EXCITATION OF NUCLEAR COLLECTIVE STATES IN CHARGED PARTICLE SCATTERING

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The differential excitation cross section of the 4^+ even-even nucleus collective level in fast nucleon small-angle scattering is derived. The cross section is strongly dependent on the absolute value and sign of the nuclear shape parameter α_4.

In the scattering of nucleons on nonspherical nuclei, excitation of the collective states of the nucleus takes place as a consequence of the process of direct interaction of the incident particle with the nuclear surface. We shall show that the angular distribution of particles scattered elastically in these processes is always strongly dependent on the nuclear shape parameter α_l entering into the equation of the surface of the nucleus $r(\mu) = R \times [1+\Sigma\alpha_l P_l(\mu)]$. For this purpose we shall compute the excitation cross section of the rotational 4^+ level of an even-even nucleus in the scattering of fast charged particles by making use of the method of diffraction theory.[1,2]

The excitation of the first rotational level 2^+ has been considered previously.[3] As before, we assume that the energy of the particles exceeds the Coulomb barrier ($kR \gg \eta = ZZ'e^2/\hbar v$), that the adiabatic condition is satisfied ($kR\Delta E/E \ll 1$), and that the nucleus is black. The differential scattering cross section with excitation of the rotational level of an even-even nucleus with moment λ is given by the equation

$$\sigma_\lambda(\theta) = \sum_\mu |\langle Y^*_{\lambda\mu}(\omega) f(\Omega, \omega) Y_{00}\rangle|^2, \quad (1)$$

which is easily generalized to the case of an odd nucleus.[3]

According to the diffraction-theory method, the amplitude is determined by the expression ($z_0 \to \infty$)

$$f(\Omega, \omega) = -\frac{ik}{2\pi}\int_0^{2\pi} d\varphi \int_{\rho(\varphi)}^{\infty} \rho d\rho e^{-i\mathbf{k}\rho}\Psi_{\mathbf{k}}(\rho, z_0) e^{-ikz_0}, \quad (2)$$

where the angles $\Omega = (\theta, \varphi')$ determine the scattering direction while the angles ω determine the direction of the symmetry axis of the nucleus; the function $\rho(\varphi)$ describes the shape of the nuclear shadow in the transverse plane; $\mathbf{k}'$ = the wave vector of the scattered particle. The wave function $\Psi_{\mathbf{k}}$ describes the scattering of charged particles on a black nucleus, and has the form[1]

$$\Psi_{\mathbf{k}}(\rho, z) = \exp\left\{i\left[kz - (\hbar v)^{-1}\int_{-z_0}^{z} U(\rho, z)\, dz\right]\right\}. \quad (3)$$

in cylindrical coordinates with the polar z axis along the wave vector of the incident particles $\mathbf{k}$. The quantity $U(\rho, z)$ represents the energy of the electrical interaction of the particle with the nucleus.

If we assume the nucleus to be uniformly charged, we then obtain in second approximation in the small parameter of nonsphericity α_l:

$$\frac{U(\rho, z)}{ZZ'e^2} = \frac{1}{r}\left[1 + 3\sum_{l'}\alpha_{l'}(C^{00}_{l'0l'0})^2\right] + \sum_{lm}\left[\alpha_l + \sum_{l'l''}\alpha_{l'}\alpha_{l''}(C^{l0}_{l'0l''0})^2\frac{2+l}{2}\right] \times \frac{12\pi R^l}{(2l+1)^2} Y^*_{lm}(\omega) Y_{lm}(\mathbf{r})\, r^{-l-1} \quad (4)$$

and similarly for the nuclear shadow function:

$$\frac{\rho(\varphi)}{R} = 1 + \sum_{lm}\frac{4\pi\alpha_l}{2l+1} Y^*_{lm}(\omega) Y_{lm}(0, \varphi) + \frac{1}{2}\sum_{Ll'mm'}\frac{(4\pi)^{3/2}\alpha_l\alpha_{l'}C^{L0}_{l0l'0}C^{Lm+m'}_{lml'm'}}{\sqrt{(2L+1)(2l+1)(2l'+1)}} Y^*_{Lm+m'}(\omega) \times \left[\frac{\partial}{\partial\mu}Y_{lm}(\mu, \varphi)\right]_{\mu=0}\left[\frac{\partial}{\partial\mu}Y_{l'm'}(\mu, \varphi)\right]_{\mu=0}, \quad (5)$$

where $C^{Lm+m'}_{lml'm'}$ are the Clebsch-Gordan coefficients.

