GALVANOMAGNETIC CHARACTERISTICS OF METALS WITH OPEN FERMI SURFACES. II

I. M. LIFSHITZ and V. G. PESCHANSKIĬ

Physico-technical Institute, Academy of Sciences, Ukrainian S.S.R.; Khar'kov State University

Submitted to JETP editor July 7, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 188-193 (January, 1960)

On the basis of a theory developed earlier of galvanomagnetic phenomena,[1,2] detailed calculations have been carried out for certain concrete types of Fermi surfaces, a particular case of which is, for example, the open surface constructed by Pippard for copper. It is shown that the stereographic projection of the resistances obtained for an open surface of this type agrees well with experimental data for gold (Alekseevskiĭ and Gaĭdukov). The peculiarities of galvanomagnetic phenomena have been studied for a surface of the "corrugated" plane type. The possibility of a new type of angular singularities has been indicated, connected with the sharp change in the direction of the open trajectories on approaching the field direction perpendicular to the corrugated plane. Possibilities are discussed of a more complete determination of the energy spectrum according to the galvanomagnetic characteristics of the metal.

METALS with open Fermi surfaces are characterized by a sharp anisotropy of the resistance and the Hall field in the region of strong magnetic field; this anisotropy increases with increase in the magnetic field.[2] As was explained earlier,[1,2] the basic character of the angular dependence of the resistance and of the Hall field is determined by the topological structure of the Fermi surfaces. In particular, it is very important to make clear whether the open trajectories of the motion of an electron in momentum space (ϵ = const, p_z = const) exists for a given direction of the magnetic field or whether such are absent.* In these two cases the asymptotic character of the tensors of conductivity σ_{ik} and resistivity ρ_{ik} in strong magnetic fields is quite different.

The general method of observation of galvanomagnetic phenomena in metals with an arbitrary Fermi surface (closed or open) was investigated in the papers mentioned. In the present communication concrete types of Fermi surfaces are considered which are of particular interest. Some of these were discovered recently by Alekseevskiĭ and Gaĭdukov.

1. By measuring the surface resistance of a single crystal of copper in a high frequency variable electromagnetic field, Pippard made it clear that the Fermi surface for copper in all probability is open.[3] The surface constructed by him can be described with a great deal of accuracy by an analytic expression containing only the first harmonics in the Fourier expansion of $\epsilon(\mathbf{p})$. This fact was noted by Moliner[4] who made the assumption that for all metals with a face-centered cubic lattice (metals of the copper group) the Fermi surface could be represented by an analytic expression of the form

$$\varepsilon(\mathbf{p}) = \lambda\{3 - \cos(ap_x/2\hbar)\cos(ap_y/2\hbar) - \cos(ap_z/2\hbar)\cos(ap_x/2\hbar) - \cos(ap_y/2\hbar)\cos(ap_z/2\hbar) + \beta[3 - \cos(ap_x/\hbar) - \cos(ap_y/\hbar) - \cos(ap_z/\hbar)]\} = \zeta_0, \quad (1)$$

a = lattice constant, $\hbar$ = Planck's constant, ζ_0 = Fermi energy. For $\beta \approx 0.1$ and $\zeta_0/\lambda \approx 3.6$, the experimental points of Pippard lie on the surface (1) with accuracy up to 1 percent.

A somewhat more general form of the expansion of $\epsilon(\mathbf{p})$, which is also suitable for simple and body-centered cubic lattices,

$$\varepsilon(\mathbf{p}) = \alpha\{3 - \cos(ap_x/\hbar)\cos(ap_y/\hbar) - \cos(ap_x/\hbar)\cos(ap_z/\hbar) - \cos(ap_y/\hbar)\cos(ap_z/\hbar) + \beta[3 - \cos(ap_x/\hbar) - \cos(ap_y/\hbar) - \cos(ap_z/\hbar)] + \delta[1 - \cos(ap_x/\hbar)\cos(ap_y/\hbar)\cos(ap_z/\hbar)]\} = \zeta_0, \quad (1a)$$

leads, as analysis will show, to the same possible types of topological structure of the surfaces as (1).

We now investigate the galvanomagnetic properties of metals whose isoenergetic surfaces are de-

*The trajectories of the motion of the electrons in the usual coordinate space are obtained from the trajectories of motion in momentum space by a rotation through the angle $\pi/2$ with a similarity coefficient eH/c. All the notation is consistent with that of reference 1.

