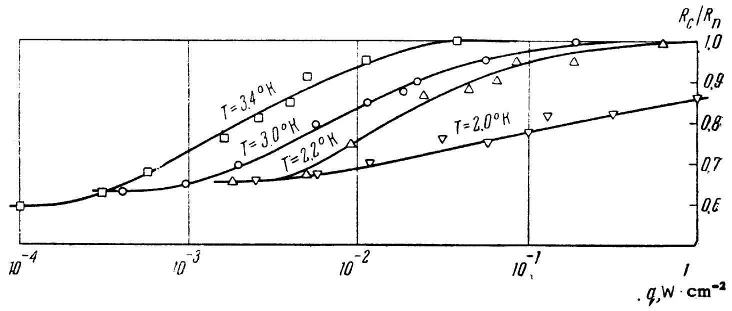

FIG. 2

The data presented thus indicates that it is only possible to elucidate the influence of surface forces (Scott) and mean free path (Kuper) if overheating of the specimen is avoided.

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DAMPING OF THE OSCILLATIONS OF A CYLINDER IN ROTATING HELIUM II

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We have previously shown[1] that the interaction of a disk oscillating in rotating helium II with the Onsager-Feynman vortex lines leads to a specific dependence of the damping upon the rotational velocity, with a characteristic maximum[2,3] which is not to be explained by consideration of the influence on the disk of the normal component of the helium II alone (even when the mutual friction between the normal and superfluid liquids is taken into account). A decisive role in the explanation of the formulas derived in reference 1 is played by the circumstance that the vortex lines, being perpendicular to the plane surface of the disk, lie with one end upon this surface. Distorted by the perpendicular displacement of the surface, they act upon it with a force which depends upon their tension. The relation between the tension of a vortex line and its circulation, moreover, determines the effective viscosity of the superfluid component (the quantity η_s in reference 1).

From what has been said, it is clear that if the disk be replaced by a cylinder whose surface is parallel to the axes of the vortex lines, then the possibility of direct interaction of the oscillating body with the vortices which form when a superfluid liquid is rotated is completely excluded. The presence of the vortices manifests itself solely in mutual friction effects.

Solving the system of hydrodynamic equations for rotating helium II,[4,1] for boundary conditions corresponding to small oscillations of an infinite cylinder rotating together with an unbounded liquid about their common axis,* one can readily verify that the force acting upon the surface of the cylinder is wholly determined by the momentum flow of the normal component.

The sum of the moments of the forces acting upon unit length of the outer and inner surfaces of a thin-walled cylinder of radius R turns out to be

$$M = -2\pi i R^3 \eta_n \Omega \varphi_0 k \left[H_2^{(1)}(kR) / H_1^{(1)}(kR) - J_2(kR) / J_1(kR) \right] e^{i\Omega t}. \quad (1)$$

Here, η_n is the viscosity of the normal component, Ω and φ_0 are the frequency and amplitude of the oscillations of the cylinder, J_p is a Bessel function, $H_p^{(1)}$ is a Hankel function, and k is the complex wave number, determined by the equation

$$k^2 = -\frac{i\Omega}{\nu_n}\left[1 + i\frac{2\omega_0}{\Omega}\beta_s\left(1 - i\frac{2\omega_0}{\Omega}\frac{\beta_n}{1 + 2i\omega_0\beta_n/\Omega}\right)\right], \quad (2)$$

with $\mathrm{Im}\, k > 0$. Here ν_n is the kinematic viscosity of the normal component, ω_0 is the angular velocity of rotation, and β_n and β_s are the coefficients for the mutual friction between the superfluid and normal components (cf. reference 1).

As was to be expected, Eqs. (1) and (2) show that the dependence of M upon the rotational velocity vanishes for $\beta_n = \beta_s = 0$. Consequently, the influence of rotation upon the damping of the oscillations of a cylinder is characteristic only of helium II. Measurements[5,6] have confirmed the absence of such an effect in a classical fluid.

Using Eq. (2) it is not difficult to show that over a broad range of frequencies ω_0 and Ω and for $R \approx 1$ cm the penetration depth of the cylindrical waves excited by the oscillations of a cylinder in rotating helium II is appreciably less than the radius of the cylinder. This makes it possible to use an asymptotic expansion of the cylindrical functions for large values of the argument.

As a result, the damping γ' at the surface of a unit length of the cylinder is

$$\gamma' = \frac{\pi R^3 \sqrt{2\eta_n \rho_n \Omega}}{I_1}\left(1 - \frac{\omega_0}{\Omega}\beta_s\right)\left(1 - \frac{3\delta_0}{R}\right), \qquad (3)$$

Where I_1 is the moment of inertia of the cylinder (per unit length), $\delta_0 = \sqrt{2\nu_n/\Omega}$ is the penetration depth in the absence of rotation, and ρ_n is the normal component density. Equation (3) is written in the linear approximation to the product of $2\omega_0/\Omega$ and the mutual friction coefficients.

To eliminate boundary effects it is convenient to measure the quantity $(\gamma_2 - \gamma_1)/(l_2 - l_1)$, which is equivalent to γ'; here γ_2 and γ_1 are the values of the damping for immersion of the cylinder to depths l_2 and l_1, respectively. (In addition, I_1 should be replaced in Eq. (3) by the moment of inertia of the suspended system I, which is presumed to be sufficiently great that the period of the oscillations is the same in both stationary and rotating helium, and for various depths of immersion.)

It can readily be seen that the ratio of the quantities $\gamma_2 - \gamma_1$ as measured in rotating and in stationary helium II is

$$(\gamma_2 - \gamma_1)/(\gamma_2 - \gamma_1)_{\omega_0 = 0} = 1 + \omega_0 \rho_s B / 2\Omega\rho, \qquad (4)$$

where ρ_s/ρ is the relative density of the superfluid component, and B is the coefficient of Hall and Vinen[7,8] ($\beta_s = -\rho_s B/2\rho$). Equations (3) and (4) are also confirmed by experiment.[6]

The authors regard it their pleasant duty to thank É. L. Andronikashvili and his colleagues in the cryogenic laboratory of the Tbilisi State University for their constant interest in this work.

*In solving this problem the necessity of using additional boundary conditions for the velocity of the superfluid liquid does not arise (cf. references 1 and 4), since its components turn out to be proportional to the corresponding components of the normal fluid velocity.

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ON THE POSSIBILITY OF MEASURING A GRAVITATIONAL FREQUENCY SHIFT IN THE SUN'S FIELD

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SEVERAL authors[1,2] have discussed the possibility of using artificial earth satellites to measure the gravitational frequency shift. However, they have considered only the shift due to the earth's field. We wish to present a calculation which shows that the frequency shift due to the sun's field can also be measured with earth satellites.

The frequency shift due to the sun is

$$\Delta\nu/\nu = -kM_\odot/c^2 r, \qquad (1)$$

where k is the gravitational constant, $M_\odot = 2.0 \times 10^{33}$ g is the mass of the sun and r is the dis-