THE FARADAY EFFECT IN SEMICONDUCTORS DUE TO FREE CARRIERS IN A STRONG MAGNETIC FIELD

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The dielectric constant tensor is determined for semiconductors with electrons with an anisotropic mass in a strong magnetic field. The Faraday effect is considered for the case of hexagonal crystals with an energy minimum in the center of the Brillouin zone and for a cubic crystal with a minimum which does not coincide with the center of the Brillouin zone. The refractive indices evaluated for different directions of the magnetic field relative to the main crystallographic directions in these crystals are found to depend strongly on the magnetic field direction. This enables us to determine the effective mass tensor by measuring the angle of rotation of the plane of polarization.

A large number of theoretical and experimental papers have been devoted in recent years to a study of the energy surfaces of the conduction electrons and to a determination of the effective mass tensor in semiconductors (see, for instance, reference 1).

One possible method of investigation is a study of the optical properties of semiconductors, in particular, the Faraday effect, i.e., the rotation of the plane of polarization in a magnetic field.

The Faraday effect in the microwave region can give information about the mobility of the conduction electrons;[2-4] the Faraday effect in the infrared region enables us, as was shown by Mitchell,[6] to determine directly the electron effective mass since one can in that region neglect the collisions between the electrons ($\omega\tau > 1$, where τ is the relaxation time). Experimental investigations by Moss, Smith, and Taylor[7] gave values for the effective mass for InSb which agreed with the results of Spitzer and Fan[8] who determined the effective mass from the coefficients of reflectivity and refraction.

The theoretical part of references 2 — 5 is based upon a classical model of free electrons with an isotropic mass in weak magnetic fields ($eH\tau/mc < 1$). Lax and Roth studied an electron with an anisotropic mass for the case of a magnetic field H in the [111] direction of a cubic crystal. This study referred, however, only to the case of a weak magnetic field. The phenomenon becomes essentially a quantum one in a strong magnetic field, showing up in a dependence of the chemical potential ζ on the magnitude and direction of the magnetic field. When there are several energy minima this leads to a non-equilibrium propagation of the electrons at those minima. The study by Lax and Roth is, however, insufficient even in the case of weak magnetic fields. The fact is that the frequency dependence of the Faraday effect depends in an important way on the direction of the magnetic field H. Generally speaking, the dielectric constant in a cubic crystal with several energy minima contains, for different directions of H, two resonance terms with sharp resonance frequencies. These frequencies coincide if the field is parallel to the [111] direction, and only this particular case was considered in reference 9.

It is the aim of the present paper to study the Faraday effect in the infrared region ($\omega\tau > 1$) for a conduction electron with an anisotropic mass and for different directions of a strong magnetic field.[10] We shall then obtain a solution of the problem of an electron with an anisotropic mass in a strong magnetic field which is considerably more convenient than Klinger and Voronyuk's solution.[11]

1. AN ELECTRON WITH AN ANISOTROPIC MASS IN A STRONG MAGNETIC FIELD

We take the magnetic field along the z axis and choose the vector potential in the form

$$\mathbf{A}^0 = (-Hy, 0, 0). \qquad (1)$$

The Hamiltonian is then

$$\hat{H} = \frac{1}{2m} \mathrm{S}\mu_{jk}\left(p_j - \frac{e}{c}A_j^0\right)\left(p_k - \frac{e}{c}A_k^0\right), \qquad (2)$$

where the μ_{jk} are the dimensionless components of the inverse mass tensor, and **S** is the symmetrization operator for non-commuting factors. We can use an effective mass representation since we are interested in the behavior of the electrons in a uniform field or in fields that vary sufficiently smoothly in space.[12]

The eigenfunctions and energy eigenvalues of the Hamiltonian (2) are of the form

$$\Psi^0_{np_xp_z} = \exp\left\{\frac{i}{\hbar}(p_x x + p_z z) - i\,\frac{\mu_{12}}{\mu_{22}}\,\frac{eH}{2\hbar c}(y-y_1)^2\right\}$$

$$\times \exp\left\{-\frac{m\Omega}{2\mu_{22}\hbar}(y-y_0)^2\right\}\mathcal{H}_n\left[\sqrt{\frac{m\Omega}{\mu_{22}\hbar}}(y-y_0)\right], \qquad (3)$$

$$E(n,\ p_z) = \hbar\Omega\left(n+\tfrac{1}{2}\right) + \eta p_z^2/2m. \qquad (4)$$

