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THE SIGNIFICANCE OF STRANGE PARTICLES IN FERMI'S STATISTICAL THEORY

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It is shown that account of resonance interaction of two π mesons in Fermi's statistical theory permits one to explain a number of experimental facts which remained inexplicable by the statistical theory which takes into account only the nucleonic "isobar" $(3/2, 3/2)$. Such facts are the mean multiplicity of π and K mesons in $p\bar{p}$ annihilations, mean multiplicity of strange particles in meson-nucleon collisions, and angular correlations between π mesons in π^-p interactions at an energy of 1.0 Bev.

BARASHENKOV and Mal'tsev[1] have reported a confirmation that account of resonance $\pi\pi$ interaction worsens the agreement of the Fermi statistical theory with experiment. In this research we shall show that there is no foundation for such a conclusion at the present time.

It is known that resonance interaction of two particles can be considered within the framework of the statistical theory of Fermi. Account of resonant interaction of a π meson with a nucleon (nucleonic "isobar") considerably improves the agreement of statistical theory with experiment.[2] However a series of experimental facts remained unexplained even in this case: 1) the mean multiplicity of π mesons in $p\bar{p}$ annihilation was shown to be significantly smaller, while the percent of cases of annihilation with the formation of K mesons was significantly larger than observed by experiment; 2) the presence of maxima in the angular distribution among the π mesons and in the momentum representation of nucleons in π^-p collisions at energies of 1.0 Bev was not explained; 3) the multiplicity of "strange" particles in meson-nucleon collisions remained significantly larger than that experimentally observed.

There is at the present time a sufficient amount of indirect experimental and theoretical indications on the presence of resonance interaction of two π mesons. By considering this resonance interaction as the presence of a π-meson "isobar" with isospin $T = 1$ and effective mass $\mu = 0.47$* we shall attempt to improve the relation of the statistical theory with the experimental facts just enumerated.

1. Resonance $\pi\pi$ interaction in a state with isospin $T = 0$ in the process of $p\bar{p}$ annihilation was considered by Eberle,[3]* who obtained the mean multiplicity of π mesons, $\bar{n}_\pi = 3.8$, and the percent of cases of annihilation with formation of the observed K mesons, $P_K = 12$ per cent. According to experimental data given by Segrê at the 1959 Ninth International Conference on High-Energy Physics in Kiev, $\bar{n}_\pi = 4.7 \pm 0.1$, $P_K = (3 \pm 1)$ per cent. In order to improve the agreement between theory and experiment, Eberle made use of the assumption of Barashenkov[5] on the creation of the K meson in its volume V_K $[V_K = (4/3)\pi r_K^3$, where $r_K = 1/M_K]$ and obtained $\bar{n}_\pi = 4.4$. Here the percentage of cases with formation of the observed K mesons appears to be greatly reduced, $P_K = 0.35$ per cent. We note that the annihilation of a strongly slowed or arrested antinucleon is the most advantageous process for verification of the assumption on the production of K mesons in the volume V_K, since only K mesons are produced from the "strange" particles in this case. By considering the resonance interaction of two π mesons in a state with isospin $T = 1$, we obtain the following statistical weight of the finite state of $p\bar{p}$ annihilation (normalized to 100 per cent):

2π	3π	4π	5π	6π	7π	$2K$	$2K\pi$	$2K2\pi$	$2K3\pi$
1.0	9.8	21.2	13.5	38.5	0.6	1.2	4.8	7.1	2.3

For the calculations, we have assumed that the K mesons, like the π mesons, are produced in the Fermi volume $V_\pi = (4/3)\pi r_\pi^3$, where r_π is the π-meson Compton wavelength. From the statistical weight of possible finite states given

*Here and below, $\hbar = c = M_{nucl} = 1$.

