neglect the possibility of decays leading to the formation of strange particles.

In peripheral interactions of this type the directions of motion of the isobar and of its decay products in the center of mass system do not differ much from the direction of motion of the nucleons before the interaction. According to the criterion chosen in reference 3 for the selection of such peripheral collisions (the presence of a slow proton in the laboratory system), we consider in the following only those "stars" in which there is a proton flying in the backward direction in the center of mass system. Taking this into account and making use of isotopic invariance, we can calculate with the help of the Clebsch-Gordan coefficients the probabilities W_{mn} for the observation of stars in which (in the center of mass system) m charged particles are emitted in the forward direction and n charged particles (including the proton) in the backward direction.

For p-p collisions we find

$$W_{11} = \tfrac{4}{15}\sigma_{pp}(X, X) + \tfrac{1}{16}(1 + \omega_1 + \tfrac{1}{27}\omega_2)\sigma_{pp}(X, Y) + \tfrac{1}{12}(1 - \omega_1 - \tfrac{1}{9}\omega_2)(1 + \omega_1 - \tfrac{1}{3}\omega_2)\sigma_{pp}(Y, Y),$$

$$W_{02} = \tfrac{1}{5}\sigma_{pp}(X, X) + \tfrac{1}{16}(1 + \omega_1 + \tfrac{1}{3}\omega_2)\sigma_{pp}(X, Y),$$

$$W_{22} = \tfrac{1}{5}\sigma_{pp}(X, X) + \tfrac{1}{16}(7 + \omega_1 - \omega_2)\sigma_{pp}(X, Y)$$

(we do not give the expressions for W_{13}, W_{31}, and W_{33}, since these cases were neglected in the analysis of the experiment[3]). For p-n collisions we find

$$W_{01} = \tfrac{2}{63}\sigma_{pn}(X, X) + \tfrac{1}{18}(1 + \omega_1 + \tfrac{1}{3}\omega_2)\sigma_{pn}(X, Y) + \tfrac{2}{15}(1 + \omega_1 + \tfrac{1}{3}\omega_2)^2\sigma_{pn}(Y, Y),$$

$$W_{21} = \tfrac{1}{63}\sigma_{pn}(X, X) + \tfrac{1}{72}(11 - \omega_1 - \tfrac{1}{3}\omega_2)\sigma_{pn}(X, Y) + \tfrac{1}{45}(5 - \omega_1 - \tfrac{1}{3}\omega_2)(1 + \omega_1 + \tfrac{1}{3}\omega_2)\sigma_{pn}(Y, Y),$$

$$W_{12} = \tfrac{2}{21}\sigma_{pn}(X, X) + \tfrac{1}{8}(1 + \omega_1 - \tfrac{7}{27}\omega_2)\sigma_{pn}(X, Y) + \tfrac{1}{30}(1 + \omega_1 - \tfrac{1}{3}\omega_2)^2\sigma_{pn}(Y, Y),$$

where $\sigma_{pp}(X, X)$ is the cross section for formation of two isobars in p-p collisions, etc.

For a more rigorous choice of cases of peripheral collisions of this type, those cases in reference 3 were selected in which there is a fast proton in addition to the slow one. We denote the corresponding probabilities by $W_{mn}^{(p)}$:

$$W_{11}^{(p)} = \tfrac{8}{45}\sigma_{pp}(X, X) + \tfrac{1}{36}(1 + \omega_1 + \tfrac{1}{3}\omega_2)\sigma_{pp}(X, Y) + \tfrac{1}{36}(1 + \omega_1 + \tfrac{1}{3}\omega_2)^2\sigma_{pp}(Y, Y),$$

$$W_{22}^{(p)} = \tfrac{1}{5}\sigma_{pp}(X, X) + \tfrac{1}{4}(1 + \omega_1 - \tfrac{1}{3}\omega_2)\sigma_{pp}(X, Y).$$

Dremin and Chernavskiĭ[4] recently made a quantitative estimate of the cross sections for these processes. Using their data and substituting the values of ω_i quoted above, we obtain characteristic numbers (see column a in the table) which can be compared with the results of the experiment.[3] In column b of the table we list for comparison the results of the calculation under the assumption that always only 2X are formed.

	Experiment	Calculation	
		a	b
$W_{22}/(W_{11}+W_{02})$	0.47	0.73	0.43
$-2W_{02}/(W_{11}+W_{02})$	−0.56	−0.88	−0.86
W_{02}/W_{11}	0.39±0.13	0.79	0.75
$W_{11}^{(p)}/W_{22}^{(p)}$	14/8=1.75±0.77	0.60	0.89
$(W_{21}-W_{12})/(W_{21}+W_{12})$	0.33	−0.46	−0.71
W_{12}/W_{21}	1.3	2.6	6

In conclusion I express my deep gratitude to I. E. Tamm and I. L. Rozental' for discussing this paper and also to D. S. Chernavskiĭ, I. M. Dremin, and the authors of reference 3 for providing me with the results of their work before publication.

[1] I. E. Tamm, Материалы конференции по физике высоких энергий в г. Киеве (Materials of the High Energy Conference at Kiev, 1959).

[2] G. Bernardini, ibid.

[3] Wang Shu-Fen, Vishki, Gramenitskiĭ, Grishin, Dalkhazhav, Lebedev, Nomofilov, Podgoretskiĭ, and Strel'tsov, ibid.