Here $\mathcal{H}_n$ is the wave function of the harmonic oscillator with frequency

$$\Omega = (eH/mc)\sqrt{M_{33}} = \Omega_0\sqrt{M_{33}}, \qquad (5)$$

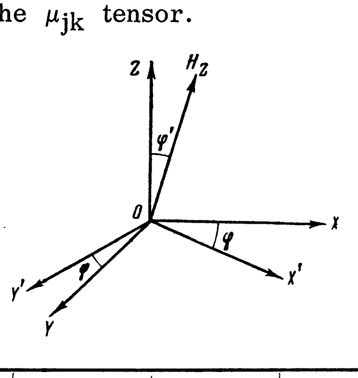
$$y_1 = -(c/eH)(p_z\mu_{23}/\mu_{12} + p_x), \qquad (6)$$

$$y_0 = -(c/eH)(p_z M_{31}/M_{33} + p_x), \qquad (7)$$

$$\eta = (M_{11}M_{33} - M_{13}^2)/\mu_{22}M_{33}, \qquad (8)$$

where M_{jk} are the cofactors of the components of the inverse mass tensor.

We shall express the μ_{jk} in terms of the principal values of the inverse mass tensor μ_j (j = 1, 2, 3). To do this we perform a rotation from a coordinate system (xyz) where H is along the z axis to the system of the principal axes of the μ_{jk} tensor (XYZ). Such a transition can be realized, for instance, using two rotations: a) a rotation around Z through an angle φ such that $H_{Y'} = 0$ in the new coordinate system, and b) a rotation around Y′ through an angle φ' such that in the new coordinate system $H_x = 0$, $H_y = 0$, $H_z = H$ (see the figure). The table of rotational cosines can be written in terms of the angles φ_i between the direction of the magnetic field and the principal axes of the μ_{jk} tensor.



	X	Y	Z
x	$\cos\varphi_1\cot\varphi_3$	$\cos\varphi_2\cot\varphi_3$	$-\sin\varphi_3$
y	$-\cos\varphi_2/\sin\varphi_3$	$\cos\varphi_1/\sin\varphi_3$	0
z	$\cos\varphi_1$	$\cos\varphi_2$	$\cos\varphi_3$

Using this table Eqs. (5) and (8) can be rewritten in the form

$$\Omega = \Omega_0\left[\mu_1\mu_2\mu_3\left(\frac{\cos^2\varphi_1}{\mu_1} + \frac{\cos^2\varphi_2}{\mu_2} + \frac{\cos^2\varphi_3}{\mu_3}\right)\right]^{1/2}, \qquad (9)$$

$$\eta = \left(\frac{\cos^2\varphi_1}{\mu_1} + \frac{\cos^2\varphi_2}{\mu_2} + \frac{\cos^2\varphi_3}{\mu_3}\right)^{-1} \qquad (10)$$

2. THE EVALUATION OF THE DIELECTRIC CONSTANT TENSOR

When light is propagated along the magnetic field, the perturbation energy is

$$\hat{H}' = -\frac{e}{2mc}\mu_{jk}\left[A_j^1\left(p_k - \frac{e}{c}A_k^0\right) + \left(p_k - \frac{e}{c}A_k^0\right)A_j^1\right], \qquad (11)$$

where

$$\mathbf{A}^1 = \frac{c}{i\omega}\left[\mathbf{E}_0 e^{i(fz-\omega t)} - \mathbf{E}_0^{+} e^{-i(fz-\omega t)}\right] \qquad (12)$$

is the vector potential of the light wave.

The current density

$$(I_j)_n = \frac{e}{2m}\left\{\Psi_n^*\mu_{jk}\left(p_k - \frac{e}{c}A_k^0\right)\Psi_n - \Psi_n\mu_{jk}\left(p_k + \frac{e}{c}A_k^0\right)\Psi_n^*\right\}$$

$$-\frac{e^2}{mc}\mu_{jk}A_k^1\Psi_n^*\Psi_n \qquad (13)$$

(n indicates here the set of values n, p_x, p_z) evaluated using the first-order perturbation-theory wave functions, must be averaged over a statistical electron distribution $\exp\{(\zeta - E)/T\}$. Taking into account that $E(n, p_z)$ is independent of p_x and that $f \ll p/\hbar$ and comparing the results of the averaging with the relation

