GREEN'S FUNCTION FOR ODD NUCLEI

Yu. T. GRIN', S. I. DROZDOV, and D. F. ZARETSKIĬ

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The techniques of many-body theory are applied to a study of pair correlations in finite systems with an odd number of particles. The Green's function is found, and perturbation theory developed.

PROPERTIES of low-excited states of Fermi systems with an even number of particles differ essentially from those of systems with an odd number of particles. If one considers the application of many-body theory to nuclei, then one encounters the problem of finding the Green's function for odd systems in order to calculate the moments of inertia, magnetic moments, excitation spectra and electromagnetic transition probabilities in odd nuclei. Migdal[1] developed techniques for studying pair correlations between particles in finite systems with an even number of particles. In the present work, the extension of these techniques to finite systems containing an odd number of particles is considered. The Green's function is calculated for such systems and perturbation theory is formulated.

For our purposes, it is convenient to write Gor'kov's system of equations[2,1] separately for the Green's functions $G^+(x_1, x_2)$ and $G^-(x_1, x_2)$ respectively for $\tau = t_1 - t_2 > 0$ and $\tau < 0$.

$$(i\partial/\partial\tau - H)G^+ - i\Delta F^+ = 0, \quad (i\partial/\partial\tau + H^* - 2\mu)F^+ + i\Delta^* G^+ = 0, \tag{1}$$

$$(i\partial/\partial\tau - H)G^- - i\Delta F^- = 0, \quad (i\partial/\partial\tau + H^* - 2\mu)F^- + i\Delta^* G^- = 0.$$

$$\Delta^* = \gamma F(\mathbf{r}_1, \mathbf{r}_2, \tau)|_{\tau=0}, \tag{2}$$

where

$$G^+(x_1, x_2) = -i(\Phi_0^N, \psi(x_1)\psi^+(x_2)\Phi_0^N),$$

$$G^-(x_1, x_2) = i(\Phi_0^N, \psi^+(x_2)\psi(x_1)\Phi_0^N),$$

$$F^+(x_1, x_2) = (\Phi_0^{N+2}, \psi^+(x_1)\psi^+(x_2)\Phi_0^N),$$

$$F^-(x_1, x_2) = -(\Phi_0^N, \psi^+(x_2)\psi^+(x_1)\Phi_0^{N-2}). \tag{3}$$

Here Φ_0^N is the wave function for the ground state of the N-particle system; $\psi(x)$, $\psi^+(x)$ are particle annihilation and creation operators in the Heisenberg representation. At $t = 0$, one has the usual commutation relations for $\psi(\mathbf{r}, t)$

$$\{\psi(\mathbf{r}_1), \psi^+(\mathbf{r}_2)\} = \delta(\mathbf{r}_1 - \mathbf{r}_2),$$
$$\{\psi(\mathbf{r}_1), \psi(\mathbf{r}_2)\} = \{\psi^+(\mathbf{r}_1), \psi^+(\mathbf{r}_2)\} = 0. \tag{4}$$

The chemical potential is defined by

$$\mu = \tfrac{1}{2}[E_0(N+2) - E_0(N)],$$

where $E_0(N)$ is the ground-state energy of the N-particle system. The parameter γ characterizes the effective interaction which leads to the pairing. We will assume Δ to be constant in our system.

We are interested in comparing properties of systems of different parity in particle number; therefore, we neglect everywhere differences in properties of neighboring systems of the same parity. Boundary conditions on G and F can be written, using their definitions (3) and commutation relations (4), in the form

$$i[G^+ - G^-]_{\tau=0} = \delta(\mathbf{r}_1 - \mathbf{r}_2); \quad [F^+ - F^-]_{\tau=0} = 0. \tag{5}$$

We expand G and F in terms of eigenfunctions of the single-particle Hamiltonian H

$$G^\pm(\mathbf{r}_1, \mathbf{r}_2, \tau) = \sum_{\lambda\lambda'} G^\pm_{\lambda\lambda'}(\tau)\varphi^*_\lambda(\mathbf{r}_1)\varphi_{\lambda'}(\mathbf{r}_2),$$

$$F^\pm(\mathbf{r}_1, \mathbf{r}_2, \tau) = \sum_{\lambda\lambda'} F^\pm_{\lambda\lambda'}(\tau)\varphi^*_\lambda(\mathbf{r}_1)\varphi_{\lambda'}(\mathbf{r}_2),$$