[4] I. M. Dremin and D. S. Chernavskiĭ, ibid.

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ON GAUGE TRANSFORMATIONS IN QUANTUM ELECTRODYNAMICS

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INVARIANCE of a quantum-mechanical theory with respect to a particular group of transformations is ordinarily associated with the existence of corresponding constants of the motion. For example, for a system with central symmetry there is conservation of the components of the angular momentum M, which generate the rotation group. Gauge transformations with a constant phase are generated by the charge operator $\hat{Q}$. Such constants of the motion do not exist, however, for the general group of gauge transformations,

$$\psi \to e^{i\Lambda}\psi, \qquad A_\mu \to A_\mu - \partial\Lambda/\partial x_\mu . \tag{1}$$

We shall show that by the introduction of additional variables into the Hamiltonian one can construct an infinite set of constants of the motion, which generate the transformations (1), and thus can include the gauge transformations in the general scheme of canonical transformations.

Let us write the Hamiltonian of quantum electrodynamics in the form

$$H = \sum_{p,\sigma} E_p (a_{p\sigma}^+ a_{p\sigma} + b_{p\sigma}^+ b_{p\sigma}) + \sum_k \omega (c_{1k}^+ c_{1k} + c_{2k}^+ c_{2k} - c_{3k}^+ c_{4k} - c_{4k}^+ c_{3k}) + \sum_k \sum_{\lambda=1}^{4} \frac{e}{\sqrt{2\omega}} \{c_{\lambda k} (e^\lambda j_k^+) + c_{\lambda k}^+ (e^\lambda j_k)\}; \tag{2}$$

$$j_k \equiv (\rho_k, \mathbf{j}_k) = \int \bar{\psi} \gamma_\mu \psi e^{-i\mathbf{kx}} d^3x, \tag{3}$$

where a and b are the operators of the electron-positron field, and c are the operators of the electromagnetic field; j_k are the Fourier components of the current vector. The integral in Eq. (3) is taken over unit volume. The polarization vectors e^λ are chosen in the following way: e^1 and e^2 are unit space vectors perpendicular to **k**, and

$$e^3 = (1, \mathbf{k}/\omega)/\sqrt{2}, \qquad e^4 = (1, -\mathbf{k}/\omega)/\sqrt{2}.$$

For the photons one introduces an indefinite metric. In accordance with this, the operators c satisfy the commutation relations

$$[c_{3k}^+, c_{4k'}] = [c_{4k}^+, c_{3k'}] = \delta_{kk'}, \qquad [c_{3k}^+, c_{3k'}] = [c_{4k}^+, c_{4k'}] = 0$$

(the remaining commutation relations are the usual ones). We now introduce "supplementary" variables α_k and β_k, which satisfy the commutation relations

$$[\alpha_k^+, \beta_{k'}] = [\beta_k^+, \alpha_{k'}] = \delta_{kk'}, \quad [\alpha_k^+, \alpha_{k'}] = [\beta_k^+, \beta_{k'}] = 0 \tag{4}$$

and commute with all the other quantities, and add to the Hamiltonian (2) the quantity

$$H_{\alpha\beta} = -\sum_k \omega (\alpha_k^+ \beta_k + \beta_k^+ \alpha_k).$$

It is easy to verify that the "total" Hamiltonian $H + H_{\alpha\beta}$ commutes with the quantities

$$R_k = \alpha_k^+ (e\rho_k/2\omega^{3/2} - c_{4k}), \qquad R_k^+ = \alpha_k (e\rho_k^+/2\omega^{3/2} - c_{4k}^+). \tag{5}$$

In terms of the operators (5) the gauge transformations (1) can be expressed in the form of a unitary operator

$$U_\Lambda = \exp\left\{ i \sum_k \left(\lambda_k R_k + \lambda_k^+ R_k^+ \right) \right\}, \tag{6}$$

where λ_k are arbitrary numbers. The function $\Lambda(x)$ for the transformation (6) is

$$\Lambda(x) = \sum_k \frac{1}{2\omega^{3/2}} (\lambda_k \alpha_k \exp\{i(\mathbf{kx} - \omega t)\} + \lambda_k^+ \alpha_k^+ \exp\{-i(\mathbf{kx} - \omega t)\}),$$

and in virtue of the relations (4) $\Lambda(x)$ can be regarded as a numerical function.

In our representation the supplementary condition $(\partial A_\mu/\partial x_\mu)\Phi = 0$ can be written in the form

$$(c_{4k} - e\rho_k/2\omega^{3/2})\Phi = 0, \qquad (c_{4k}^+ - e\rho_k^+/2\omega^{3/2})\Phi = 0. \tag{7}$$

Comparing Eqs. (7) and (5), we see that for the allowed states the quantities R_k and $R_k^\dagger$ are equal to zero:

$$R_k \Phi = R_k^+ \Phi = 0. \tag{8}$$

Obviously the conditions (8) single out the Maxwellian electrodynamics from among all the theories described by the Hamiltonian (2).

The variables α and β are of the nature of two additional components of the electromagnetic field. Since these components do not interact with charges, this scheme is entirely equivalent to the usual electrodynamics.

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THE RELATIVISTIC PHOTOEFFECT IN THE L SHELL

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THE problem of a theory of the nonrelativistic photoeffect in the L shell was solved a long time ago.[1,2] The relativistic aspects of this problem, however, have only been remarked upon. In view of the successful development of β-ray spectrom-