*Resonance $\pi\pi$ interaction was considered by Goto.[4] However, in this research the calculation of the statistical weights of all processes was carried out according to the same very approximate formula.

above, it follows that the mean multiplicity of the π mesons is $\bar{n}_\pi = 4.5$, while the percentage of cases with formation of observed K mesons is $P_K = 7.7$. The distribution of charged products of $p\bar{p}$ annihilation was also computed by us according to the number of prongs:

0-prong cases — 2 (5); 2-prong cases — 38 (40); 4-prong cases — 56 (50); 6-prong cases — 4 (5); 8-prong cases — 0 (0).

Here the figures in parentheses correspond to the experimental data,[6] normalized to 100 per cent. The statistical weight of cases with the formation of two charged π mesons ($p + \bar{p} \rightarrow \pi^+ + \pi^-$) was shown to be less than one per cent. The results that were obtained were in reasonable agreement with the experimental data given by Segrè.[6]

Taking into account the resonance $\pi\pi$ interaction in the state with $T = 1$, $S = 1$, and effective mass $\mu = 0.60$, we obtain better agreement with experiment: $\bar{n}_\pi = 4.9$, $P_K \approx 4$ per cent.

2. In the majority of researches on the statistical theory of multiple production of particles, correlations between the directions of motion of the secondary particles have not been taken into consideration. At the same time, an investigation of this problem would permit one to decide on the necessity of taking into account resonance interaction of two particles, since this interaction ought to lead to the appearance of definite angular correlations. In the general case, this problem is very complicated and evidently cannot be solved without consideration of the law of conservation of momentum, which is neglected at the present time. However, in the case of $\pi^- p$ interaction in the energy region of 1.0 Bev, where the processes

$$\pi + N \rightarrow N + \pi + \pi, \tag{1}$$

$$\pi + N \rightarrow N' + \pi \rightarrow N + \pi + \pi, \tag{2}$$

$$\pi + N \rightarrow \Pi + N \rightarrow N + \pi + \pi \tag{3}$$

play the principal role, with relative statistical weights 41.9, 41.9, and 16.2, the distribution over angles among the π mesons can easily be found. For this purpose it suffices to assume that the characteristic features of this distribution are brought about by the resonance processes (2) and (3), and therefore one can neglect angular correlations in the process (1).

In the process (2) (here the symbol N′ denotes the nucleonic "isobar"), the meson moves in a direction which is strongly opposed to the direction of the "isobar." Therefore, the angle φ between the direction of motion of the "isobar" and the direction of motion of the π meson from decay of the "isobar" will be connected with the angle θ between the directions of motion of both π mesons by the relation $\varphi + \theta = \pi$. Making use of the angular distribution of π mesons from the decay of the "isobar," given in the review of Rozental',[7] one can easily obtain the distribution over the angle θ for the secondary π mesons.

The angular distribution over the π mesons in the process (3) (here the symbol Π denotes the π-meson "isobar") can be obtained if use is made of the dependence of the angle θ between the two π mesons from the decay of the π-meson "isobar" on the energy of the π mesons E_1.[8] Transforming from the energy distribution of one of the π mesons to the angular distribution between them, we obtain

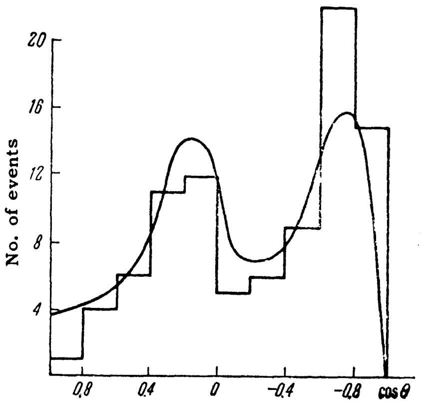
$$N(\theta)\, d\cos\theta = d\cos\theta \Big\{ M^2 \cos\theta \sqrt{M^4/4 - E_0^2 \mu^2 \cos^2\theta} - (\cos^2\theta + 1)\left(\tfrac{1}{4}M^4 - E_0^2\mu^2\sin^2\theta\right) - \cos^2\theta \sin^2\theta E_0^2 \mu^2 \Big\} (4 p_c \gamma v \sin^4\theta)^{-1} \times \left(\tfrac{1}{4}M^4 - E_0^2\mu^2\sin^2\theta\right)^{-1/2} \left(\tfrac{1}{4}E_0^2 + \mu^2 - \frac{M^2}{2\sin^2\theta} + \frac{\cos\theta}{\sin^2\theta}\sqrt{M^4/4 - E_0^2\mu^2\sin^2\theta}\right)^{-1/2},$$

