

*CHARGED PION PRODUCTION IN INTERACTIONS OF 9-Bev PROTONS WITH EMULSION NUCLEI*

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Nuclear disintegrations produced in photographic emulsions by 9 Bev protons and involving not less than three fast particles were studied. The charged  $\pi$ -meson energy spectrum, measured up to 540 Mev and extrapolated to the high-energy region, is presented. The angular distributions of fast  $\pi$  mesons and protons in the laboratory system were measured. The average number of  $\pi$  mesons and fast protons per disintegration was found. The fraction of primary proton energy carried away by  $\pi$  mesons is estimated. The ratio of the numbers of  $\pi$  and K mesons in the velocity range  $\beta = 0.5 - 0.8$  has been derived.

### 1. EXPERIMENTAL CONDITIONS

SOME general information concerning the interaction of 9-Bev protons with emulsion nuclei<sup>2-5</sup> has been obtained after the proton synchrotron of the High-Energy Laboratory of the Joint Institute for Nuclear Research<sup>1</sup> had begun operation. The present article is an attempt to study the spectrum and the angular distribution of  $\pi$  mesons produced in the interaction of 9-Bev protons with emulsion nuclei.

An emulsion chamber consisting of a hundred layers of the NIKFI-R emulsion 450  $\mu$  thick, 10  $\times$  10 cm in size, was irradiated by the internal proton beam. The emulsion layers were scanned under a microscope along the tracks of primary protons, using 630  $\times$  magnification. For the study of the spectrum of  $\pi$  mesons produced in interactions of primary protons with complex emulsion nuclei, nuclear disintegrations with a number of fast particles  $n_s \geq 3$  were chosen. (Particles with an ionization  $J \leq 1.4 J_0$  were considered fast, where  $J_0$  is the ionization along the tracks of primary protons. This ionization corresponds to a pion kinetic energy higher than 80 Mev and a proton kinetic energy higher than 500 Mev.) Such a choice made it possible to select events in which several  $\pi$  mesons are produced. Events recognized as proton-nucleon collisions<sup>6,7</sup> were excluded.

Measurements of multiple Coulomb scattering and ionization were used for identification of the secondary particles. The multiple scattering measurements were carried out for tracks with ionization  $J \leq 2 J_0$  and length in one emulsion

layer  $l \geq 6$  mm. In constructing the spectrum, a geometrical correction was applied, because of the finite thickness of an emulsion layer and the limitation in the dip angle of the tracks (condition  $l \geq 6$  mm). This correction was calculated according to the formula

$$\eta = \pi / [\arcsin(h_1/l \sin \theta) + \arcsin(h_2/l \sin \theta)].$$

Here  $l$  is constant and equal to 6 mm,  $\theta$  is the spatial angle between the track and the direction of motion of the primary proton, and  $h_{1,2}$  are the distances from the center of the star to the emulsion surface and to the glass respectively. Under the assumption of azimuthal symmetry of the angular distribution of secondary particles with respect to the direction of the primary protons, this correction yields the total number of particles at a given angle  $\theta$ , i.e., takes into account also those tracks for which the scattering was not measured because of large dip angles.

From the distribution of the stars to which the selected tracks belong with respect to the number of fast particles  $n_s$  in the star, it was found, taking into account the geometrical correction, that the average multiplicity  $\bar{n}_s$  for the selected class of stars is  $4.60 \pm 0.14$ . It should be noted that the value  $\bar{n}_s$  obtained should be somewhat greater than the corresponding value  $\bar{n}_s$  determined directly from the distribution of stars with respect to  $n_s$  found in scanning along the track,<sup>4</sup> as given by

$$\bar{n}_s' = \bar{n}_s + D/\bar{n}_s,$$

where  $D$  is the dispersion of star distribution with respect to  $n_s$ . The quantity  $\bar{n}_s'$  obtained for

stars with  $n_s \geq 3$  from data of reference 2 was found to be equal to  $4.78 \pm 0.10$ . It can be seen that this value is, within the limits of error, in agreement with the value of the mean multiplicity for our selection criterion. This shows that, in selecting tracks with  $l \geq 6$  mm from stars with  $n_s \geq 3$ , there was no bias against some stars and relativistic tracks.

