COLLECTIVE PROPERTIES OF Si³⁰, Si³¹, AND Ne²³, AND REDUCED WIDTHS IN STRIPPING REACTIONS

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Within the framework of the unified model with strong coupling, we investigate the collective properties of Si^{30} , Si^{31} and Ne^{23} , by analyzing the experimentally observed reduced widths for neutron capture in (d, p) stripping reactions. The analysis shows that Si^{31} apparently has an oblate shape with a Nilsson parameter $\delta < 0$, but it is not possible to establish the shape of the deformation for Ne^{23} , though it does confirm that this nucleus is highly deformed. The result for Si^{30} does not agree with the theoretical estimates, which raises doubts concerning the applicability of the strong coupling scheme to this nucleus.

T is known that the low-lying levels of light nuclei in the region up to A = 40 in many cases show a well developed rotational structure, which is describable by the uniform model of the nucleus¹ in the form proposed by Nilsson.² Analyses within the framework of this model have been made of the experimental data for Al^{25} and Mg^{25} , $3 Si^{29}$, 4for nuclei in the region 4 < A < 32,⁵ etc. In particular, it was shown⁴ that Si²⁹ apparently has an oblate shape, with deformation parameter δ ≈ -0.15 , although the question of the existence of oblate nuclei remains an open question. It therefore seems desirable to further investigate the collective properties of light nuclei. In this paper we consider the possibility of experimental confirmation of the deformation of the nuclei Si^{30} , Si^{31} and Ne²³, which have been studied in stripping reactions,^{6,7} by using the measured values of the reduced widths for neutron capture.

The expressions for the reduced widths for capture into rotational states of deformed nuclei were calculated by Satchler.⁸ In the case of strong coupling with the surface, the motion of the odd nucleon in the spheroidal potential is described by the single-particle wave function $\chi_{\Omega} = \Sigma_j C_j \psi_j$, where j is the angular momentum of the external nucleon and Ω its projection on the symmetry axis of the nucleus. The coefficients C_j were computed by Nilsson. Because of the orthogonality of the singleparticle wave functions, the reduced widths γ^2 , in the reaction A (d, p) B, for capture of the neutron into an orbit with definite j, l will be proportional to $|C_j|^2$:

$$\gamma^2 \sim \left[\left(2I_A + 1 \right) / \left(2I_B + 1 \right) \right] \Sigma_j \left| \langle I_A j K_A \Omega \left| I_B K_B \rangle \right|^2 \right| C_j \right|^2.$$

For an even-odd initial nucleus,

$$|C_j|^2 \sim (2I_B+1) \gamma^2 \delta_{j, I_B} \delta_{\Omega, K_B}$$

where δ is the delta function, and K is the projection of the total angular momentum of the nucleus on the symmetry axis.

The nucleus Si³¹ has $j = \frac{3}{2}$ in its ground state, and is a convenient example for investigating the sign of the deformation, since in the uniform model the potential energy of a nucleus with an odd particle having angular momentum $j = \frac{3}{2}$ is degenerate, with values $\Omega = \frac{3}{2}$, $\delta < 0$, and $\Omega = \frac{1}{2}$, $\delta > 0$. Within the framework of this model, the sign of the deformation for Si³¹ can be established from knowledge of the value of $\Omega = K$. Analysis¹ of the *l*-forbidden β transition $Si^{31} \rightarrow P^{31}$ gives $j = \Omega = \frac{3}{2}$ for the state of the external nucleon in Si^{31} . The value of $\Omega = K$ can also be determined from the quantum numbers of the sequence of levels in the rotational band, as predicted by the uniform model. Unfortunately, the spins and parities are known only for the ground and first excited states of Si³¹. The determination of the quantum numbers of the next two levels at 1.70 and 2.32 Mev would give still another possibility for determining the sign of the deformation in Si^{31} . A direct comparison of the experimental values of reduced widths from the $Si^{30}(d, p)Si^{31}$ reaction with the values of $|C_j|^2$ does no good in this case, since the reaction was studied at a deuteron energy of 4.3 Mev, which is almost equal to the Coulomb barrier for this nucleus, so that the values of γ^2 are greatly reduced by Coulomb and nuclear interactions.⁶ For this reason, the table compares the ratios of reduced widths, multiplied by appropriate