where E_0 and v are the energy and velocity of the π-meson "isobar" in the center of mass of the meson and nucleon; $\gamma = (1 - v^2)^{-1/2}$; M is the mass of the π-meson "isobar," equal to 0.47; μ is the mass of the π meson; p_c is the momentum of the π meson from the decay of the π-meson "isobar" in its rest system; θ is the angle between the two π mesons from the decay of the π-meson "isobar." The limits of variation of the angle θ are easily obtained from the usual formulas given in a review by Rozental'.[7]

Distribution over the angles between the π mesons in the reactions

$$\pi^- + p \rightarrow p + \pi^- + \pi^0, \qquad \pi^- + p \rightarrow n + \pi^+ + \pi^- \tag{4}$$

was computed by us for an energy of 1.0 Bev. The resultant distribution is plotted in the drawing, where the experimental data of reference 9 are represented by the histogram, while our calculations are given by the solid curve. As is seen from the drawing, the results obtained here are in excellent qualitative agreement with experiment. The maximum is brought about by process (3) when $\cos\theta = 0.2$ and by process (2) when $\cos\theta = -0.8$. A certain quantitative discrepancy comes about here by the fact that we have neglected angular correlations in process (1).



Distribution according to the angles between the π mesons in reactions (4) at an energy of 1.0 Bev. Histogram from the experiment in reference 9; smooth curve – calculated according to the statistical theory with account of resonance interactions (nuclear and π-meson "isobar").

The effect of consideration of resonance $\pi\pi$ interaction on the momentum distribution of the π mesons and nucleons, obtained theoretically for the reactions considered above, has been investigated by us in reference 10; therefore, we shall not dwell on it. We only note that consideration of resonance interaction of two π mesons allows one to explain the characteristic maximum in the momentum distribution of nucleons in π^-p collisions at 1.0 Bev. (The momentum distribution of the secondary particles, calculated with account of the π-meson "isobar" in a state with isospin $T = 0$, differs slightly from the momentum distribution computed with account of the π-meson "isobar" in a state with isospin $T = 1$.)

3. We shall further consider the effect of resonance $\pi\pi$ interaction on the multiplicity of strange particles in meson-nucleon collisions. It is known that the statistical theory of Fermi, which takes into account only the nucleonic isobar ($^3/_2$, $^3/_2$), gives a ratio of the cross section of production of strange particles to the cross section of production of π mesons, $\sigma_{str}/\sigma_\pi = 0.3$.[2] By taking into account (along with the nucleonic isobar) the resonance $\pi\pi$ interaction in the state with isospin $T = 1$, it is possible to lower this ratio to 0.08.[11] However, both in reference 2 and in reference 11, calculation of the statistical weights of processes with four and more particles was carried out by approximate formulas. Therefore, some of the reactions with strange particles were not considered in these researches. More exact calculations increase the ratio given above by a factor of more than 2.5. Therefore, although calculation of the resonance $\pi\pi$ interaction significantly lowers the ratio σ_{str}/σ_π, this ratio still remains larger than that experimentally observed.[12]

In the first part of this paper, we showed that account of the resonance interaction of two π mesons in a state with isospin $T = 1$ in the case of $p\bar{p}$ annihilation, where only K mesons are produced from the strange particles, allows one to obtain a reasonable multiplicity of strange particles. Starting out from this situation, one can attempt to improve the agreement of the statistical theory with experiment by assuming that in the case of meson-nucleon collisions, the statistical weight of all strange particles (hyperons and K mesons) is determined by the statistical weight of processes with the production of only meson pairs $K + \bar{K}$. The scheme of elementary particles advanced by Goldhaber (see reference 13) lends some support to this assumption; in this scheme the hyperons are the product of a medium-strong interaction of secondary $\bar{K}$ mesons with secondary nucleons. The given assumption makes it possible to lower the ratio σ_{str}/σ_π to 0.14. In the calculation of this ratio, reactions previously omitted[11] have been taken into account.* The experimental value is $\sigma_{str}/\sigma_\pi = 0.04$.[12] This ratio is known to be too low, since the strange particles in reference 12 were established only by the decay. Therefore, the ratio that we have found of $\sigma_{str}/\sigma_\pi = 0.14$ can be regarded as not very far from the correct one.