## 2. SCATTERING AND IONIZATION MEASUREMENTS. PARTICLE IDENTIFICATION

The particles were identified by measuring the multiple Coulomb scattering and ionization. The scattering was measured using a Koristka MS-2 microscope. The table noise was measured by the multiple-beam interference method, and amounted to 0.03, 0.07, and  $0.04 \mu$  for cells of 100, 500, and  $1000 \mu$  respectively. The scattering measurements were carried out by the coordinate method.<sup>8</sup> False scattering<sup>9</sup> for different cell lengths was measured in several layers. The average value of false scattering  $\bar{D}_f$  over a cell of  $400 \mu$  was found to be equal to  $0.29 \mu$ , which corresponds to the Coulomb scattering of a singly-charged particle with  $p\beta = 1$  Bev/c. Therefore, a quantitative energy computation was carried out only for particles for which the value of  $p\beta$ , after correcting for false scattering, was not higher than 650 Mev/c; the maximum correction for false scattering was then less than 15%.

Preliminary measurements were made on each track (20 readings with a cell of  $250 \mu$ ), from which

the optimum cell length  $t$  was determined. (It was required that  $\bar{D}_t/\bar{D}_n \geq 2$ , the average value of the measurement noise  $D_n$  being equal to about  $0.2 \mu$ .) The final measurements were carried out with a cell shorter by a factor of two than the optimum one, in order to exclude the measurement noise (grain noise and reading noise). The value  $p\beta$  was calculated according to the formula

$$p\beta = \frac{k}{0.573 \cdot D_{100}} \text{ Mev/c.}$$

$$D_{100} = \sqrt{\frac{(D_t^2 - D_{t/2}^2) - (D_{t/2}^2 - D_{t/4}^2)}{t^3 - (t/2)^3}} \mu.$$

The scattering constant as a function of the particle velocity and cell size was taken according to Gottstein et al.,<sup>10</sup> and did not vary greatly in the measurements described here, its value being  $k = 24 - 25$ . To take into account the distortion of tracks, calculation of third, and where necessary, fourth differences, was carried out. The statistical error in the scattering measurements, calculated according to the formula  $\sigma = 0.75/\sqrt{N}$ , where  $N$  is the number of cells, was not greater than 13%. For several tracks, the values of  $p\beta$  were calculated taking false scattering into account, according to the maximum likelihood method described by Solntsev.<sup>11</sup> The values of  $p\beta$  thus obtained did not differ, within the limits of error, from the results obtained by the method described above.

As a result of scattering measurements on the selected 204 tracks, it was found that 78 tracks have  $p\beta \leq 650$  Mev/c and 126 tracks  $p\beta > 650$

