensor $B_{\alpha\beta}$ has appeared before, in the theory of the anomalous skin effect.[9]* Its order of magnitude is Re B ~ $e^2N/p_0k$. Hence, for acoustic wavelengths satisfying the condition

$$\lambda \gg \delta \approx [p_0c/e^2N\omega]^{1/3}, \qquad (17)$$

the imaginary part of $B_{\alpha\beta}$ can be completely neglected. It should be mentioned that in the inequality (17) $\delta$ represents the depth of the skin layer corresponding to the frequency $\omega$ in the case of the anomalous skin-effect. What is important is that if condition (17) is fulfilled, both the first term on the right hand side of (16) and the second term, which depends on the transverse field, are of the same order of magnitude, generally speaking. On the other hand, in the region of acoustic wavelengths small compared with the skin depth, for ordinary metals when $\omega/2\pi > 10^9$ sec$^{-1}$, the imaginary part of the tensor $B_{\alpha\beta}$ turns out to be considerably larger than the real part. In this connection, the contribution due to the transverse field turns out to be relatively small, and the second term in (16) can be neglected.

It can thus be stated that in both the regions $\lambda \gg \delta$ and $\lambda \ll \delta$ the sound-absorption coefficient is proportional to the first power of the frequency. However, the ratio $q^t/\omega$, which is a constant for each of these regions, must in general decrease noticeably upon changing from a wavelength greater than the corresponding skin depth to one which is smaller. We note that this picture is nearly the same as the one obtained by Pippard in the isotropic model for the absorption of transverse sound. The essential difference is that in our case the absorption coefficient, generally speaking, does not remain constant when $\lambda \ll \delta$, as it does in Pippard's model. The reason for this is chiefly our allowance for the anisotropy of the Fermi surface.

*In the region of wavelength which we are considering, the inequalities $kl \gg 1$ and $kv_0 \gg \omega$, characteristic for the anomalous skin effect in the radio frequency region, are satisfied. In this case the complex dielectric constant of the metal has a spatial dispersion; in other words, it depends not only on the frequency but also on the wave vector. Under these circumstances the longitudinal component of the dielectric-constant tensor is negligibly small in comparison with the transverse components, for which the following expression is valid:

$$\varepsilon_{\alpha\beta} = -(4\pi i/\omega)\,\pi e^2 < v^{\perp}_{0\alpha}v^{\perp}_{0\beta}\delta\,(kv_0) >.$$

From this it is plain that the real part of the tensor $B_{\alpha\beta}$ is none other than the conductivity tensor of the metal, corresponding to the region of anomalous skin effect.

## APPENDIX A

## ROTATION OF THE PLANE OF POLARIZATION OF SOUND

Consider a non-absorbing medium. Then, from (9),

$$k_jk_m\lambda_{ijlm} = k_jk_m\lambda^{*}_{lmij},$$

$$k_jk_m\lambda'_{ijlm} = k_jk_m\lambda'_{lmij},\quad k_jk_m\lambda''_{ijlm} = -\lambda''_{lmij}k_jk_m.$$

For an antisymmetric imaginary part, the expressions can be rewritten in terms of the dual vector $\rho_m k^2\mathbf{G}$

$$k_jk_r\lambda''_{ijlr} = e_{irl}G_r\rho_mk^2.$$

We choose the coordinate axes along the principal axes of the tensor

$$k_jk_r\lambda'_{ijlr} = \delta_{il}\rho_m\omega_i^2.$$

(On the right hand side of this equation, as well as in the following one, (4″), there is no summation over repeated indices!) Then (4′) takes on the form

$$(\omega^2 - \omega_i^2)\,u_i + ik^2[\mathbf{u}\times\mathbf{G}]_i = 0. \qquad (4'')$$

It is easy to verify that if all three values of $\omega_i$ are quite different from each other, then the changes in the natural frequencies and, consequently, the rotation of the plane of polarization, will be quadratic in $\mathbf{G}$. A different picture emerges if two values of $\omega_i$ coincide. An isotropic medium is a trivial case. For oscillations in the plane corresponding to the coinciding $\omega$, we have $\omega^2 = \omega_1^2 \pm G_\perp k^2$, where $G_\perp$ is the projection of $\mathbf{G}$ on the axis perpendicular to the plane of the coinciding principal values. The corresponding angle of rotation per unit length is equal to $\frac{1}{2}(k^3/\omega_1^2)\,G_\perp$. The most obvious reason for the occurrence of the gyration is the presence of a magnetic field.[10]

Another possible cause of rotation of the plane of polarization of the sound is natural optical activity.* In particular, in optically-active media one would expect a considerable rotation of the plane of polarization of sound (comparable in order of magnitude with the optical rotation), under conditions in which the acoustic wavelength is equal to (or smaller than) the wavelength of a light beam

*Attention has already been called to this possibility by V. L. Ginzburg, who has kindly informed me that a study of this question has also been undertaken by A. A. Andronov.

which undergoes a noticeable rotation of the plane of polarization. Such conditions can be realized at frequencies $\omega/2\pi \sim 10^9$ to $10^{10}$ sec$^{-1}$.