$$I_j = i\omega E_k(\varepsilon_{jk} - \varepsilon_{jk}^0)/4\pi, \qquad (14)$$

which is well known from electrodynamics, we get for the dielectric-constant tensor

$$\varepsilon_{jk} = \varepsilon_{jk}^0 + \frac{4\pi e^2 N}{m\omega^2}\,\frac{\omega^2\mu_{jk} - i(e/mc)\,\omega\,\delta_{jkl}M_{ls}H_s - (e/mc)^2\Delta H_j H_k}{(e/mc)^2 H_\alpha H_\beta M_{\alpha\beta} - \omega^2}, \qquad (15)$$

where ϵ_{jk}^0 is the dielectric-constant tensor caused by the bound charges, δ_{jkl} the antisymmetric unit tensor, and Δ the determinant of the inverse mass tensor. The conduction electron density $N = \exp(\zeta/T)Z/V$ must be determined from the neutrality condition. Here, ζ is the chemical potential and Z the partition function:

$$Z = Z_0\hbar\Omega/2T\sqrt{\mu_1\mu_2\mu_3}\,\sinh(h\Omega/2T),$$

$$Z_0 = 2(2\pi)^{-3}(2\pi mT/\hbar^2)^{3/2}. \qquad (16)$$

One should note that Eq. (15) remains valid for an arbitrary orientation of the light wave relative to the magnetic field.

Hexagonal crystals. We shall consider the case where the bands are unconnected, where the energy minimum corresponds to the center of the Brillouin zone, and where the energy surface is an ellipsoid of revolution. If the axis of revolution is along Z, we must put in (15) $\mu_1 = \mu_2$. In hexagonal crystals the principal axes of the ϵ_{jk} and the μ_{jk} tensor are the same. If there is no magnetic field ϵ^0_{jk} will have three non-vanishing components $\epsilon^0_{xx} = \epsilon^0_{yy} \neq \epsilon^0_{zz}$. We get from Eq. (16) and the neutrality condition for the electron density in an impurity semiconductor (in the limiting case $\hbar\Omega > T$)

$$N = \frac{e^{\zeta/T}Z}{V} = \left(\frac{Z_0}{2V}\frac{\hbar\Omega}{2T}\frac{1}{\mu_1\sqrt{\mu_3}}N_1\right)^{1/2}\exp\left\{-\frac{\Delta\varepsilon}{2T}-\frac{\hbar\Omega}{4T}\right\}, \tag{17}$$

where N_1 is the impurity concentration and $\Delta\epsilon$ is the energy gap between the impurity level and the bottom of the conduction band.

Using the table we can write ϵ_{jk} in a coordinate system where the z axis is chosen along the magnetic field H and the optical axis of the crystal makes an angle φ_3 with H:

$$\varepsilon_{xx} = \left(\varepsilon^0_x + \frac{\nu^2\mu_1}{\Omega^2-\omega^2}\right)\cos^2\varphi_3 + \left(\varepsilon^0_z + \frac{\nu^2\mu_3}{\Omega^2-\omega^2}\right)\sin^2\varphi_3,$$

$$\varepsilon_{yy} = \left(\varepsilon^0_x + \frac{\nu^2\mu_1}{\Omega^2-\omega^2}\right),$$

$$\varepsilon_{zz} = \left(\varepsilon^0_x + \frac{\nu^2\mu_1}{\Omega^2-\omega^2}\right)\sin^2\varphi_3 + \left(\varepsilon^0_z + \frac{\nu^2\mu_3}{\Omega^2-\omega^2}\right)$$

$$\times\cos\varphi_3 - \frac{\nu^2\Omega_0^2\Delta}{\omega^2(\Omega^2-\omega^2)},$$

$$\varepsilon_{xy} = -i\nu^2\Omega^2/\Omega_0\omega(\Omega^2-\omega^2),$$

$$\varepsilon_{xz} = \left[\varepsilon^0_x + \frac{\nu^2\mu_1}{\Omega^2-\omega^2} - \varepsilon^0_z - \frac{\nu^2\mu_3}{\Omega^2-\omega^2}\right]\cos\varphi_3\sin\varphi_3,$$

$$\varepsilon_{yz} = -i\nu^2\Omega_0\mu_1(\mu_3-\mu_1)\cos\varphi_3\sin\varphi_3/\omega(\Omega^2-\omega^2), \tag{18}$$

where

$$\nu^2 = \frac{4\pi e^2 N}{m}, \qquad \Omega = \Omega_0\left[\mu_1^2\mu_3\left(\frac{\sin^2\varphi_3}{\mu_1}+\frac{\cos^2\varphi_3}{\mu_3}\right)\right]^{1/2}$$