where $H\varphi_\lambda = \epsilon_\lambda\varphi_\lambda$. If Δ is constant, only the diagonal terms remain in these expansions.[1] Therefore, instead of Eqs. (1), (2), and (5) we obtain

$$(i\partial/\partial\tau - \varepsilon_\lambda)G^\pm_\lambda - i\Delta F^\pm_\lambda = 0, \quad (i\partial/\partial\tau + \varepsilon_\lambda - 2\mu)F^\pm_\lambda + i\Delta^* G^\pm_\lambda = 0; \tag{6}$$

$$\Delta^* = \gamma \sum_\lambda F_\lambda(0)\varphi^*_\lambda(\mathbf{r})\varphi_\lambda(\mathbf{r}); \tag{7}$$

$$i[G^+_\lambda(0) - G^-_\lambda(0)] = 1, \quad F^+(0) - F^-_\lambda(0) = 0. \tag{8}$$

The solution of (6) has the form

$$G^\pm_\lambda(\tau) = C^\pm_{1\lambda}\exp\{i(E_\lambda - \mu)\tau\} + C^\pm_{2\lambda}\exp\{-i(E_\lambda + \mu)\tau\}; \tag{9a}$$

$$F^\pm_\lambda(\tau) = \frac{i\Delta}{E_\lambda - \varepsilon_\lambda}C^\pm_{1\lambda}\exp\{i(E_\lambda - \mu)\tau\} - \frac{i\Delta}{E_\lambda + \varepsilon_\lambda}C^\pm_{2\lambda}\exp\{-i(E_\lambda + \mu)\tau\}. \tag{9b}$$

In (9b) and in the following, ϵ_λ is measured from μ as origin.

Using the initial conditions, Eqs. (8), it is possible to eliminate two constants:

$$C_{1\lambda}^{+} = C_{1\lambda}^{-} - i(E_\lambda - \varepsilon_\lambda)/2E_\lambda, \quad C_{2\lambda}^{+} = C_{2\lambda} - i(E_\lambda + \varepsilon_\lambda)/2E_\lambda.$$

In order to determine two other constants, we compare the solution obtained with the exact expression for the Green's function of the N-particle system. We consider a nucleus in which there are N_a particles with positive projection of angular momentum $(+\lambda)$ and N_b particles with negative projection $(-\lambda)$. An exact expression for the diagonal part of the Green's function can be put in the form:[3]

$$G_\lambda^{+} = i\sum_s |(a_\lambda^{+})_{s0}|^2 \times \exp\{-i[E_s(N_a+1, N_b) - E_0(N_a, N_b)]\tau\},$$

$$G_\lambda^{-} = -i\sum_s |(a_\lambda)_{s0}|^2 \times \exp\{i[E_s(N_a-1, N_b) - E_0(N_a, N_b)]\tau\},$$

$$G_{-\lambda}^{+} = i\sum_s |(a_{-\lambda}^{+})_{s0}|^2 \times \exp\{-i[E_s(N_a, N_b+1) - E_0(N_a, N_b)]\tau\},$$

$$G_{-\lambda}^{-} = -i\sum_s |(a_{-\lambda})_{s0}|^2 \times \exp\{i[E_s(N_a, N_b-1) - E_0(N_a, N_b)]\tau\}, \tag{10}$$

where $E_s(N\pm1)$ is the energy of the s-th state of the system of $N\pm1$ particles. We use the notation $2E_0(N_a+1, N_b) - E_0(N_a+1, N_b+1) - E_0(N_a, N_b) = 2\tilde{\Delta}$. Comparing exponents in (9a) and (10) for G_λ^{+} and $G_{-\lambda}^{+}$, we obtain

$$E_\lambda + \tilde{\Delta} = -\varepsilon_s^{(1)}(N_a+1, N_b), \tag{11a}$$

$$E_\lambda - \tilde{\Delta} = \varepsilon^{(2)}(N_a+1, N_b), \tag{11b}$$

$$E_\lambda + E_0(N_a, N_b+1) - E_0(N_a+1, N_b) + \tilde{\Delta} = -\varepsilon_s^{(3)}(N_a, N_b+1), \tag{11c}$$

$$E_\lambda - E_0(N_a, N_b+1) + E_0(N_a+1, N_b) - \Delta = \varepsilon_s^{(4)}(N_a, N_b+1). \tag{11d}$$

Comparison of exponents for G_λ^{-} and $G_{-\lambda}^{-}$ gives equations coinciding with Eqs. (11). The quantity $\epsilon_s(N_a, N_b) = E_s(N_a, N_b) - E_0(N_a, N_b)$ is positive by definition. We will show that the condition $\epsilon_s \geq 0$ sets definite restrictions on the solutions of Eqs. (9a) and (9b).