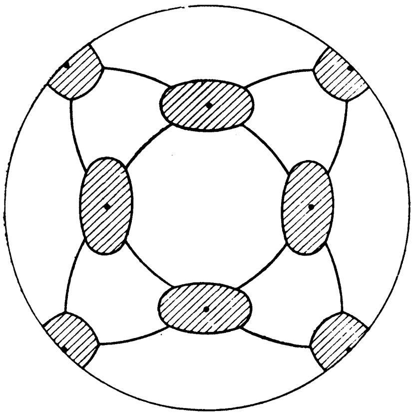

FIG. 1. Stereographic projection of the directions of the magnetic field (shaded regions and continuous lines), which lead to open trajectories ε = const, p_z = const for a Fermi surface consisting of corrugated cylinders directed along the three-dimensional diagonals of the cell of the reciprocal lattice.

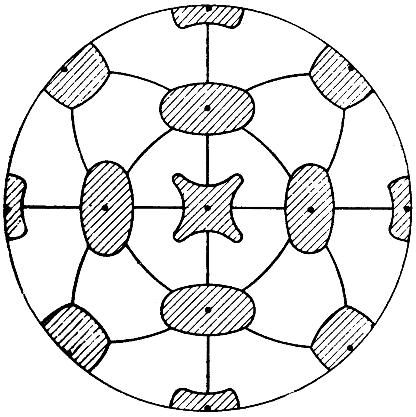

FIG. 2. Stereographic projection of the directions of the magnetic field (shaded regions and continuous lines), for which open plane intersections exist for surfaces formed by corrugated cylinders whose axes are directed along the three-dimensional diagonals and the principal crystallographic axes.

scribed by the expression (1a) and (1) with arbitrary values of the parameters δ, ζ_0/α and β.

For fixed values of α, β and δ, the region of possible values of ζ_0 is limited ($\epsilon_{min} \le \zeta_0 \le \epsilon_{max}$; $\epsilon_{min} = 0$). If $\zeta_0 = \epsilon_{min}$ or $\zeta_0 = \epsilon_{max}$, Eq. (1a) describes a set of isolated points. When $\zeta_0 - \epsilon_{min}$ is not large or is close to its maximum value, the surface (1a) is a small closed surface (sphere), whose dimensions increase in proportion to the departure of ζ_0 from its minimum or maximum possible value. For intermediate values of ζ_0, the Fermi surface (1a) can be either closed or open.

For $2\alpha(\beta+2) < \zeta_0 < 4\alpha(\beta+1)$, $\delta \ll 1$, the surface (1a) is an open surface of the type "three-dimensional grid," consisting of "corrugated" cylinders, directed along the principal crystallographic axes [100], [010], [001]. For $\zeta_0 > 4\alpha(\beta+1)$, the surface (1a) again represents closed regions located around the centers of the cubic cells of the reciprocal lattice. Quasiparticles with such values of the energy behave as positively charged particles — "holes." (For simplicity we consider $\alpha > 0$ and $\beta > 0$.)

For $\beta < 1$,

$$\zeta_0/\alpha > [3(1+\beta/2)^2 + \delta(1+\beta^3/8)] \qquad \text{(1a')}$$

the surface (1a) is a "three-dimensional grid" of corrugated cylinders whose axes can be parallel to the three-dimensional diagonals, the diagonals of the boundaries and the edges of the cubic cell of the reciprocal lattice.[5] For values of ζ_0/α close to

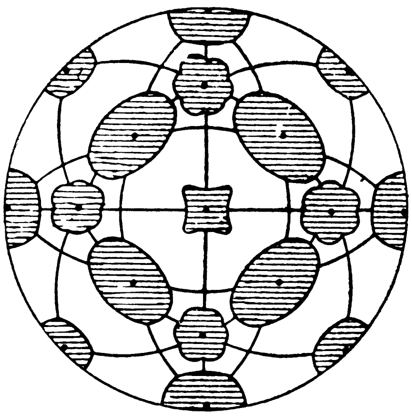

FIG. 3. Stereographic projection of the directions of the magnetic field (shaded region and continuous lines), which lead to the appearance of open plane intersections of surfaces consisting of corrugated cylinders whose axes are parallel to the three-dimensional diagonals and the diagonals of the boundaries of the cubic lattice.