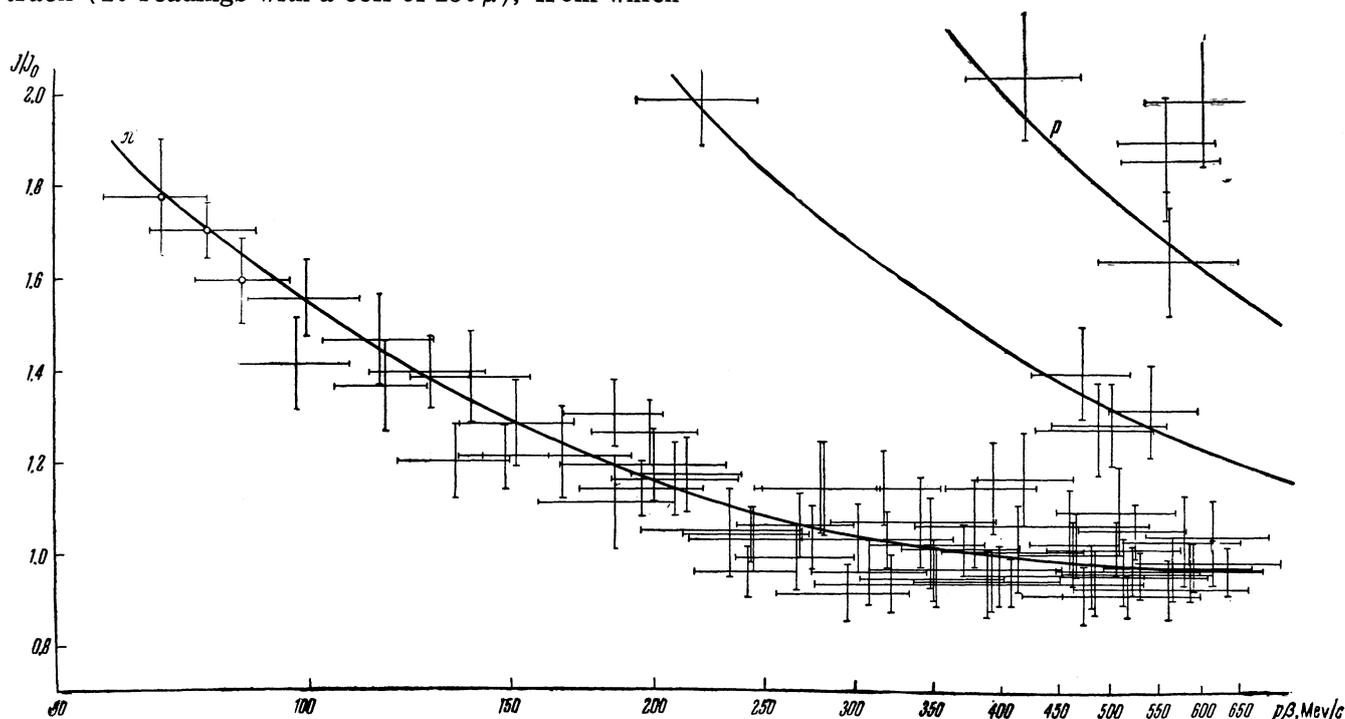


FIG. 1. Variation of ionization with  $p\beta$  for the NIKFI-R emulsion.

Data on K mesons produced in collisions between 9 Bev protons and nuclei\*

$\theta$ , deg	$\eta$	$J/J_0$	$p\beta$ , Mev/c	$E_K$ , Mev	Type of parent star	
					$n_s$	$N_h$
32	22.5	1.4	475±46	290	4	3
35	24.0	1.3	506±58	315	4	4
26	18.3	1.3	490±57	300	5	6
3	2.0	2.0	222±26	121	6	6
27	19.0	1.3	546±45	345	7	6

\* $N_h$  — number of particles with ionization  $J > 1.4 J_0$  in the star;  $E_K$  — kinetic energy of the K meson.

Mev/c. Ionization was measured by counting of gaps and blobs above a certain size along the given track.<sup>12</sup> Statistical errors of the measurement of the relative ionization were not greater than 7%.

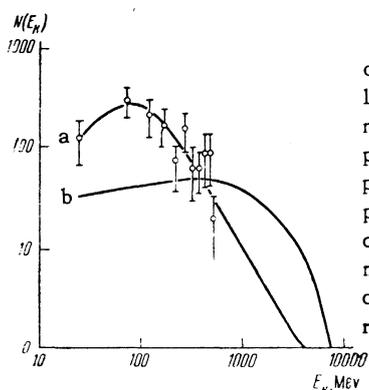


FIG. 2. Energy spectrum of  $\pi$  mesons produced in collisions of 9-Bev protons with nuclei; a — data of the experiments, the curve corresponds to the empirical dependence (1); b — theoretical curve for spectrum of  $\pi$  mesons produced in N-N collisions according to reference 13.