We define the ground state of a system with particle number $N+1 = (N_a+1, N_b)$, setting $\epsilon_s^{(2)} = E_{\lambda_0} - \tilde{\Delta} = 0$, where λ_0 is a state near to the Fermi surface. Then the conditions Eqs. (11) take the form

$$E_\lambda + E_{\lambda_0} = -\varepsilon_s^{(1)}(N_a+1, N_b), \tag{12a}$$

$$E_\lambda - E_{\lambda_0} = \varepsilon_s^{(2)}(N_a+1, N_b), \tag{12b}$$

$$E_\lambda + E_0(N_a, N_b+1) - E_0(N_a+1, N_b) + E_{\lambda_0} = -\varepsilon_s^{(3)}(N_a, N_b+1), \tag{12c}$$

$$E_\lambda - E_0(N_a, N_b+1) + E_0(N_a+1, N_b) - E_{\lambda_0} = \varepsilon_s^{(4)}(N_a, N_b+1). \tag{12d}$$

The quantity $E_0(N_a, N_b+1) - E_0(N_a+1, N_b) = \Delta E$ can be considered to be the excitation energy of the nucleus (N_a+1, N_b). Then the conditions (12a) and (12c) cannot be satisfied for any λ. In accordance with Eq. (12b) we set $\Delta E = 0$. Then the conditions (12b) and (12d) coincide, and are satisfied for arbitrary λ. Therefore, in Eqs. (9a) and (9b) we should set

$$C_{1\pm\lambda}^{+} = 0, \quad C_{2\pm\lambda}^{-} = 0.$$

The functions G and F become

$$G_{\pm\lambda}^{+}(\tau) = -i\frac{E_\lambda + \varepsilon_\lambda}{2E_\lambda}\exp\{-i(E_\lambda + \mu)\tau\},$$

$$G_{\pm\lambda}^{-}(\tau) = i\frac{E_\lambda - \varepsilon_\lambda}{2E_\lambda}\exp\{i(E_\lambda - \mu)\tau\},$$

$$F_{\pm\lambda}^{+}(\tau) = -\frac{\Delta}{2E_\lambda}\exp\{-i(E_\lambda + \mu)\tau\},$$

$$\times F_{\pm\lambda}^{-}(\tau) = -\frac{\Delta}{2E_\lambda}\exp\{i(E_\lambda - \mu)\tau\}. \tag{13}$$

Since $-iG_\lambda^{-}(0) = \rho_\lambda$ is the density matrix of the particles, it is easy to see that the solution found corresponds to an even system $(\rho_\lambda = \rho_{-\lambda})$. At the same time, from Eq. (12b) we determine the excitation spectrum for odd nuclei:

$$\varepsilon_s = E_\lambda - E_{\lambda_0},$$

where the quantities E_λ, E_{λ_0} contain the Δ of the even system.

In a Fourier representation for τ, the functions G_λ and F_λ take the form

$$G_\lambda(\omega) = \frac{(E_\lambda + \varepsilon_\lambda)/2E_\lambda}{\omega - E_\lambda + i\delta} + \frac{(E_\lambda - \varepsilon_\lambda)/2E_\lambda}{\omega + E_\lambda - i\delta}, \tag{14}$$

$$F_\lambda(\omega) = -\frac{i\Delta}{2E_\lambda}\left[\frac{1}{\omega - E_\lambda + i\delta} - \frac{1}{\omega + E_\lambda - i\delta}\right]. \tag{15}$$

An analogous result is obtained if we determine the ground state of the $N+1$ particle system from Eq. (11c).