We note that if the scheme of elementary particles of Goldhaber is correct, then in those cases in which the energy of the emitted π mesons in meson-nucleon collisions is below the threshold of formation of two K mesons (1.34 Bev), formation of the latter is possible only virtually in an intermediate state. Consequently, formation of strange particles up to this energy region cannot be considered within the framework of the statistical theory of Fermi. It is quite evident that all other consequences of the Goldhaber scheme remain valid under such considerations.

In conclusion, we take up the principal objection against calculation in the statistical Fermi theory of resonance $\pi\pi$ interaction, put forward in the work of Barashenkov and Mal'tsev.[1] This objection is that in the case of consideration of a π-meson "isobar" the theoretically determined distribution of charged products of π^-p collisions according to the number of prongs differs from that experimentally observed. As seen from reference 11, the distribution computed with account of the π-meson "isobar" is in better agreement with experiment than that which takes into account only the nucleon isobar if in this case we can take into account the formation of strange particles. The departure from

*We are grateful to V. S. Barashenkov and V. M. Mal'tsev for making it possible for us to use their calculations of the statistical weights of the reactions omitted by us in reference 11, and for discussion of the problem.

the experimentally observed distribution is evidently brought about by the crudeness of the statistical theory.

Actually, for example, the state with isospin $T = 3/2$ and $T = -1/2$ can be resolved into two π mesons and a nucleon in two ways:

$$[3/2, -1/2] = \sqrt{2/5}\,[1/2, -1/2]\,[2, 0] + \sqrt{3/5}\,[1/2, 1/2]\,[2, -1],$$
$$[3/2, -1/2] = \sqrt{2/3}\,[1/2, -1/2]\,[1, 0] + \sqrt{1/3}\,[1/2, 1/2]\,[1, -1].$$

Within the framework of the statistical theory, these two distributions are considered equally probable. Account of the interaction matrix element H_{if} should lead to a deviation from this equality of probability. Processes taking place close to threshold (see, for example, the appendix to reference 2) represent a trivial example of such a discrepancy.

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[1] V. S. Barashenkov and V. M. Mal'tsev, JETP **37**, 884 (1959), Soviet Phys. JETP **10**, 630 (1959).

[2] Belen'kiĭ, Maksimenko, Nikishev, and Rozental', Usp. Fiz. Nauk **62**, 1 (1957).

[3] E. Eberle, Nuovo cimento **8**, 610 (1958).

[4] T. Goto, Nuovo cimento **8**, 625 (1958).

[5] V. S. Barashenkov, JETP **34**, 1016 (1958), Soviet Phys. JETP **7**, 701 (1958).

[6] E. Segrè, Ninth International Conference on High-Energy Physics, Kiev, 1959.

[7] I. L. Rozental', Usp. Fiz. Nauk **54**, 405 (1954).

[8] G. I. Kopylov, preprint P-166, Dubna, 1958.

[9] Walker, Hushfar, and Shephard, Phys. Rev. **104**, 526 (1956).

[10] V. I. Rus'kin, JETP **37**, 105 (1959), Soviet Phys. JETP **10**, 74 (1960).

[11] V. I. Rus'kin, JETP **36**, 164 (1959), Soviet Phys. JETP **9**, 113 (1959).

[12] Maenchen, Fowler, Powell, and Wright, Phys. Rev. **108**, 850 (1957).

[13] M. A. Markov, Гипероны и *K*-мезоны (Hyperons and K Mesons) Fizmatgiz, 1958.

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