| Nucleus | Level energy, Mev | $I_B = j, \pi$ | γ² Mev cm ×10 ^{−13} | 8 | $\left \frac{\gamma^{2}(2I_{B}+1)}{\gamma^{\prime 2}(2I_{B}+1)} \right $ | $\delta < 0$ | $\delta > 0$ |
|--------------------------------------|-------------------------|--|------------------------------------|------|---|--|--|
| Si ³¹ Si ³¹ | 0 0.76 | ³ / ₂ + 1/ ₂ + | 0.034 * 0.047 * | 0.15 | 3.7-5.2 | $\frac{ C_j _{K=3_2}^2}{ C_j _{K'=3_2}^2} = 5.2$ | $\frac{ C_j _{K=\frac{1}{2}}^2}{ C_j _{K'=\frac{1}{2}}^2} = 1.8$ |
| Ne ²³ Ne ²³ | 0 0. 98 | ⁵ /2 ⁺ 1/2 ⁺ | 0.023 0.017 | 0.30 | 4.0-6.4 | $\frac{ C_j _{K=\frac{5}{2}}^2}{ C_j _{K'=\frac{1}{2}}^2} = 6.2$ | $\frac{ C_j _{K=5/2}^2}{ C_j _{K'=5/2}^2} = 6.2$ |
| Si ³⁰ Si ³⁰ | 0 2.24 | 0+ 2+ | 0.087* 0.057* | 0.15 | 0.3-0.2 | | |

*Because of a numerical error, incorrect values of γ^2 (and of θ^2) were given in reference 6. However, this does not change the conclusions of the paper.

statistical factors, with the ratios of the coefficients $|C_i|^2$ for the Nilsson orbits for the ground and first excited states, for the cases $\delta < 0$ and $\delta > 0$ (columns 7 and 8 of the table). $\Omega = K = \frac{3}{2}$ corresponds to the eighth Nilsson orbit, while Ω = K = $\frac{1}{2}$ is the eleventh orbit. The absolute values of the equilibrium deformation $|\delta|$ given in column 5 are taken from reference 5. Column 4 of the table gives the values of γ^2 computed omitting and including the isotropic part of the angular distributions. The ratio of the $|C_j|^2$'s for $\delta < 0$ is just bracketed by these values. A similar situation occurs for Ne²³, which was studied in the reaction Ne²² (d, p) Ne²³. For this nucleus, $j = \frac{5}{2}$ (the fifth Nilsson orbit), but the ratio of the $|C_i|^{2}$'s no longer determines the sign of the deformation, though it is known that in the region A = 20 - 25the nuclei are deformed and have a prolate shape $(\delta > 0)$. It is possible that, in both cases, part of the isotropic angular distribution, which is usually attributed to a reaction which proceeds via compound nucleus formation, may be caused by the stripping mechanism. Thus the analysis of reduced widths from stripping shows that the nucleus Si³¹ apparently has an oblate shape with deformation parameter $\delta < 0$, but it is not possible to establish the sign of the deformation for Ne^{23} , although it is confirmed that this nucleus is highly deformed.

In the case of even-even final nuclei, the probabilities of capture to levels of the rotational band differ only in their statistical factors and the corresponding Clebsch-Gordan coefficients $\langle I_B \Omega | I_A j I_A \Omega - I_A \rangle$.⁸ For ₁₄Si³⁰ the experimental ratio of reduced widths from the reaction Si²⁹ (d, p) Si³⁰, multiplied by the corresponding statistical factors, is equal to 0.3 to 0.2, whereas the theory gives 1 for the case of strong coupling. In the experiment⁶ it was not possible to separate completely the proton groups corresponding to the transition to the 2.24 Mev level of Si³⁰, and to the ground state of Si²⁹ from the reaction on Si²⁸ which is present as an impurity in the target. As a result the ratio of the reduced widths is apparently reduced by 25 - 30%. However, this cannot explain all of the difference between the experimental and theoretical values. It may be that, for Si³⁰ as for certain other nuclei in the region $28 \le A \le 32$, the strong coupling scheme is not valid. To clarify this question it will be necessary to determine the spins and parities of higher excited states of Si³⁰.

Very recently doubts have been raised concerning the validity of using reduced widths for analysis of nuclear structure,⁹ since in addition to the usual stripping reaction there may be other direct reactions, such as the ejection of a proton by the deuteron with capture of the deuteron. However, when the usual stripping process is allowed, the cross section for other direct processes, especially when the deuteron energy is around the value of the Coulomb barrier for the target nucleus, will be probably much smaller than the stripping cross section. The use of reduced widths for analysis therefore seems to be justified.

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