We now consider another possibility. Let the ground state of the $N+1$ particle system be determined from the relation

$$-\varepsilon_s^{(1)}(N_a+1, N_b) = E_{\lambda_0} + \tilde{\Delta} = 0.$$

Then the conditions [Eqs. (11)] take the form

$$E_\lambda - E_{\lambda_0} = -\varepsilon_s^{(1)}(N_a+1, N_b), \tag{16a}$$

$$E_\lambda + E_{\lambda_0} = \varepsilon_s^{(2)}(N_a+1, N_b), \tag{16b}$$

$$E_\lambda + E_0(N_a, N_b+1) - E_0(N_a+1, N_b) - E_{\lambda_0} = -\varepsilon_s^{(3)}(N_a, N_b+1), \tag{16c}$$

$$E_\lambda - E_0(N_a, N_b+1) + E_0(N_a+1, N_b) + E_{\lambda_0} = \varepsilon_s^{(1)}(N_a, N_b+1). \tag{16d}$$

Condition (16a) is satisfied only for $\lambda = \lambda_0$, and condition (16b), for all λ. The quantity $\Delta E = E_0(N_a, N_b+1) - E_0(N_a+1, N_b)$ corresponds to the minimum excitation energy of the nucleus (N_a+1, N_b). Therefore, in accordance with Eq. (16b), $\Delta E = E_{\lambda_0} + E_{\lambda_1}$, where λ_1 is a state near in energy to the state λ_0. Then Eq. (16c) is not satisfied for any λ, and Eq. (16d) becomes

$$E_\lambda - E_{\lambda_1} = \varepsilon_s(N_a, N_b+1).$$

In deformed nuclei, the energy levels are doubly degenerate, and this relation is satisfied for all $\lambda \neq \lambda_0$. Therefore, in the solutions to Eqs. (9a) and (9b) we should set

$$C_{1\lambda}^{+} = C_{2-\lambda}^{-} = 0, \ \lambda \neq \lambda_0,$$
$$C_{1-\lambda}^{+} = C_{2\lambda}^{-} = 0, \ C_{2-\lambda_0}^{+} = C_{1\lambda_0}^{-} = 0.$$

In this case, G and F take the form

$$G_\lambda^{+}(\tau) = -i\frac{E_\lambda - \varepsilon_\lambda}{2E_\lambda}\delta_{\lambda\lambda_0}\exp\{i(E_\lambda - \mu)\tau\} - i\frac{E_\lambda + \varepsilon_\lambda}{2E_\lambda}(1 - \delta_{\lambda-\lambda_0})\exp\{-i(E_\lambda + \mu)\tau\}$$

$$G_\lambda^{-}(\tau) = i\frac{E_\lambda - \varepsilon_\lambda}{2E_\lambda}(1 - \delta_{\lambda\lambda_0})\exp\{i(E_\lambda - \mu)\tau\} + i\frac{E_\lambda + \varepsilon_\lambda}{2E_\lambda}\delta_{\lambda-\lambda_0}\exp\{-i(E_\lambda + \mu)\tau\},$$

$$F_\lambda^{+}(\tau) = \frac{\Delta}{2E_\lambda}\delta_{\lambda\lambda_0}\exp\{i(E_\lambda - \mu)\tau\} - \frac{\Delta}{2E_\lambda}(1 - \delta_{\lambda-\lambda_0})\exp\{-i(E_\lambda + \mu)\tau\},$$

$$F_\lambda^{-}(\tau) = -\frac{\Delta}{2E_\lambda}(1 - \delta_{\lambda\lambda_0})\exp\{i(E_\lambda - \mu)\tau\} + \frac{\Delta}{2E_\lambda}\delta_{\lambda-\lambda_0}\exp\{-i(E_\lambda + \mu)\tau\}. \tag{17}$$

In calculating the density matrix ρ_λ, we see that $\rho_\lambda = \rho_{-\lambda}$ $(\lambda \neq \pm\lambda_0)$, $\rho_{\lambda_0} = 0$, $\rho_{-\lambda_0} = 1$. Thus, the solution obtained corresponds to a nucleus with an odd number of particles where the odd particle is in state $-\lambda_0$ with probability 1, and the corresponding state λ_0 is empty. If we determine the ground state of the system from Eq. (11c), then we obtain an analogous result with the odd particle in state λ_0. Equation (16b) gives the excitation spectrum of the even system

$$\varepsilon_s = E_\lambda + E_{\lambda_0}.$$

It should be noted that in this case the Δ of the odd system enters into E_λ and E_{λ_0}. Therefore, Δ of the odd nucleus can be determined, knowing the value of the energy gap in the excitation spectra of the neighboring even-odd nucleus.