We consider two cases:

a) $\varphi_3 = 0$, the magnetic field is along the optical axis. Then $\epsilon_{xx} = \epsilon_{yy}$; $\epsilon_{xz} = \epsilon_{yz} = 0$, $\Omega = \Omega_0\mu_1$. The refractive indices for left and right hand polarized light are $n^2_\pm = \epsilon_{xx} \pm i\epsilon_{xy}$ (see reference 13). The angle of rotation is determined by the formula

$$\varphi = \frac{\omega}{c}\left(\frac{n_+ - n_-}{2}\right)d, \tag{19}$$

where d is the thickness of the layer.

b) $\varphi_3 = 90°$, the magnetic field is perpendicular to the optical axis.

$$\varepsilon_{xx} \neq \varepsilon_{yy};\quad \varepsilon_{xz} = \varepsilon_{yz} = 0, \qquad \Omega = \Omega_0\sqrt{\mu_1\mu_3}.$$

There are now two elliptically polarized waves with refractive indices

$$n^2_\pm = \frac{1}{2}(\varepsilon_{xx}+\varepsilon_{yy}) \pm \sqrt{\frac{1}{4}(\varepsilon_{xx}-\varepsilon_{xy})^2 + \varepsilon_{xy}\varepsilon_{yx}} \tag{20}$$

which pass through the crystal and we measure the angle between the major axis of the ellipse of the transmitted wave and the direction of the polarization of the incident wave.

Cubic crystals. To be specific, we shall consider the Faraday effect in silicon. It is well known from cyclotron resonance experiments[1] that the conduction electron energy surface in quasi-momentum space consists of six ellipsoids of revolution which are situated along the fourth order axes of the cubic crystal.

The dielectric constant tensor has the form (15) for each ellipsoid, if we put $\epsilon^0_{jk} = \epsilon^0\delta_{jk}$, $\mu_1 = \mu_2$ and determine the electron concentration from the neutrality condition:[11] $\Sigma N_i + N_0 = N_1$, where N_0 is the density of electrons in the impurity levels, N_1 the impurity concentration, and

$$N_i = \left(\frac{Z_0}{V}\frac{\hbar\Omega_0}{2T}N_1\right)^{1/2}(\sin^2\varphi_i/\mu_1 + \cos 2\varphi_i/\mu_3)\exp\left(-\frac{\hbar\Omega_i}{2T}\right)\Big/ 2$$

$$\times\sum_{t=1}^{3}(\sin^2\varphi_t/\mu_1 + \cos^2\varphi_t/\mu_3)^{1/2}\exp\left(-\frac{\hbar\Omega_t}{2T}\right)$$

is the electron density for the i-th ellipsoid.

Since the electromagnetic field cannot transfer electrons from one energy minimum to another because the wave vector is small, we get for the dielectric constant of a cubic crystal a sum over all ellipsoids taking their relative position in the Brillouin zone into account. We shall take the coordinate axes along the four-fold axes of the cubic crystal, and then

$$\varepsilon_{XX} = \varepsilon_0 + \frac{1}{3}\left\{\frac{\mu_1\nu_3^2}{\Omega_3^2-\omega^2} + \frac{\mu_1\nu_2^2}{\Omega_2^2-\omega^2} + \frac{\mu_3\nu_1^2}{\Omega_1^2-\omega^2} - \left(\frac{\Omega_0}{\omega}\right)^2\right.$$

$$\left.\times\Delta\cos^2\varphi_3\sum_i\frac{\nu_i^2}{\Omega_i^2-\omega^2}\right\},$$

$$\varepsilon_{XY} = -i\frac{\Omega_0}{3\omega}\left\{\frac{\mu_1^2\nu_3^2}{\Omega_3^2-\omega^2} + \frac{\mu_1\mu_3\nu_2^2}{\Omega_3^2-\omega^2} + \frac{\mu_1\mu_3\nu_1^2}{\Omega_1^2-\omega^2}\right\}\cos\varphi_3$$

$$-\frac{1}{3}\left(\frac{\Omega_0}{\omega}\right)^2\Delta\cos\varphi_2\cos\varphi_1\sum\frac{\nu_i^2}{\Omega_i^2-\omega^2}.$$