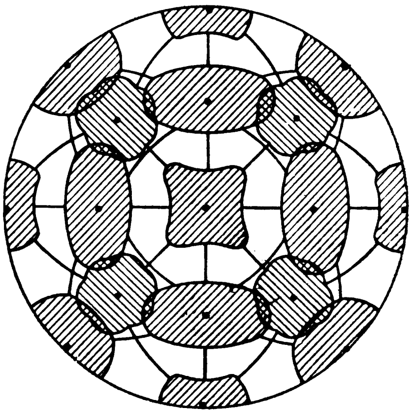

FIG. 4. Stereographic projection of the directions of the magnetic field (shaded regions and continuous lines), which lead to the appearance of open plane intersections of an isoenergetic surface of the type "three-dimensional grid," formed from corrugated cylinders whose axes have thirteen different directions (the three-dimensional diagonals, the diagonals of the boundaries and edges of the cell). The doubly shaded regions are the directions of the magnetic field for which the layers of open trajectories with different average directions exist.

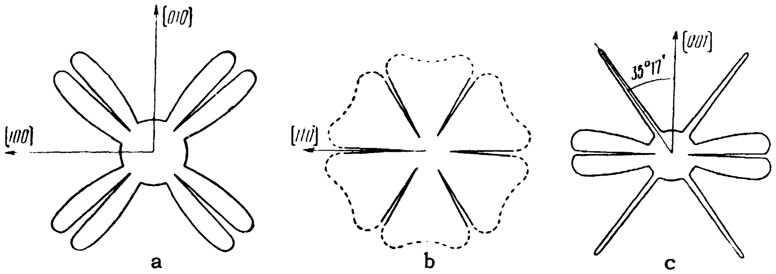

FIG. 5. Angular dependence of the resistivity $\rho = \rho(\vartheta)$ (diagram of rotation of $\mathbf{H} \perp \mathbf{j}$) of metals with a Fermi surface consisting of corrugated cylinders directed along the three-dimensional diagonals of the cubic cell: a – electric current j directed along the [001] axis, b – electric current j directed along the [111] axis, c – j directed along the [110] axis.

the maximum value, the surface (1a) again consists only of closed regions distributed about the centers of the walls ($\beta < \delta$) or the middle of the edges ($\beta > \delta$) of the cells of the reciprocal lattice. [As before, we shall take $\alpha > 0$ for simplicity. The case $\alpha < 0$ does not lead to any new qualitative singularities of the open surfaces of (1).]

Thus, in all, five different types of open surfaces are possible.[5] The simplest topological case of an open surface (1a) ("three-dimensional grid") of corrugated cylinders whose axes are parallel to the principal crystallographic axes [100], [010], [001] has been studied in detail previously. The stereographic projections of the directions of the magnetic field which lead to open plane intersections $\epsilon = \text{const}$, $p_z = \text{const}$ with the remaining four varieties of open surfaces of (1a) are shown in Figs. 1–4. For surfaces described by Eq. (1), Fig. 3 takes place for $-\frac{1}{4} < \beta < 0$, $3 + 6\beta < \zeta_0/\lambda < 3 + 4\beta$, while Fig. 4 holds for $0 < \beta < 1$, $\zeta_0/\lambda > 3(1 + 2\beta)$.

The stereographic projections of particular directions of the magnetic field for gold and silver constructed experimentally by Alekseevskiĭ and Gaĭdukov[6] are very close to the sterographic projections shown in Fig. 3. Figure 5 shows polar diagrams ($\mathbf{H} \perp \mathbf{j}$) of the angular dependence of the resistance of metals whose Fermi surface is an open surface of the type "three-dimensional grid" formed from corrugated cylinders whose axes are directed along the three-dimensional diagonals of the cell of the reciprocal lattice. Figure 6 shows polar diagrams for the resistance of metals with an open Fermi surface consisting of corrugated cylinders whose axes are parallel to the three-dimensional diagonals, the diagonals of the walls and the edges of the cells of the reciprocal lattice. (For the stereographic projection, see Fig. 4.) The Fermi surface for copper has a very similar topological structure.