The variation of ionization with  $p\beta$  obtained in the experiment is shown in Fig. 1. The circles denote  $\pi$  mesons stopping in the emulsion chamber. Their energy, as found from multiple scattering, is in agreement with the energy determined from range measurements. Curves for K mesons and protons are constructed from the  $\pi$  meson curve. It is evident that the experimental points are grouped around these curves. Among the particles with  $p\beta \leq 650$  Mev/c and  $J \leq 2 J_0$ , 69  $\pi$  mesons, 4 protons, and 5 K mesons were identified. Data concerning K mesons and their parent star are given in the table. In order to detect associated production of charged strange particles, all black and grey tracks ( $J > 1.4 J_0$ ) emerging from stars in which K mesons were identified were scanned. No strange particles were found among the scanned tracks.

Particles with  $p\beta > 650$  Mev/c and  $J \leq 1.4 J_0$  were not identified; it was assumed that this group of particles consists only of  $\pi$  mesons and protons, and the particles with  $p\beta > 650$  Mev/c and ionization in the range  $1.4 J_0 \leq J \leq 2 J_0$  were considered to be protons. After introducing geometrical corrections, the following effective numbers

of particles were obtained;  $\pi$  mesons — 1326, protons — 266, K mesons — 86, unidentified particles — 879.

In scanning the tracks with ionization  $J > 2 J_0$ , one event of  $\Sigma$ -hyperon decay was detected among the tracks stopping in the chamber. The hyperon was produced in a parent star of the type  $4 + 2 p$ . The emission angle of the hyperon was  $29^\circ$ , ionization  $J = 6.5 J_0$ , energy determined from ionization and scattering  $\sim 60$  Mev. The hyperon traversed 7.7 mm and decayed in flight according to the scheme  $\Sigma^\pm \rightarrow n + \pi^\pm$ ; the ionization of the  $\pi$  meson  $J = 1.1 J_0$  and the energy was approximately 100 Mev. The angle of emission of the  $\pi$  meson in the c.m.s. was  $\theta_\pi^* = 65^\circ$ .

### 3. ENERGY SPECTRUM OF $\pi$ MESONS

The energy spectrum of  $\pi$  mesons is shown in Fig. 2. As mentioned above, the measurement of the energy of particles was limited by the value  $p\beta \leq 650$  Mev/c (corresponding  $\pi$  meson energy 540 Mev). Moreover, the scattering was measured only for tracks with  $J \leq 2 J_0$ , which limited the spectrum from the low-energy end. For the determination of the number of slow  $\pi$  mesons emerging from stars with  $n_s \geq 3$ , all tracks with ionization greater than  $2 J_0$  were followed in some of the stars. From the number of  $\pi$  mesons with  $J \geq 2 J_0$  found in this fraction of stars, the total number of slow  $\pi$  mesons in all selected stars with  $n_c \geq 3$  was determined.

The following empirical formula was found to fit the experimental points of the spectrum in the measured energy range:

$$N(E_k) = E_k / (a - bE_k^2). \quad (1)$$

where  $E_k$  is the kinetic energy of  $\pi$  mesons in Mev. The coefficients  $a$ ,  $b$ , and  $\alpha$  were determined by the least-squares method\* and found to

\*The computations were carried out using the "Ural" computer. The authors are grateful to N. N. Govorun for carrying these out, and also to V. A. Meshcheryakov for help in reducing the data.

be  $a = 0.17 \pm 0.07$ ,  $b = (1.2 \pm 1.4) \times 10^{-6}$ ,  $\alpha = 2.60 \pm 0.35$  (see curve a in Fig. 2). Information on the spectrum in the meson energy range  $> 540$  Mev was obtained by extrapolating the empirical function found up to the energy of 7.6 Bev (maximum energy which can be attained by  $\pi$  mesons in nucleon-nucleon collisions in three-pion production by 9 Bev incident nucleons). The spectrum of  $\pi$  mesons measured in cosmic rays up to the energy of 2 Bev by Baradzei et al.<sup>14</sup> for primary particle energy of about 10 Bev is of the same form. The number of  $\pi$  mesons with  $p\beta > 650$  Mev/c, obtained by extrapolating the spectrum, was found to be equal to 400, and hence the number of protons with  $p\beta > 650$  Mev/c equals 479.

