THE SCHWINGER EFFECT WITH ACCOUNT OF THE SCREENING OF THE NUCLEUS BY THE ATOMIC ELECTRONS

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AN important effect in the scattering of neutrons by heavy nuclei at small angles ($\theta \lesssim 10°$) is the interaction of the magnetic moment of the neutron with the electric field of the nucleus. This effect was first considered by Schwinger,[1] who found that the additional cross section for an unpolarized current, $\Delta\sigma(\theta)$, has, in first order perturbation theory, the form

$$\Delta\sigma(\theta) = \frac{\varepsilon^2}{16}\cot^2\frac{\theta}{2}, \quad \varepsilon = 2|\mu_n| Z\frac{e^2}{\hbar c}\frac{\hbar}{mc}, \tag{1}$$

where the zero order functions are taken to be those corresponding to the absence of the scattering center (first Born approximation). We note that this cross section is infinite at the origin. Later Sample[2] calculated $\Delta\sigma(\theta)$ using in zeroth approximation functions corresponding to the scattering from a hard sphere. Sample's cross section is also infinite at the origin and differs very little from that obtained by Schwinger. Both Schwinger and Sample used for the perturbing potential

$$V_1 = \varepsilon \mathrm{LS}/r^3. \tag{2}$$

Experimentally this effect was observed by Voss and Wilson[3] for U (energy ~ 100 Mev) and Aleksandrov and Bondarenko[4] for Pb (energy ~ 3 — 4 Mev). Later Aleksandrov[5] also measured this effect for Cu, Sn, Pb, Bi, U, and Pu at the energy ~ 2 Mev. According to references 4 and 5 the cross section increases as the angle becomes smaller in the region of small θ; this dependence is well described by formula (1) in the cases of Cu, Sn, Pb, and Bi. There is some disagreement for U and Pu, but even in these cases formula (1) does reproduce the general behavior of the cross section.

Owing to the screening of the nucleus by the atomic electrons the electric field of the atom is different from the Coulomb field. Let $\Phi(r)$ describe the screening of the potential U, i.e., $U(r) = (Ze/r)\Phi(r)$. Then $\mathbf{E}(r) = -(Ze/r^3) \times \alpha(r)\mathbf{r}$, where $\alpha(r) = \Phi(r) - r\Phi'(r)$, and

$$V_1 = \varepsilon\alpha(r)\,\mathrm{LS}/r^3. \tag{3}$$

For $\Phi(r) = 1$ formula (3) coincides with (2). If $\psi_l(r)$ are the zero order functions, i.e., solutions of the radial equation for the scattering from the unperturbed potential and δ_l the corresponding phases, the following formulas hold for the additional cross section in the first order of perturbation theory:

$$\Delta\sigma(\theta) = |a_1(\theta) + a_2(\theta)|^2,$$

$$a_1(\theta) = -\frac{\varepsilon}{4}\cot\frac{\theta}{2}\cdot K\int_0^\infty \Phi(r)\sin Kr\,dr,$$

$$a_2(\theta) = \frac{\varepsilon}{2}\sum_{l=1}^{\infty}(2l+1)\Bigg[\int_0^\infty \frac{j_\nu^2(kr)}{r}\alpha(r)\,dr - e^{i2\delta_l} \times \int_0^\infty \frac{[\psi_l(r)]^2}{r}\alpha(r)\,dr\Bigg]P_l^{(1)}(\cos\theta), \tag{4}$$

where $K = 2k\sin(\theta/2)$, k is the wave number, and the $j_\nu = j_{l+1/2}$ are spherical Bessel functions. The ψ_l are normalized by the condition $\psi_l(r) \to \sin(kr - l\pi/2 + \delta_l)/kr$ for $r \to \infty$. It follows from these formulas that

$$\Delta\sigma(0) = 0. \tag{5}$$

The screening factor $\Phi(r)$ can be obtained from the statistical Thomas-Fermi model of the atom:

$$\Phi(r) = \varphi(x), \qquad x = r/\mu,$$

$$\mu = {}^1\!/_4\,(9\pi^2/2Z)^{1/3}\,(\hbar^2/4\pi^2 m_e e^2), \tag{6}$$

where φ is the solution of the Thomas-Fermi equation.[6]