By substituting (3) — (5) in (2), we can compute the amplitude $\langle Y^*_{\lambda\mu}(\omega) f(\Omega, \omega) Y_{00}\rangle$ corresponding to excitation of the rotational state of the nucleus $Y_{\lambda\mu}(\omega)$ in the form of an expansion in powers of α_l ($\alpha_l kR^\theta \ll 1$). For simplicity, we shall assume that the nuclear shape parameter $\alpha_4 \ll \alpha_2$. Then the amplitude of the excitation of the state 4^+ of the even-even nucleus in the scattering of charged particles has the form ($\xi = \alpha_4/\alpha_2^2$, $a = kR^\theta$, $\lambda = 4$):

$$i^{-\mu}e^{-i\mu\varphi'}(2\lambda+1)^{1/2}\alpha_2^{-2}k\,(kR)^{-2(1+i\eta)}\langle Y^{*}_{\lambda-\mu}(\omega)\,f(\Omega,\,\omega)\,Y_{00}\rangle$$
$$= 2\eta\,(A^{(3)}_{\mu} - A^{(2)}_{\mu})\,J_{\mu}(a) + \eta\,(\xi B^{(2)}_{\mu} + A^{(5)}_{\mu})\,\mathrm{Re}\,(F_{4\mu} - \Phi_{4\mu})$$
$$+ \eta^2 A^{(4)}_{\mu}\,\mathrm{Im}\,(F_{4\mu} - \Phi_{4\mu}) + i\,[(\xi B^{(1)}_{\mu} + A^{(1)}_{\mu} + A^{(2)}_{\mu})\,J_{\mu}(a)$$
$$+ A^{(2)}_{\mu}aJ'_{\mu}(a) + \eta\,(\xi B^{(2)}_{\mu} + A^{(5)}_{\mu})\,\mathrm{Im}\,(F_{4\mu} - \Phi_{4\mu})$$
$$- \eta^2 A^{(4)}_{\mu}\,\mathrm{Re}\,(F_{4\mu} - \Phi_{4\mu})]. \qquad (6)$$

Here $A^{(i)}_{\mu}$, $B^{(i)}_{\mu}$ are numbers representing combinations of the Clebsch-Gordan coefficients and the spherical harmonics, for example,

$$A^{(2)}_{\mu} = \frac{4}{10}\,\pi C^{40}_{2020}\sum_{mm'} C^{4\mu}_{2m2m'}Y_{2m}(0,0)\,Y_{2m'}(0,0).$$

These equations are given in the table; they differ from 0 only for even μ and do not depend on the sign of μ.

μ	$B^{(1)}_{\mu}$	$B^{(2)}_{\mu}$	$A^{(1)}_{\mu}$	$A^{(2)}_{\mu}$	$A^{(3)}_{\mu}$	$A^{(4)}_{\mu}$	$A^{(5)}_{\mu}$
0	0.375	0	−0.514	0.0964	0.0257	0.0206	0
2	−0.395	0	0.407	−0.102	−0.0407	0	0
4	0.523	0.159	0	0.134	0.108	0.0861	0.246

The complex functions $F_{\lambda\mu}(a, \eta)$, $\Phi_{\lambda\mu}(a, \eta)$ entering into (6) depend on a and η. These functions can be computed from the following formulas:

$$\Phi_{\lambda\mu}(a, \eta) = a^{\lambda-2(1+i\eta)}2^{1-\lambda+2i\eta}\frac{\Gamma(i\eta + (\mu-\lambda)/2 + 1)}{\Gamma(-i\eta + (\mu+\lambda)/2)},$$

$$F_{\lambda\mu}(a, \eta) = \sum_{m=0}^{\infty}\frac{a^{m}J_{\mu+m}(a)}{2^{m+1}}\,\frac{\Gamma(i\eta + (\mu-\lambda)/2+1)}{\Gamma(i\eta + (\mu-\lambda)/2 + m+2)}. \qquad (7)$$

For $\eta = 0$, we get the amplitude of inelastic scattering of neutrons ($\lambda = 4$) from Eq. (6):

$$i^{-\mu-1}e^{-i\mu\varphi'}(2\lambda+1)^{1/2}\alpha_2^{-2}k\,(kR)^{-2}\langle Y^{*}_{\lambda-\mu}(\omega)\,f(\Omega,\,\omega)\,Y_{00}\rangle$$
$$= [A^{(1)}_{\mu} + A^{(2)}_{\mu} + \xi B^{(1)}_{\mu}]\,J_{\mu}(a) + A^{(2)}_{\mu}aJ'_{\mu}(a). \qquad (8)$$

In a fashion similar to what was done earlier[3] for the amplitude of excitation of the 2^+ state, one can divide the amplitude of inelastic scattering (6), corresponding to excitation of the 4^+ state, into two components: the amplitude of electrical excitation E4 and the part of the amplitude connected with nuclear interaction. It is not difficult to show that the terms in (6) containing the function $\Phi_{\lambda\mu}$ give the amplitude of the Coulomb excitation E4. The remaining terms correspond to the nuclear part of the scattering amplitude n4. Thus the excitation cross section of the level 4^+ can be represented in the form of a sum of the Coulomb excitation cross section and σ_{E4}, the nuclear part of the cross section σ_{n4} and the interference term $\sigma_{int\,4}$:

$$\sigma_4(\theta) = \sigma_{E4}(\theta) + \sigma_{n4}(\theta) + \sigma_{int4}(\theta). \qquad (9)$$