All other components of the ϵ_{jk} tensor can easily be written down from symmetry considerations. Here

$$\nu_i^2 = \frac{4\pi e^2 N_i}{m}, \qquad \Omega_i = \Omega_0\left[\mu_1^2\mu_3\left(\frac{\sin^2\varphi_i}{\mu_1}+\frac{\cos^2\varphi_i}{\mu_3}\right)\right]^{1/2}.$$

Using the rotation determined by the table we can write down ϵ_{jk} in the coordinate system where the z axis is taken along the magnetic field. We shall consider the following cases:

a) H ∥ [001], $\varphi_3 = 0$. We have then

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_0 + \frac{1}{3}\left\{\frac{\mu_1 v_3^2}{\Omega_3^2 - \omega^2} + \frac{(\mu_1 + \mu_3) v_1^2}{\Omega_1^2 - \omega^2}\right\},$$

$$\varepsilon_{xy} = -\frac{i}{3\Omega_0\omega}\left\{\frac{\Omega_3^2 v_3^2}{\Omega_3^2 - \omega^2} + \frac{2\Omega_1^2 v_1^2}{\Omega_1^2 - \omega^2}\right\},$$

$$\varepsilon_{xz} = \varepsilon_{yz} = 0, \quad \Omega_1 = \Omega_0\sqrt{\mu_1\mu_3}, \quad \Omega_3 = \Omega_0\mu_1.$$

In this case the angle of rotation is given by Eq. (19).

b) H ∥ [111], $\varphi_1 = \varphi_2 = \varphi_3$. Then

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_0 + (2\mu_1 + \mu_3) v_3^2 / 3(\Omega_3^2 - \omega^2),$$

$$\varepsilon_{xy} = -(i/\Omega_0\omega)\,\Omega_3^2 v_3^2/(\Omega_3^2 - \omega^2),$$

$$\varepsilon_{xz} = \varepsilon_{yz} = 0, \quad \Omega_3 = \Omega_0\left[\frac{1}{3}(2\mu_1\mu_3 + \mu_1^2)\right]^{1/2}.$$

Here again the angle of rotation is determined using (19) and the expressions for the refractive indices $n_\pm^2 = \epsilon_{xx} \pm i\epsilon_{xy}$ are the same as those found in reference 9.

c) H ∥ [110], $\varphi_3 = 90°$. Then

$$\varepsilon_{xx} = \varepsilon_0 + \frac{1}{3}\left[\frac{\mu_3 v_3^2}{\Omega_3^2 - \omega^2} + \frac{2\mu_1 v_1^2}{\Omega_1^2 - \omega^2}\right],$$

$$\varepsilon_{yy} = \varepsilon_0 + \frac{1}{3}\left[\frac{\mu_1 v_3^2}{\Omega_3^2 - \omega^2} + \frac{(\mu_1 + \mu_3) v_1^2}{\Omega_1^2 - \omega^2}\right],$$

$$\varepsilon_{xy} = -i\frac{1}{3\Omega_0\omega}\left[\frac{\Omega_3^2 v_3^2}{\Omega_3^2 - \omega^2} + \frac{2\Omega_1^2 v_1^2}{\Omega_1^2 - \omega^2}\right],$$

$$\varepsilon_{xz} = \varepsilon_{yz} = 0, \quad \Omega_1 = \Omega_0\left[\frac{1}{2}\mu_1(\mu_3 + \mu_1)\right]^{1/2}, \quad \Omega_3 = \Omega_0\sqrt{\mu_1\mu_3}.$$

The refractive indices are determined in this case from (20), and we measure the angle between the rotation of the ellipse of the transmitted wave and the direction of the polarization of the incident wave.

We see that in all cases considered, the dependence of the indices of refraction and thus also of the angle of rotation of the polarization ellipse on the frequency of the light has a resonance character, and that the magnitude of the resonance for the frequencies depends on the direction of the magnetic field. In the case H ∥ [111] there is one resonance maximum. If, however, the field is parallel to the [001] or the [110] direction there are two resonance maxima. This makes it possible to determine the effective mass of conduction electrons both at the cyclotron resonance of the angle of rotation and also outside the resonance.

We need only take the quantum motion of the electrons into account for the evaluation of the electron concentration N_i, which leads to an exponential dependence of N_i on H.

In the limiting case of an isotropic mass our formulae go over into those of Rau and Caspari,[3] if we neglect in the latter the collision frequencies. In strong magnetic fields there is an additional exponential dependence of the electron concentration on the field.

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