An estimate of the mean total pion energy, taking into account the inaccuracy due to the extrapolation, leads to a value  $\bar{E} = (0.7 \pm 0.2)$  Bev for the whole spectrum and  $\bar{E} = (0.8 \pm 0.2)$  Bev for fast mesons. The estimates of the mean energy of  $\pi$  mesons given in references 4 and 5 are in agreement with these results.

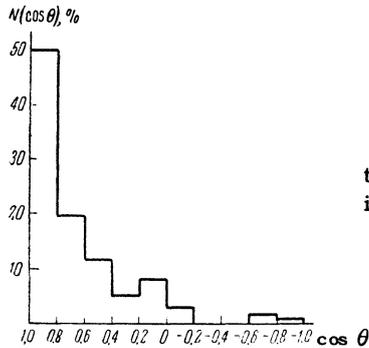


FIG. 3. Angular distribution of  $\pi$  mesons with ionization  $J \leq 1.4 J_0$ .

The ratio of the number of fast  $\pi$  mesons to the number of fast protons was found to be equal to  $3.2 \pm 1.9$ , and the fast-particle multiplicity  $\bar{n}_S = 4.37 \pm 0.10$ . Hence, one can determine the mean number of fast  $\pi$  mesons ( $n_\pi$ ) and fast protons ( $n_p$ ) per interaction. For the stars under consideration, these numbers are  $n_\pi = 3.3 \pm 0.5$ ;  $n_p = 1.0 \pm 0.5$ . The number of  $\pi$  mesons with  $E_k \leq 80$  Mev per interaction as determined from the spectrum equals  $0.6 \pm 0.2$ . The number of all charged  $\pi$  mesons  $N_\pi$  (fast and slow) per interaction equals  $3.9 \pm 0.5$ . In order to determine the mean energy of  $\pi$  mesons separately for light and heavy nuclei, all  $\pi$  mesons were divided into two groups:  $\pi$  mesons from stars with  $N_h > 8$  (60% of the total number of  $\pi$  mesons), and from stars with  $N_h \leq 8$  (40% of the total number of  $\pi$  mesons). It was found that, in the measured spectral region, the mean energy of  $\pi$  mesons from heavy nuclei ( $N_h > 8$ ) is smaller by roughly 100 Mev than the energy of  $\pi$  mesons from the group of nuclei with  $N_h \leq 8$ .

In order to find the variation of the mean  $\pi$ -meson energy with the number of fast particles in a star, all stars were divided into two groups: one with  $n_S = 3$  or 4, and the second with  $n_S > 4$ . The number of stars in the groups was found to be roughly the same. The mean energy of  $\pi$  mesons for these groups over the measured part of the spectrum does not change, within the limits of error.

#### 4. ANGULAR DISTRIBUTION OF FAST $\pi$ MESONS AND PROTONS

The angular distribution of fast  $\pi$  mesons ( $J \leq 1.4 J_0$ ) in the laboratory system is shown in Fig. 3.  $\pi$  mesons with energy greater than 540 Mev were included in the first interval ( $\cos \theta = 1.0 - 0.8$ ), as found from the angle-energy relation for  $\pi$  mesons shown in Fig. 4. The points in

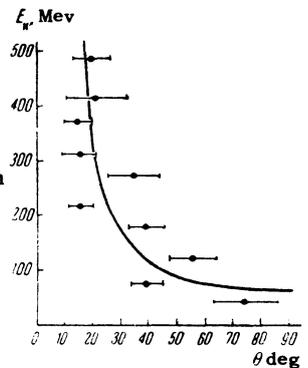


FIG. 4. Variation of  $\pi$  meson energy with emission angle.