2. As a second example, let us consider an open Fermi surface of the type "corrugated plane" (see Figs. 3 and 5 of reference 2 and Fig. 7 of the present paper) which can evidently exist for metals with hexagonal, tetragonal, and rhombic crystallographic lattices. For such a surface, open trajectories exist for any direction of the magnetic field. Their average direction p_x, generally speaking, coincides with the lines of intersection of the plane $p_z = \text{const}$ and the plane $\nu\xi$, tangent to the Fermi surface (ν, ξ, ζ are coordinate axes connected with the crystallographic axes of the reciprocal lattice). When these planes (plane $\nu\xi$ and the plane $p_z = \text{const}$) are parallel to each other, i.e., the direction of the magnetic field coincides with the ζ axis, the thickness of the layer of open trajectories vanishes (for metals with hexagonal and tetragonal crystallographic lattices), or these trajectories have a perfectly definite direction ξ, determined exclusively by the symme-

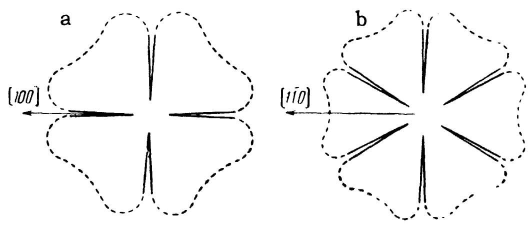

FIG. 6. Angular dependence of the resistivity $\rho(\vartheta)$ (diagram of rotation $\mathbf{H} \perp \mathbf{j}$) of metals with a Fermi surface formed from corrugated cylinders whose axes have thirteen different crystallographic directions: a – j directed along the [001] axis, b – j directed along the [111] axis.

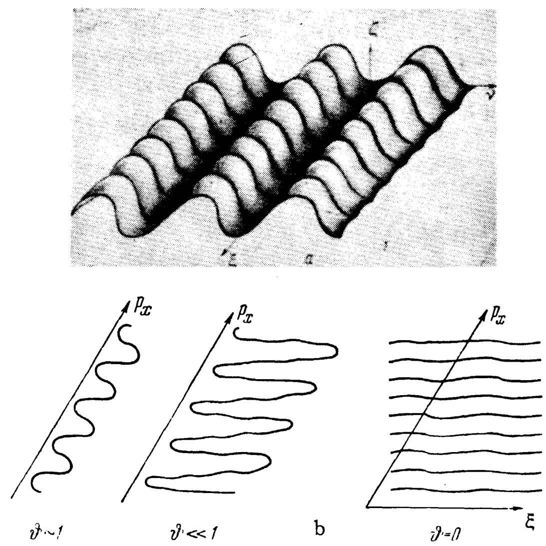

FIG. 7. a – constant energy surface of the type "corrugated plane," b – its intersection with the plane $p_z = \text{const}$.

try of the corrugated surface (for metals with rhombic crystallographic lattice). In particular, this takes place if the "ripples" on the surface have the form of parallel waves (see Fig. 7). (The ν axis is the preferred direction of the corrugation. In the direction of the ξ axis, the surface is weakly corrugated.)

For this surface particular interest is attached to the case in which the direction of the magnetic field is close to the ζ axis. For $\vartheta \lesssim \gamma_0$ (ϑ is the angle between the direction of the magnetic field and the axis ζ), the open trajectories are greatly extended in the direction of the ξ axis, although their average direction evidently coincides with the p_x axis. For $\vartheta = 0$, the direction of the open trajectories changes over from the p_x axis to the ξ axis (see Fig. 7b). It is clear that for $\vartheta \sim \gamma_0$, it is not possible to make use of the solution of the kinetic equation in powers of $\gamma_0 = H_0/H$ for the determination of the asymptotic value of the tensor σ_{ik} (see reference 1), since the average value $\overline{v}_x$ over a time of the order of the time of the mean free path of the electron ($\Delta\tau \sim 1/\gamma_0$) can turn out to be materially different from zero, whereas on all trajectories $\overline{v}_x = 0$.

A similar situation arises in the case of a surface of the "corrugated cylinder" type, when the direction of the magnetic field is almost perpendicular to the axis of the cylinder (see reference 1). In the simplest case, when the x and ξ axes are orthogonal, we obtain the following expression for σ_{ik} ($\eta = \gamma_0/\vartheta$):

$$\sigma_{ik} = \begin{pmatrix} a_{xx}(\eta) & \gamma_0 a_{xy}(\eta) & a_{xz}(\eta) \\ \gamma_0 a_{yx}(\eta) & \gamma_0^2 a_{yy}(\eta) & \gamma_0 a_{yz}(\eta) \\ a_{zx}(\eta) & \gamma_0 a_{zy}(\eta) & a_{zz}(\eta) \end{pmatrix}. \qquad (2)$$

