result of superposition of fission products of uranium nuclei before and after evaporation of the nucleons. The fission asymmetry of nuclei with small excitation energy is determined by the influence of certain factors, among which the shell effect can play a large role. At large excitation energy the influence of these factors apparently does not manifest itself during the instant of fission. If it is assumed that fission asymmetry of such nuclei is determined by Eq. (1), so that the most probable is symmetrical fission, and that the relative contribution of the asymmetrical form of fission increases with increasing excitation energy, then the change in shape of the mass curve of the fission products of uranium with increasing particle energy becomes understandable. Fairhall et al.[8] have shown that nuclear fission near bismuth at excitation energies up to ~ 40 Mev occurs prior to neutron evaporation. The increase in the fraction of asymmetrical fission with increasing temperature, in accordance with (1), agrees qualitatively with the observed broadening of the mass curves of the fission products of bismuth with increasing excitation energy.[4,9] The broadening of the mass curve of the fission products with increasing atomic number of the target, found in bombardment by 450-Mev protons,[10] may be, in particular, the rèsult of the increase in the average excitation energy with increasing atomic number of the target.

Among the experiments performed up to now on the fission of nuclei, two groups can be segregated. In accordance with the experiments of the first group, the fission of nuclei near uranium competes successfully with the evaporation of neutrons over a broad range of excitation energies.[11,12] The investigations of the second group[13] are evidence that the fission of uranium bombarded with high energy protons occurs essentially after the excitation energy has been removed by nucleon evaporation. The "cold" nucleus can have a large angular momentum. Consequently, and also as a result of the change in the composition of the nucleons, the fission characteristics of such a nucleus, including the asymmetry, may differ from the characteristics of nuclear fission in the case of small particle energies.

In conclusion, the author expresses his gratitude to Prof. N. A. Perfilov for interest in this work.

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ON THE CROSS SECTION FOR COMPOUND NUCLEUS FORMATION IN THE INTERACTION OF ATOMIC NUCLEI

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THE available data[1] on the nuclear reactions induced by multiply charged ions indicate that one of the basic processes in these reactions is the formation of a compound nucleus with high energy of excitation and its subsequent decay.

The cross section for compound nucleus formation, $\sigma(E)$, can be calculated on the basis of a model in which the colliding nuclei have a sharp spherical boundary and are completely absorptive for particles entering the sphere of nuclear interaction.

The fact that the problem contains the large parameter* $\eta = Z_1 Z_2 e^2/\hbar v \gg 1$ indicates that in this model the cross section $\sigma(E)$ must be close to the classical value $\sigma(E) = \pi R^2 (1 - B/E)$ for ion energies above the Coulomb barrier $B = Z_1 Z_2 e^2/R$ ($R = R_1 + R_2$ is the radius of the region of nuclear interaction). The condition of quasi-classical motion for an ion with angular momentum $l\hbar$ near the boundary $r = R$, $|d\lambda\!\!\!^{-}_l(r)/dr| \ll 1$, can be written in the form (we take advantage of the fact that in reactions induced by multiply charged ions the parameter $\rho = R/\lambda\!\!\!^{-}(\infty) \gg 1$)†

$$1 - \beta - \gamma_l^2 \gg \left(\frac{2-\beta}{9\rho}\right)^{2/3}, \qquad \beta = \frac{2\eta}{\rho}, \qquad \gamma_l = \frac{l+1}{\rho}. \quad (1)$$

According to the classical model of a black sphere the values of l satisfying condition (1) with an equality sign give the following contribution to the cross section:

$$\sigma = \pi R^2 \left[1 - \beta - \left(\frac{2-\beta}{9\rho}\right)^{2/3}\right]. \quad (2)$$

The contribution from angular momenta which do not satisfy condition (1) can be calculated for arbitrary values of β with the help of a simple quantum mechanical model close to the classical model.‡ In view of the large excitation energy and, hence, of the high level density of the compound nucleus it is appropriate to use the resonanceless theory with a nuclear potential in the form of a square well with radius R. The depth U plays practically no role in reactions with multiply charged ions. Since the nucleons in the ion are bound more strongly than the nucleons in the ground state of the compound nucleus (for target nuclei with $A > 50$), part of the kinetic energy of the ion is used for the "unpacking," i.e., the lowest bound state level in the well lies above zero. Therefore $U/E < \pi^2/4\rho^2 \ll 1$, and the well depth can be neglected in comparison with the energy E.

Keeping this remark in mind, we now consider the known[2] expression for the partial reaction cross section in this model:

$$\sigma_l = \pi\lambda\!\!\!^{-2} (2l+1)\, 4 s_l \rho / [\Delta_l^2 + (s_l + \rho)^2]. \quad (3)$$

The quantities s_l and Δ_l are expressed in terms of the Coulomb functions $G_l(\rho)$ and $F_l(\rho)$ and their derivatives $G_l'(\rho)$ and $F_l'(\rho)$. The presence of the large parameters ρ and η in the problem allows us to obtain simple analytic expressions for s_l and Δ_l and, hence, for σ_l in the form of asymptotic expansions in terms of inverse powers of these parameters. We use contour integral representations[3] in the complex plane for G_l and F_l and shift the parameter η to the exponent under the integral. By the method of steepest descent we then obtain

$$\left.\begin{aligned} s_l &= \frac{\pi 3^{1/3} (2-\beta)^{1/3}}{\Gamma^2(1/3)} \rho^{2/3} [1 + O(\rho^{-1/3})], \\ \Delta_l &= -\frac{\pi (2-\beta)^{1/3}}{3^{1/3}\Gamma^2(1/3)} \rho^{2/3} [1 + O(\rho^{-1/3})], \end{aligned}\right\} |1 - \beta - \gamma_l^2| \ll \left(\frac{2-\beta}{9\rho}\right)^{2/3};$$

$$s_l = \rho a \left[\frac{(a+\gamma_l)^2 + 1}{(a-\gamma_l)^2 + 1}\right]^{1/2} \exp\left[-\rho\left(\gamma_l \ln\frac{(a+\gamma_l)^2+1}{(a-\gamma_l)^2+1} + \beta \tan^{-1}\frac{2a}{2-\beta} - 2a\right)\right] [1 + O(\rho^{-1/2})], \quad (4)$$

$$\Delta_l = -\rho a [1 + O(\rho^{-1})], \quad a = \gamma_l^2 + \beta - 1 \gg \left(\frac{2-\beta}{9\rho}\right)^{2/3}. \quad (5)$$