Calculating $F_\lambda(0)$, we obtain from Eq. (7) the equation for Δ of the odd system given earlier in reference 1:

$$1 = -\gamma\sum_\lambda \frac{1 - \delta_{\lambda\lambda_0} - \delta_{\lambda-\lambda_0}}{2E_\lambda}\varphi_\lambda^*(\mathbf{r})\varphi_\lambda(\mathbf{r}). \tag{18}$$

In the Fourier representation for τ, the Green's function of the odd system with the odd particle in state λ_0 has the form

$$G_\lambda(\omega) = \frac{(E_\lambda + \varepsilon_\lambda)/2E_\lambda}{\omega - E_\lambda + i\delta\theta_{\lambda\lambda_0}} + \frac{(E_\lambda - \varepsilon_\lambda)/2E_\lambda}{\omega + E_\lambda - i\delta\theta_{\lambda-\lambda_0}}, \tag{19}$$

$$F_\lambda(\omega) = -\frac{i\Delta}{2E_\lambda}\left(\frac{1}{\omega - E_\lambda + i\delta\theta_{\lambda\lambda_0}} - \frac{1}{\omega + E_\lambda - i\delta\theta_{\lambda-\lambda_0}}\right), \tag{20}$$

where

$$\theta_{\lambda\lambda_0} = \begin{cases} 1, & \lambda \neq \lambda_0. \\ -1, & \lambda = \lambda_0. \end{cases}$$

The case of spherical nuclei needs separate consideration. In this case the levels are multiply degenerate, and, consequently, $E_\lambda = E_{\lambda_0}$ for all states λ corresponding to a given energy level. Then Eqs. (16d) and (16a) are satisfied for any of these λ. Therefore, in order to fill in the missing supplementary condition, it is necessary to turn to considerations connected with conservation of the total angular momentum of the system

$$J_z = \sum_\lambda (j_z)_\lambda \rho_\lambda.$$

Assuming that the nucleons pair off into states of zero angular momentum, we find $J_z = 0$ for even systems and $J_z = (j_z)_{\lambda_0}$ for odd ones. From this it follows that $\rho_{\lambda_0} = 1$ and, consequently, $G_{\lambda_0}^{+}(0) = 0$. Therefore, if one exponent is missing in $G_{\lambda_0}^{+}$, then, on account of the initial condition, the other one will also be absent. From the condition $\rho_\lambda = \rho_{-\lambda}$ for all $\lambda \neq \pm\lambda_0$, it follows that the coefficient in front of the exponential, according to condition (16a), is zero. Then the expressions (19) and (20) can also be applied to the case of spherical nuclei.

Migdal[1] showed that for small, time-independent perturbations, corrections to the functions G and F take the form

$$G'_{\lambda\lambda'} = G_\lambda V_{\lambda\lambda'} G_{\lambda'} + F_\lambda V^*_{\lambda\lambda'} F_{\lambda'} + iG_\lambda \Delta'_{\lambda\lambda'} F_{\lambda'} + iF_\lambda \Delta'^*_{\lambda\lambda'} G_{\lambda'}, \tag{21}$$

$$F'_{\lambda\lambda'} = -D_\lambda V^*_{\lambda\lambda'} F_{\lambda'} + F_\lambda V_{\lambda\lambda'} G_{\lambda'} + iF_\lambda \Delta'_{\lambda\lambda'} F_{\lambda'} - iD_\lambda \Delta'^*_{\lambda\lambda'} G_{\lambda'}; \tag{22}$$

$$\Delta'^*(\mathbf{r}) = \gamma\sum_{\lambda\lambda'} F'_{\lambda\lambda'}\varphi_\lambda(\mathbf{r})\varphi^*_{\lambda'}(\mathbf{r}). \tag{23}$$

For an odd nucleus, G and F are determined by (19) and (20), and D is

$$D_\lambda = \frac{(E_\lambda - \varepsilon_\lambda)/2E_\lambda}{\omega - E_\lambda + i\delta\theta_{\lambda\lambda_0}} + \frac{(E_\lambda + \varepsilon_\lambda)/2E_\lambda}{\omega + E_\lambda - i\delta\theta_{\lambda-\lambda_0}}. \tag{24}$$