$a_{ik}(\eta)$ can be represented by the following extrapolated formulas which characterize the behavior of σ_{ik} for $\eta \ll 1$ and $\eta \gg 1$:

$$a_{ik}(\eta) = (\alpha_{ik}^{(0)} + \alpha_{ik}^{(1)}\eta + \alpha_{ik}^{(2)}\eta^2)/(\beta_{ik} + \eta^2),$$
$$\alpha_{xx}^{(0)} = \alpha_{xx}^{(1)} = \alpha_{xz}^{(0)} = \alpha_{zx}^{(0)} = \alpha_{yy}^{(1)} = \alpha_{yz}^{(1)} = 0,$$
$$\beta_{yy} = \beta_{yz} = \beta_{zy} = 0. \qquad (3)$$

All $a_{ik}(\eta)$ tend to finite values $a_{ik}(\infty) = \alpha_{ik}^{(2)}$ as $\eta \to \infty$.

The transverse resistivity ρ in the limiting cases $\vartheta \ll \gamma_0$ and $\vartheta \gg \gamma_0$ has the following form:

$$\rho = b(H/H_0)^2\cos^2\alpha + A; \quad \vartheta \gg \gamma_0$$
$$\rho = (H/H_0)^2[b_1\cos^2(\alpha+\varphi) + b_2\vartheta^2\cos^2\alpha] + A_1; \quad \vartheta \ll \gamma_0, \qquad (4)$$

where α is the angle between the direction of the electric current and the x axis; φ is the angle between the x and ξ axes; b_1, b_2, b, A, A_1 are constants of approximately the same order. (For $\vartheta \ll \gamma_0 \ll 1$, the z axis virtually coincides with the ζ axis.)*

It can easily be seen from Eq. (4) that the transverse resistivity increases quadratically with increase in magnetic field over the whole polar diagram $\rho(\vartheta)$ if the direction of the electric field lies in the $\nu\xi$ plane and does not coincide with the ν axis. When the electric current is directed along the ζ axis, the transverse resistivity in strong magnetic fields reaches saturation independently of the orientation of the direction of the magnetic field, since all the open trajectories are perpendicular to the direction of the electric current. In all remaining cases two sharp minima exist on the polar diagram of $\rho(\vartheta)$ for the resistivity (at the point of a minimum, $\rho \approx$ const); the width of the minimum decreases with decrease in the magnetic field as 1/H. As Alekseevskiĭ and Gaĭdukov have shown,[6] the Fermi surface of gallium has evidently a similar topological structure.

In conclusion, we note that by means of polar diagrams obtained experimentally for different orientations of the electric field, it is not only possible to determine the region of the direction of the magnetic field for which there are open trajectories ϵ = const, p_z = const, but also to construct a stereographic projection of the direction of these trajectories. In certain cases, very interesting information on the Fermi surface can be obtained by an investigation of the Hall effect. For example, if at a given direction of the magnetic field, there is a layer of open trajectories ϵ = const, p_z = const with different average directions, then the Hall "constant" decreases with increase of magnetic field as $1/H^2$. Simultaneous investigation of the anisotropy of the resistivity and the Hall effect permits a more detailed study of the topological structure of the Fermi surface.

We thank N. E. Alekseevskiĭ and Yu. P. Gaĭdukov for acquainting us with their experimental results before publication.

[1] Lifshitz, Azbel', and Kaganov, JETP **31**, 63 (1956); Soviet Phys. JETP **4**, 41 (1957).

[2] I. M. Lifshitz and V. G. Peschanskiĭ, JETP **35**, 1251 (1958), Soviet Phys. JETP **8**, 875 (1959).

[3] A. B. Pippard, Phil. Trans. Roy. Soc. (London) **A250**, 325 (1957).

*In a rectangular crystal lattice the orientations of the corresponding axes in real and reciprocal space are identical.

[4] F. G. Moliner, Phil. Mag. **3**, 207 (1958).
[5] V. G. Peschanskiĭ, Dissertation, Kharkov State University, 1958.
[6] N. E. Alekseevskiĭ and Yu. P. Gaĭdukov, JETP **36**, 447 (1959), Soviet Phys. JETP **9**, 311 (1959); Yu. P. Gaĭdukov, JETP **37**, 1281 (1959), Soviet Phys. JETP **10**, 913 (1960).

Translated by R. T. Beyer
30