Summing over all l with the corresponding extrapolation to the values l determined by the equalities

$$1 - \beta - \gamma_l^2 = \pm\left(\frac{2-\beta}{9\rho}\right)^{2/3},$$

we find** for the different regions of the ion energy ($\beta = B/E$, terms $\sim \rho^{-4/3}$ are neglected)

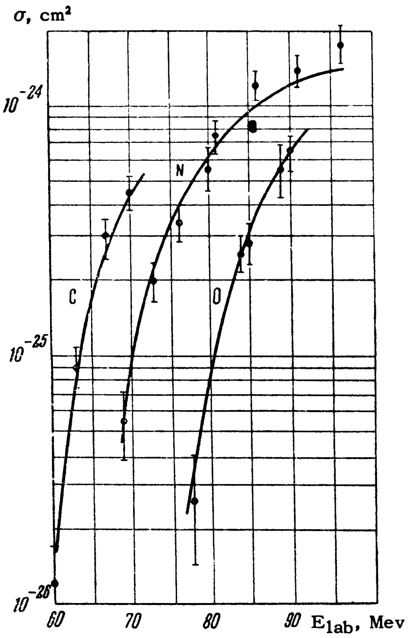
$$\sigma = 2\pi R^2 \rho^{-1} \beta \exp\left[-\rho\left(\beta \tan^{-1}\frac{2\sqrt{\beta-1}}{2-\beta} - 2\sqrt{\beta-1}\right)\right], \quad \beta - 1 \gg \left(\frac{2-\beta}{9\rho}\right)^{2/3};$$

$$\sigma = 2\pi R^2 \rho^{-1} \left\{\exp\left(-\frac{4}{27}\right) + \frac{2\pi}{3\Gamma^2(1/3)}\left[1 + \left(\frac{2-\beta}{9\rho}\right)^{-2/3}(1-\beta)\right]\right\}, \quad |1 - \beta| \ll \left(\frac{2-\beta}{9\rho}\right)^{2/3};$$

$$\sigma = \pi R^2 \left[1 - \beta - \left(\frac{2-\beta}{9\rho}\right)^{2/3} + \frac{2(2-\beta)}{\rho}\left(\exp\left(-\frac{4}{27}\right) + \frac{4\pi}{3\Gamma^2(1/3)}\right)\right], \quad 1 - \beta \gg \left(\frac{2-\beta}{9\rho}\right)^{2/3}. \quad (6)$$

The comparison of the cross sections (6) with the experimental data of Druin and Polikanov[4] on the fission cross sections of bismuth bombarded by the ions of carbon, nitrogen, and oxygen, which practically coincide with the corresponding cross sections for the formation of a compound nucleus, leads to good agreement (see the figure) for the following choices of the parameter R:

$$C^{12},\ R = 1.17 \cdot 10^{-12}\ \text{cm};$$
$$N^{14},\ R = 1.24 \cdot 10^{-12}\ \text{cm};$$
$$O^{16},\ R = 1.27 \cdot 10^{-12}\ \text{cm}.$$



Thomas[5] and Piliya[6] also considered the cross section for compound nucleus formation caused by heavy ions. Thomas calculated the cross sections numerically for several ions and target nuclei, using formula (3) with a definite choice of the nuclear potential parameters R and U, which makes it difficult to use his results in the case of arbitrary nuclei (see also footnote‡). The results of Piliya are quite different from those quoted above, since he made use of an incorrect asymptotic expansion.

In conclusion I express my gratitude to G. N. Flerov for his interest in this work and also to G. N. Vyalov and S. M. Polikanov for fruitful remarks.

*The whole discussion is in the center-of-mass system of the colliding nuclei.

†Condition (1) differs from $|d\lambda\!\!\!^{-}(R)/dR| \ll 1$ by a coefficient ~ 1 which was introduced to be used in connection with condition (4).

‡The condition of complete absorption at the nuclear boundary in the quantum mechanical description (only incoming waves in the region $r < R$) is different from the analogous condition in the classical model. The use of the quantum description for all angular momenta would lead to a smaller cross section which does not go over into the classical result for $\hbar \to 0$ ($\rho \to \infty$).

**In the same way we can obtain expressions for the average moment of inertia of the nucleus.

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*PHOTOPRODUCTION OF POSITIVE PIONS IN HYDROGEN NEAR THRESHOLD**

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EXPERIMENTS on photoproduction of pions from nucleons near threshold play an essential role in testing the meson theory based on dispersion relations. In particular, there is great interest in the behavior of the square of the matrix element for the photoproduction of positive pions near threshold, since according to the theory the direct interaction of the photon with the meson current leads to an increase in the square of the matrix element as the photon energy decreases. Besides this, a comparison of the π^+ photoproduction cross section for hydrogen near threshold with the π^- photoproduction cross section for neutrons[1,2] allows us to match the experimental data with the predictions of meson theory about the quantity σ^-/σ^+ near threshold. Our work is devoted to clarifying these questions.