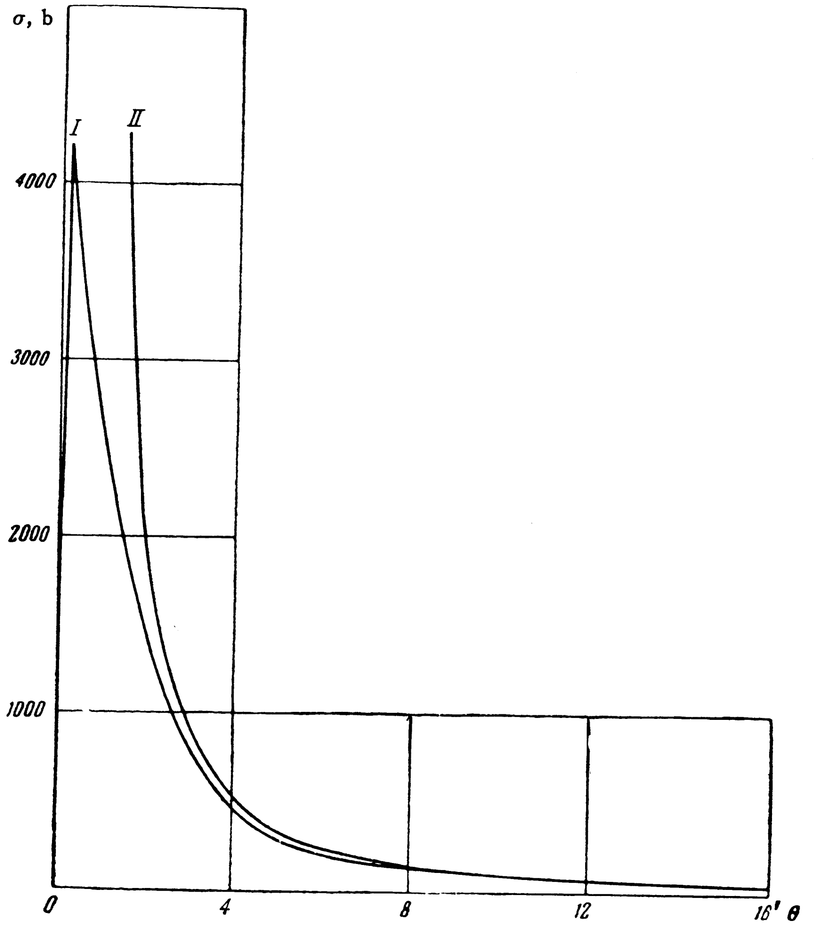
For the calculation of $\Delta\sigma(\theta)$ according to formulas (4) and (6) with $Z = 92$ we used as the unperturbed potential

$$V_0(r) = -V_0(1 + i\zeta)/\{1 + \exp[(r - R)/a]\}$$

with $V_0 = 44$ Mev, $R = 7.72 \times 10^{-13}$ cm, $a = 0.5 \times 10^{-13}$ cm, $\zeta = 0.075$; the energy is 2.5 Mev. The contribution from the term $a_2(\theta)$ which is determined by the functions and phases of the zeroth approximation appeared to be negligible for $\theta \lesssim 10°$: $|a_2(\theta)| \ll |a_1(\theta)|$ for $\theta \lesssim 10°$. The screening changes the angular distribution considerably at very small angles. The behavior of the cross section for $\theta < 16'$ is shown in the figure. The relative magnitude of the correction to the screening,

$$\left[\Delta\sigma(\theta) - (\varepsilon^2/16)\cot^2\frac{\theta}{2}\right]/(\varepsilon^2/16)\cot^2\frac{\theta}{2}$$

is 2.6% for $\theta = 20'$. For larger angles it decreases as $\sin^{-3/2}(\theta/2)$. In the region of angles in which the measurements[4,5] were carried out the correction to the screening is somewhat smaller than the experimental error.



Curve I: cross section of Schwinger scattering with screening; curve II: the same cross section without screening.

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INVESTIGATION OF BETA RADIATION OF Nb^{95} *AND* Ce^{144} *BY THE METHOD OF ABSORPTION IN AIR*

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1. Beta radiation of Nb^{95}. A spectroscopic investigation of this radiation has been the subject of many papers. The values of the β radiation energy of Nb^{95}, obtained by different authors, range over sufficiently wide limits, 0.140 – 0.171 Mev, i.e., the outermost values differ by 20%.[1-4] These investigations were performed with spectrometers of different constructions.