Substituting (19) and (20) in (21), and integrating over ω, we obtain the correction to the density matrix:

$$\rho'_{\lambda\lambda'} = \frac{V_{\lambda\lambda'}(\varepsilon_\lambda\varepsilon_{\lambda'} - E_\lambda E_{\lambda'}) - \Delta^2 V^*_{\lambda\lambda'} + \Delta(\varepsilon_\lambda \Delta'_{\lambda\lambda'} + \varepsilon_{\lambda'}\Delta'^*_{\lambda\lambda'})}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})} + \delta_{\lambda'-\lambda_0}\frac{V_{\lambda\lambda'}(E_{\lambda'} - \varepsilon_\lambda)(\varepsilon_{\lambda'} - E_{\lambda'}) + \Delta^2 V^*_{\lambda\lambda'} + \Delta[\Delta'^*_{\lambda\lambda'}(E_{\lambda'} - \varepsilon_{\lambda'}) + \Delta'_{\lambda\lambda'}(E_{\lambda'} - \varepsilon_\lambda)]}{2E_{\lambda'}(E^2_\lambda - E^2_{\lambda'})}$$

$$+ \delta_{\lambda'\lambda_0}\frac{V_{\lambda\lambda'}(E_{\lambda'} + \varepsilon_\lambda)(E_{\lambda'} + \varepsilon_{\lambda'}) - \Delta^2 V^*_{\lambda\lambda'} + \Delta[\Delta'^*_{\lambda\lambda'}(E_{\lambda'} + \varepsilon_{\lambda'}) + \Delta'_{\lambda\lambda'}(E_{\lambda'} + \varepsilon_\lambda)]}{2E_{\lambda'}(E^2_{\lambda'} - E^2_\lambda)}$$

$$+ \delta_{\lambda-\lambda_0}\frac{V_{\lambda\lambda'}(E_\lambda - \varepsilon_\lambda)(\varepsilon_{\lambda'} - E_\lambda) + \Delta^2 V^*_{\lambda\lambda'} + \Delta[\Delta'^*_{\lambda\lambda'}(E_\lambda - \varepsilon_{\lambda'}) + \Delta'_{\lambda\lambda'}(E_\lambda - \varepsilon_\lambda)]}{2E_\lambda(E^2_{\lambda'} - E^2_\lambda)}$$

$$+ \delta_{\lambda\lambda_0}\frac{V_{\lambda\lambda'}(E_\lambda + \varepsilon_\lambda)(E_\lambda + \varepsilon_{\lambda'}) - \Delta^2 V^*_{\lambda\lambda'} + \Delta[\Delta'^*_{\lambda\lambda'}(E_\lambda + \varepsilon_{\lambda'}) + \Delta'_{\lambda\lambda'}(E_\lambda + \varepsilon_\lambda)]}{2E_\lambda(E^2_\lambda - E^2_{\lambda'})}$$

for $E_\lambda \neq E_{\lambda'}$;

$$\rho'_{\lambda\lambda'} = \frac{\varepsilon_\lambda\Delta(\Delta'_{\lambda\lambda'} + \Delta'^*_{\lambda\lambda'}) - \Delta^2(V_{\lambda\lambda'} + V^*_{\lambda\lambda'})}{4E^3_\lambda} + (\delta_{\lambda'-\lambda_0} + \delta_{\lambda\lambda_0})\frac{\Delta^2(V_{\lambda\lambda'} + V^*_{\lambda\lambda'}) + \Delta[\Delta'^*_{\lambda\lambda'}(E_\lambda - \varepsilon_\lambda) - \Delta'_{\lambda\lambda'}(E_\lambda + \varepsilon_\lambda)]}{8E^3_\lambda}$$

$$+ (\delta_{\lambda'\lambda_0} + \delta_{\lambda-\lambda_0})\frac{\Delta^2(V_{\lambda\lambda'} + V^*_{\lambda\lambda'}) + \Delta[\Delta'_{\lambda\lambda'}(E_\lambda - \varepsilon_\lambda) - \Delta'^*_{\lambda\lambda'}(E_\lambda + \varepsilon_\lambda)]}{8E^3_\lambda} \tag{25}$$

for $E_\lambda = E_{\lambda'}$.