## APPENDIX B

## INFLUENCE OF LIQUID EFFECTS

Let us consider what changes would be introduced, by comparison with the results of Sec. 4, by taking into account the fact that the electrons in a metal form a liquid rather than a gas. For this purpose we use Landau's theory of a Fermi liquid.[11] Then instead of Eq. (2) we must use a kinetic equation of the form

$$\frac{\partial \delta f}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}\left(\delta f - \frac{\partial f_0}{\partial \varepsilon}\delta\varepsilon\right) + e\mathbf{E}\frac{\partial f_0}{\partial \mathbf{p}} - \Lambda_{jl}\frac{\partial^2 u_j}{\partial r \partial x_l}\frac{\partial f_0}{\partial \mathbf{p}} = 0. \tag{2'}$$

Here $\delta\epsilon(\mathbf{p}, \mathbf{r}) = \int d\mathbf{p}'\,\Phi(\mathbf{p}, \mathbf{p}')\,\delta f(\mathbf{p}', \mathbf{r})$. We also denote by $R(\mathbf{p}, \mathbf{p}')$ the operator which enables the solution of the equation $\delta f - \delta\epsilon\,\partial f_0/\partial\epsilon = A(\mathbf{p})\,\partial f_0/\partial\epsilon$ to be expressed in the form

$$\delta f = (\partial f_0/\partial\varepsilon)\int R(\mathbf{p}, \mathbf{p}')\,A(\mathbf{p}')\,d\mathbf{p}'\partial f_0/\partial\varepsilon'.$$

Because of the fact that $\Phi(\mathbf{p}, \mathbf{p}') = \Phi(\mathbf{p}', \mathbf{p})$, there is also an analogous identity for the operator just introduced: $R(\mathbf{p}, \mathbf{p}') = R(\mathbf{p}', \mathbf{p})$.

Continuing the treatment in a manner analogous to that in Sec. 4, we obtain, instead of (15) and (16)

$$k_m k_l(\lambda'_{imjl} - \lambda^{(0)}_{imjl}) = k_m k_l\{\langle\Lambda_{im}(\mathbf{p})R(\mathbf{p},\mathbf{p}')\Lambda_{jl}(\mathbf{p}')\rangle_2 - \langle R(\mathbf{p},\mathbf{p}')\rangle_2^{-1}(\langle\Lambda_{im}R\rangle_2 + N\delta_{im})(\langle\Lambda_{il}R\rangle_2 + N\delta_{jl})\}, \tag{15'}$$

$$k_m k_l\lambda''_{imjl} = \pi\omega\langle L_i^R(\mathbf{p})R(\mathbf{p},\mathbf{p}')\delta(\mathbf{kv}')R(\mathbf{p}',\mathbf{p}'')L_j^R(\mathbf{p}'')\rangle_3 - \omega e^2\left\{\left\langle\frac{v_\alpha^\perp}{\mathbf{kv}}R(\mathbf{p},\mathbf{p}')L_i^R(\mathbf{p}')\right\rangle_2 + \delta_{i\alpha}N\right\}\mathrm{Im}(iB^{-1}_{\alpha\beta}) \times\left\{\left\langle\frac{v_\beta^\perp}{\mathbf{kv}}R(\mathbf{p},\mathbf{p}')L_j^R(\mathbf{p}')\right\rangle_2 + \delta_{j\beta}N\right\}. \tag{16'}$$

Here we have used the notation

$$\langle F(\mathbf{p}_1,\ldots,\mathbf{p}_n)\rangle_n = \int d\mathbf{p}_1\ldots d\mathbf{p}_n\frac{\partial f_0}{\partial\varepsilon_1}\ldots\frac{\partial f_0}{\partial\varepsilon_n}F(\mathbf{p}_1,\ldots,\mathbf{p}_n),$$

$$L_j^R = k_l\left\{\Lambda_{jl} - \frac{\langle R\Lambda_{jl}\rangle_2}{\langle R\rangle_2} - \frac{N\delta_{jl}}{\langle R\rangle_2}\right\}.$$

It follows from formulas (15') and (16') that in cases where the electron pair correlation characterized by the function $\Phi(\mathbf{p}, \mathbf{p}')$ is not small, the influence of the liquid effects cannot be small either.

In conclusion, I should like to express my gratitude to V. L. Ginzburg for discussing the results of the present work.

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