The angular distributions of protons and neutrons with energies of 20 and 30 Mev scattered with excitation of the state 4^+ of the nucleus Gd^{160}_{64} are shown in Figs. 1 — 3. As is seen in Fig. 1, the role

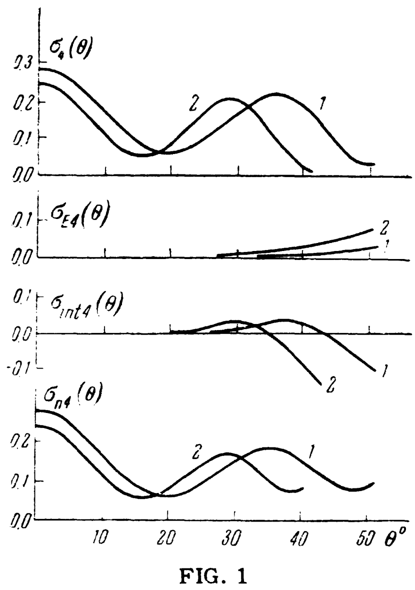

FIG. 1

FIG. 1. The functions $\sigma_4(\theta)$, $\sigma_{E4}(\theta)$, $\sigma_{int4}(\theta)$, $\sigma_{n4}(\theta)$, in units of $\alpha_2^4(kR)^4/9k^2$, describing the angular distribution of protons with energy 20 Mev (curve 1) and 30 Mev (curve 2), scattered on the Gd^{160}_{64} nucleus with excitation of its rotational level 4^+. The calculation was made at $\alpha_4 = 0$ and $R = 1.3 \times 10^{-13}A^{1/3}$ cm.

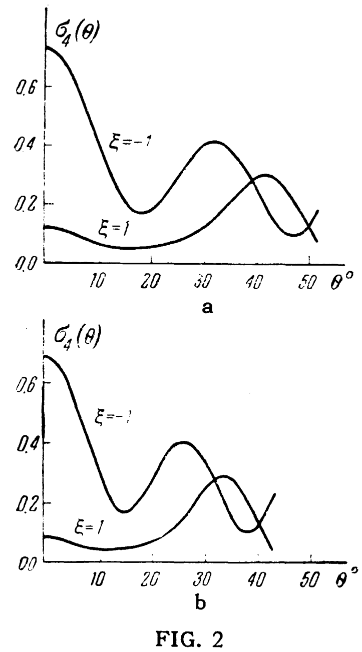

FIG. 2

FIG. 2. The function $\sigma_4(\theta)$ in units of $\alpha_2^4(kR)^4/9k^2$, describing the angular distribution of protons with energies of 20 Mev (a) and 30 Mev (b), inelastically scattered by the Gd^{160}_{64} nucleus. The calculation was made for $\xi = \alpha_4/\alpha_2^2 = \pm 1$.

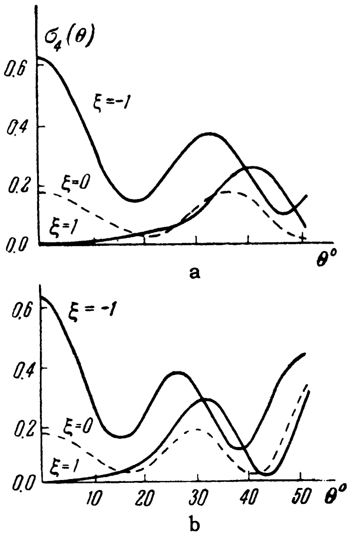

FIG. 3. Graph of the same quantity as in Fig. 2, for the case of inelastic scatterings of neutrons (a – 20 Mev, b – 30 Mev). The calculation was made for $\xi = 0, \pm 1$.

of Coulomb excitation E4 is small in small angle scattering $\theta \ll 1$, inasmuch as in this region of angles, $\sigma_{E4}(\theta)$ is proportional to θ^4. It should be noted that in the excitation of the level 2^+, the role of Coulomb excitation is quite significant.[3]

The form of the angular distribution $\sigma_4(\theta)$ for neutrons and protons is strongly dependent on the absolute value and the sign of the nuclear shape parameter (Figs. 2 – 3); for $\xi = \alpha_4/\alpha_2^2 \approx -1$, there is a large maximum in the angular distribution, corresponding to forward scattering; for $\xi \approx 1$, this maximum has essentially vanished.

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