Analogously, we find the correction to the function F:

$$F'_{\lambda\lambda'} = \frac{\Delta(\varepsilon_\lambda V_{\lambda\lambda'} + \varepsilon_\lambda V^*_{\lambda\lambda'}) - \Delta'^*_{\lambda\lambda'}(E_\lambda E_{\lambda'} + \varepsilon_\lambda\varepsilon_{\lambda'}) + \Delta^2\Delta'_{\lambda\lambda'}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})} + \delta_{\lambda'-\lambda_0}\frac{\Delta[(E_{\lambda'} - \varepsilon_{\lambda'})V_{\lambda\lambda'} - (E_{\lambda'} + \varepsilon_\lambda)V^*_{\lambda\lambda'}] - \Delta'^*_{\lambda\lambda'}(E_{\lambda'} + \varepsilon_\lambda)(E_{\lambda'} - \varepsilon_{\lambda'}) - \Delta^2\Delta'_{\lambda\lambda'}}{2E_{\lambda'}(E^2_\lambda - E^2_{\lambda'})}$$

$$+ \delta_{\lambda'\lambda_0}\frac{\Delta[(E_{\lambda'} + \varepsilon_{\lambda'})V_{\lambda\lambda'} - (E_{\lambda'} - \varepsilon_\lambda)V^*_{\lambda\lambda'}] + \Delta'^*_{\lambda\lambda'}(E_{\lambda'} - \varepsilon_\lambda)(E_{\lambda'} + \varepsilon_{\lambda'}) + \Delta^2\Delta'_{\lambda\lambda'}}{2E_{\lambda'}(E^2_{\lambda'} - E^2_\lambda)}$$

$$+ \delta_{\lambda-\lambda_0}\frac{\Delta[(E_\lambda - \varepsilon_{\lambda'})V_{\lambda\lambda'} - (E_\lambda + \varepsilon_\lambda)V^*_{\lambda\lambda'}] - \Delta'^*_{\lambda\lambda'}(E_\lambda + \varepsilon_\lambda)(E_\lambda - \varepsilon_{\lambda'}) - \Delta^2\Delta'_{\lambda\lambda'}}{2E_\lambda(E^2_{\lambda'} - E^2_\lambda)}$$

$$+ \delta_{\lambda\lambda_0}\frac{\Delta[(E_\lambda + \varepsilon_{\lambda'})V_{\lambda\lambda'} - (E_\lambda - \varepsilon_\lambda)V^*_{\lambda\lambda'}] + \Delta'^*_{\lambda\lambda'}(E_\lambda - \varepsilon_\lambda)(\varepsilon_{\lambda'} + E_\lambda) + \Delta^2\Delta'_{\lambda\lambda'}}{2E_\lambda(E^2_\lambda - E^2_{\lambda'})}$$

for $E_\lambda \neq E_{\lambda'}$;

$$F'_{\lambda\lambda'} = \frac{\Delta\varepsilon_\lambda(V_{\lambda\lambda'} + V^*_{\lambda\lambda'}) - \Delta'^*_{\lambda\lambda'}(E^2_\lambda + \varepsilon^2_\lambda) + \Delta^2\Delta'_{\lambda\lambda'}}{4E^3_\lambda} + (\delta_{\lambda'-\lambda_0} + \delta_{\lambda\lambda_0})\frac{\Delta(E_\lambda - \varepsilon_\lambda)(V_{\lambda\lambda'} + V^*_{\lambda\lambda'}) + (E_\lambda - \varepsilon_\lambda)^2\Delta'^*_{\lambda\lambda'} - \Delta^2\Delta'_{\lambda\lambda'}}{8E^3_\lambda}$$

$$+ (\delta_{\lambda'\lambda_0} + \delta_{\lambda-\lambda_0})\frac{-\Delta(E_\lambda + \varepsilon_\lambda)(V_{\lambda\lambda'} + V^*_{\lambda\lambda'}) + (E_\lambda + \varepsilon_\lambda)^2\Delta'^*_{\lambda\lambda'} - \Delta^2\Delta'_{\lambda\lambda'}}{8E^3_\lambda} \tag{26}$$

for $E_\lambda = E_{\lambda'}$.

Substituting (26) in (23) and using the equality

$$\Delta'^*(\mathbf{r}) = -\gamma\sum_{\lambda\lambda'}\Delta'^*_{\lambda\lambda'}\varphi_\lambda(\mathbf{r})\varphi^*_{\lambda'}(\mathbf{r})\frac{1 - \delta_{\lambda\lambda_0} - \delta_{\lambda-\lambda_0}}{2E_\lambda} = -\gamma\sum_{\lambda\lambda'}\Delta'^*_{\lambda\lambda'}\varphi_\lambda\varphi^*_{\lambda'}\frac{1 - \delta_{\lambda'\lambda_0} - \delta_{\lambda'-\lambda_0}}{2E_{\lambda'}},$$

we obtain an integral equation for $\Delta'(\mathbf{r})$:

$$\sum_{\lambda\lambda'}\varphi_\lambda(\mathbf{r})\varphi^*_{\lambda'}(\mathbf{r})\frac{\Delta'^*_{\lambda\lambda'}[2\Delta^2 + (\varepsilon_\lambda - \varepsilon_{\lambda'})^2] + 2\Delta^2\Delta'_{\lambda\lambda'} + 2\Delta(\varepsilon_{\lambda'}V_{\lambda\lambda'} + \varepsilon_\lambda V^*_{\lambda\lambda'})}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})} + \sum_{\substack{\lambda \\ (E_\lambda \neq E_{\lambda_0})}}\frac{1}{E_{\lambda_0}(E^2_{\lambda_0} - E^2_\lambda)}\{\varphi_{\lambda_0}(\mathbf{r})\varphi^*_\lambda(\mathbf{r})[2\Delta(E_{\lambda_0} + \varepsilon_\lambda)V_{\lambda_0\lambda}$$

$$- 2\Delta(E_{\lambda_0} - \varepsilon_{\lambda_0})V^*_{\lambda_0\lambda} + 2\Delta^2\Delta'_{\lambda_0\lambda} + \Delta'^*_{\lambda_0\lambda}2(E_{\lambda_0} + \varepsilon_\lambda)(E_{\lambda_0} - \varepsilon_{\lambda_0}) + \Delta'^*_{\lambda_0\lambda}(E^2_\lambda - E^2_{\lambda_0})] + \varphi_\lambda(\mathbf{r})\varphi^*_{\lambda_0}(\mathbf{r})[2\Delta(E_{\lambda_0} + \varepsilon_{\lambda_0})V_{\lambda\lambda_0}$$

$$- 2\Delta(E_{\lambda_0} - \varepsilon_\lambda)V^*_{\lambda\lambda_0} + 2\Delta^2\Delta'_{\lambda\lambda_0} + \Delta'^*_{\lambda\lambda_0}2(E_{\lambda_0} - \varepsilon_\lambda)(E_{\lambda_0} + \varepsilon_{\lambda_0}) + \Delta'^*_{\lambda\lambda_0}(E^2_\lambda - E^2_{\lambda_0})]\} + \sum_{\substack{\lambda \\ (E_\lambda = E_{\lambda_0})}}\frac{1}{2E^3_{\lambda_0}}\{\varphi_{\lambda_0}(\mathbf{r})\varphi^*_\lambda(\mathbf{r})$$

$$\times[\Delta(E_{\lambda_0} - \varepsilon_{\lambda_0})(V_{\lambda_0\lambda} + V^*_{\lambda_0\lambda}) - \Delta^2\Delta_{\lambda_0\lambda} - \Delta'^*_{\lambda_0\lambda}(2E_{\lambda_0}\varepsilon_{\lambda_0} + \Delta^2)] + \varphi_\lambda(\mathbf{r})\varphi^*_{\lambda_0}(\mathbf{r})[-\Delta(E_{\lambda_0} + \varepsilon_{\lambda_0})(V_{\lambda\lambda_0} + V^*_{\lambda\lambda_0})$$

$$- \Delta^2\Delta'_{\lambda\lambda_0} + \Delta'^*_{\lambda\lambda_0}(2E_{\lambda_0}\varepsilon_{\lambda_0} - \Delta^2)]\} = 0. \tag{27}$$

As a check, Eqs. (25) — (27) were also obtained by using Eq. (1) without going to the Fourier representation.

In the present work, a theorem about the form of the Green's function of a nonspherical nucleus was proved. It turned out that, in spite of the effect of pair correlations, the odd particle was in a definite state with probability 1, and that the conjugate state was completely empty. However, the pairing of particles in the nucleus leads to the excitation spectrum of the nucleus differing essentially from that of the usual one-particle one connected with excitations of the odd particle: $\epsilon_s = E_\lambda - E_{\lambda_0}$, $E_\lambda = \sqrt{\Delta^2 + \epsilon_\lambda^2}$. It is easy to see that for small excitations ($|\epsilon_\lambda| < \Delta$) the density of levels of the odd nucleus turns out to be roughly $2\Delta/|\epsilon_\lambda|$ times larger than in the one-particle model. The formulae (25) — (27) obtained from perturbation theory at the end of the article are essential for application of the theory considered. They make it possible to calculate effects connected with the influence of the odd particle, e.g., moments of inertia of odd nuclei, or magnetic moments.

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