Fig. 4 have been obtained by averaging the data over all emission angles of  $\pi$  mesons in the energy interval. The half angle of fast  $\pi$  mesons (the angle which contains half of all particles)  $\theta_{1/2}^\pi = (36.5 \pm 8.8)^\circ$ . The angular distribution of fast protons in the laboratory system is shown in Fig. 5.

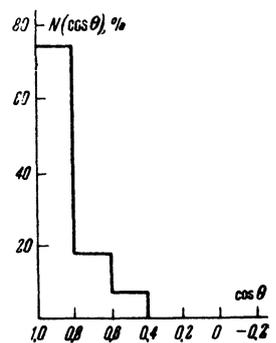


FIG. 5. Angular distribution of protons with ionization  $J \leq 1.4 J_0$ .

Since none of the fast protons ( $p\beta > 650$  Mev/c) have been identified, in order to construct their angular distribution, fast  $\pi$  mesons ( $p\beta > 650$  Mev/c) were subtracted from the angular distribution of all fast unidentified particles. (All the  $\pi$  mesons were subtracted from the interval  $\cos \theta = 1.0 - 0.8$ , as found from the angle-energy relation.)

The half angle for fast protons is  $\theta_{1/2}^p \sim 29^\circ$ . The half angle of all fast particles for stars with  $n_s \geq 3$  equals  $\theta_{1/2}^{\pi p} = (30.5 \pm 5.0)^\circ$ .

## 5. DISCUSSION OF RESULTS

1. A comparison of the energy spectrum obtained (Fig. 2) with the spectrum of  $\pi$  mesons obtained in cosmic rays<sup>14</sup> shows that, within the limits of error, the spectra are not different. The spectrum obtained in the present experiment has a maximum near 100 Mev. Spectra of secondary  $\pi$  mesons originating in collisions of  $\pi$  mesons with energies of 1.5 and 4.2 Bev with emulsion nuclei, as obtained by other authors,<sup>15,16</sup> also have a maximum in the range 100 – 150 Mev. Thus, independent of the nature of the incident particle, and over a wide energy range, the position of the maximum in the spectrum does not change. This, evidently, indicates that the majority of  $\pi$  mesons, at least in the 100 – 150 Mev energy range, are essentially produced as a result of secondary collisions inside the nucleus. This is also confirmed by a comparison of the spectrum obtained with the spectrum of  $\pi$  mesons calculated<sup>13</sup> for nucleon-nucleon collisions according to the statistical theory of multiple particle production and using the isobar method (see curve b in Fig. 2). It is evident that the observed energy spectrum is shifted towards lower energies. It is natural to assume that the difference between the observed spectrum of  $\pi$  mesons emitted in collisions of protons with nuclei and the theoretical spectrum of  $\pi$  mesons in nucleon-nucleon collisions is due to secondary processes inside the nucleus. (It is also quite possible that the statistical theory does not predict the spectrum of  $\pi$  mesons from N-N collisions accurately enough.)

As has been mentioned above, the mean energy of  $\pi$  mesons originating in stars with  $N_h > 8$  in the measured part of the spectrum is, by roughly 100 Mev, lower than for the group of stars with  $N_h \leq 8$ . In addition, if  $\pi$  mesons with  $p\beta < 650$  Mev/c are emitted roughly with the same frequency from stars with  $N_h > 8$  and  $N_h \leq 8$ , then the particles with  $p\beta > 650$  Mev/c are emitted from the stars with  $N_h \leq 8$  three times more frequently than from stars with  $N_h > 8$ . These results can be explained by a large number of collisions in heavy nuclei, as compared with the number of collisions in light nuclei. In fact, it is natural to assume that the number of collisions in the nucleus is proportional to the radius of the nucleus, and, consequently, the number of collisions in heavy nuclei of the emulsion should be twice of that in light nuclei. On the other hand, the mean

number of fast particles in the groups of stars under investigation does not differ greatly. Thus, for stars with  $N_h \leq 8$  and  $N_h > 8$ , these numbers are  $4.24 \pm 0.13$  and  $4.47 \pm 0.16$  respectively. This can be explained by the fact that, in heavy nuclei, the multiplicity of relativistic particles produced in secondary collisions decreases rapidly with the development of the cascade.

2. The results obtained in the experiment make it possible to find the mean energy spent for the production of  $\pi$  mesons in the collision of 9 Bev protons with nuclei. Using the value of the mean energy of  $\pi$  mesons in the spectrum and the mean number  $N_\pi$  of charged  $\pi$  mesons per interaction for the chosen class of stars, one can find that the following energy is, on the average, spent for meson production

$$E_\pi = \frac{3}{2} \bar{E} N_\pi = (4.1 \pm 1.3) \text{ Bev,}$$

which amounts to  $(45 \pm 14)\%$  of the energy of the primary proton. (It is assumed here that  $\pi^0$  mesons carry away half of the energy transferred to charged  $\pi$  mesons.)

In experiments carried out using the proton synchrotron at 9-Bev proton energy,<sup>3-5</sup> the energy fraction carried away by fast  $\pi$  mesons equalled  $(44 \pm 9)\%$ ,  $(27 \pm 8) - (33 \pm 8)\%$ , and  $(33 \pm 9)\%$  respectively.

It should be mentioned that the energy carried away by  $\pi$  mesons obtained in the present experiment should be higher than the average for all disintegrations found in scanning along tracks, since events with  $n_s \geq 3$ , in which a large number of  $\pi$  mesons is produced, have been selected. Using the value for the energy fraction spent for meson production as obtained in the present article and that of the disintegration energy of nuclei from reference 3, we obtain the value  $0.43 \pm 0.14$  for the energy fraction carried away by the proton after colliding with a nucleus. For nucleon-nucleon collisions, the energy fraction carried away by the proton is, on the average, equal to  $0.65$ .<sup>7</sup>

From these results, it follows that the primary proton undergoes approximately two collisions with an average nucleus of the emulsion.

3. The measurements of the values of  $p\beta$  and of the ionization along tracks of particles produced in collisions of primary protons with nuclei made it possible to identify five  $K^\pm$  mesons in the ionization range  $1.2 J_0 \leq J \leq 2 J_0$  (corresponding to a velocity interval  $\beta = 0.8 - 0.5$ ). The energy interval for  $\pi$  mesons with such velocities is 22 – 95 Mev. Using the effective number of K mesons (86) and the number of  $\pi$  mesons calculated from

the spectrum (409), we obtain the value  $5.0 \pm 2.5$  for the ratio  $n_\pi/n_K$  in the energy range under consideration. It should be noted that Kostanashvili and Shakhulashvili<sup>17</sup> obtained  $n_\pi/n_K = 6$  for the range  $\beta \leq 0.67$ .

## CONCLUSIONS

1. The energy spectrum of charged  $\pi$  mesons produced in interactions between 9-Bev protons and emulsion nuclei, with a number of charged particles  $n_S \geq 3$ , is described by the empirical formula (1).

2. The mean total  $\pi$ -meson energy  $\bar{E} = (0.70 \pm 0.2)$  Bev, and the mean total energy of a fast  $\pi$  meson is equal to  $(0.8 \pm 0.2)$  Bev.

3. The mean number of fast  $\pi$  mesons and protons per interaction is  $3.3 \pm 0.5$  and  $1.0 \pm 0.5$  respectively. For the mean number of  $\pi$  mesons with energy smaller than 80 Mev, the value of  $0.6 \pm 0.2$  has been obtained.

4. The energy fraction carried away by  $\pi$  mesons (taking  $\pi^0$  mesons into account) amounts to  $(45 \pm 14)\%$ .

5. The ratio of the number of charged  $\pi$  mesons and K mesons in the velocity interval  $\beta = (0.5 - 0.8)$  equals  $5.0 \pm 2.5$ .

6. The experimental data obtained do not contradict the assumption that the interaction of 9-Bev protons with complex nuclei represents, for the stars under consideration, a series of consecutive collisions.

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