ELASTIC SCATTERING OF POSITIVE PIONS BY CARBON AT ENERGIES OF 5 — 22 Mev

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Elastic scattering of positive 5 — 22 Mev pions by carbon was investigated in a propane bubble chamber. Phase-shift analysis of the angular distribution shows that a repulsive potential acts on the meson in the S state in the nucleus.

INTRODUCTION

An investigation of the scattering of very slow positive pions yields interesting data on the interaction between pions and nuclei practically in the S state, since the contribution of waves with $l \neq 0$ is found to be small at ~ 10 Mev. Analogous information for negative pions can be obtained when measuring the level shifts in π-mesic atoms. However, the pion energy does not exceed 0.13 Mev even at the K level of carbon and therefore, at energies greater than this value, a study of the scattering of pions by carbon is the only direct experimental method of investigating the interaction between pions and nuclei.

Investigations of the scattering of slow pions (with energies on the order of 20 Mev and greater) were undertaken with the aid of photoemulsions.[1-4] These papers give, in particular, data on the elastic and inelastic scattering cross sections. However the accuracy of the data given, together with the fact that there scattering by a mixture of heavy nuclei (for which the Coulomb scattering is large) takes place in the emulsion does not permit an analysis of the character of the pion-nucleon interaction in elastic scattering.

In an earlier paper[5] we reported preliminary data on the scattering of slow positive pions by carbon. The cross sections given in reference 5 include nuclear scattering and interference between the Coulomb and nuclear scattering.

In the present paper we give a data-reduction method and an analysis, based on greater statistical material, of the angular distributions of elastic scattering of positive pions at energies 5 — 22 Mev.

EXPERIMENTAL CONDITIONS

The scattering was investigated in a propane bubble chamber. Some of the material was obtained with the chamber described by Kotenko, Kuznetsov, and Popov.[6] Another portion of the material was obtained with the chamber described by Pershin,[7] and was graciously made available to us by the author, for which we take this opportunity to express our gratitude. The material obtained with these chambers was used earlier to determine the correlation in $\pi \rightarrow \mu \rightarrow e$ decay.[8,9] The chambers were irradiated in the pion beam of the Joint Institute for Nuclear Research, the pions were produced by the external proton beam on a polyethylene target 70 cm thick. The 170-Mev mesons emitted from the target at an angle of 70° were guided into the collimator by a deflecting magnet. In front of the chamber itself the pions were slowed down with an absorber so that most pions of the beam were stopped within the working volume of the chamber.

PROCESSING OF THE EXPERIMENTAL DATA

We investigated pions scattered and stopped in the chamber (the stopping of a positive pion was identified by the characteristic $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ decay). In scanning the statistical material, an important role can be played by subjective errors, connected with miscounting of stopped positive pions. It can be stated that a positive pion stopped after prior scattering of the pion (particularly at a large angle) is observed in scanning more frequently than a stopping without scattering. Thus, the current of stopped particles may be underestimated and the scattering cross section may be artificially overestimated. To avoid this error as much as possible, half of the entire statistical material was scanned three times, the first two times being scanned only for the purpose of detecting the stopping of a positive pion as identified by the $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ decay. The number of stopped positive pions after the first and second scannings was 5,306 and 6,670 respectively, i.e., the number of stoppings missed in the first scanning was 11.5%

TABLE I

Energy intervals, ΔE, Mev	Angular intervals, $\Delta\varphi$, deg	Observed number of scattering events, ΔN_{obs}	Number of scattering cases after introducing the correction, ΔN_c	$d\sigma/d\varphi$, mb/sterad: Experiment	Solutions I	Solutions II	Solutions III	Data of Byers[13]
					$\eta_0=-8°$, $\eta_1=-1°45'$	$\eta_0=33°$, $\eta_1=11°35'$	—	$\eta_0=-3°$, $\eta_1=1°$
5–8	15–20	25	20.8	3612±754	2702	1335	—	2303
	20–60	18	18.4	393±100	510	369	—	349
	60–120	9	9.5	141±47	72	157	—	39
	120–180	1	1.1	16±16	25	3	—	24
					$\eta_0=-7°30'$, $\eta_1=-1°$	$\eta_0=5°$, $\eta_1=-5°$	$\eta_0=20°$, $\eta_1=14°$	$\eta_0=-4°30'$, $\eta_1=2°40'$
8–15	15–20	24	20.5	853±181	835	987	539	727
	20–60	26	30.3	159±33	148	148	136	84
	60–120	4	6.2	22±12	29	5	28	15
	120–180	6	6.9	25±10	22	19	51	23
					$\eta_0=-4°$, $\eta_1=2°$	$\eta_0=12°$, $\eta_1=-0°45'$	$\eta_0=20°$, $\eta_1=0°45'$	$\eta_0=-6°20'$, $\eta_1=6°$
15–22	15–20	11	9.8	273±88	297	324	203	262
	20–60	7	10.5	36±15	46	60	27	22
	60–120	4	9.2	23±12	9	16	20	13
	120–180	3	4.5	11±7	11	15	19	29

will decay in flight, W_2 the probability that the positive muon will enter the investigated range of angles in the $\pi^+ \to \mu^+$ decay, and W_3 is the probability that the positive muon will imitate the $\pi^+ \to \mu^+$ decay at the end of its range (single scattering greater than 15°). W_1, W_2, and W_3 can be calculated.

c) Calculation of the error in the measurement of the angles and of the multiple scattering. In determining the limit of angle intervals and in plotting the angular distributions, it is very important to account correctly for the excess count of scatterings in a given angular interval due to the error in the measurement of the angles and due to multiple scattering. Assuming the distribution of the deviations in the angle measurement to be Gaussian within two standard deviations, the correction desired is obtained in the form

$$W_{\text{ang}} = \int\limits_{\varphi_{\text{lim}}-2\sigma}^{\varphi_{\text{lim}}} F(\varphi)\, d\varphi \frac{1}{\sqrt{2\pi}} \int\limits_{\varphi_{\text{lim}}-\varphi}^{\infty} e^{-t^2/2} dt - \times \int\limits_{\varphi_{\text{lim}}}^{\varphi_{\text{lim}}+2\sigma} F(\varphi)\, d\varphi \frac{1}{\sqrt{2\pi}} \int\limits_{\varphi-\varphi_{\text{lim}}}^{-\infty} e^{-t^2/2} dt, \quad (4)$$

where $F(\varphi)$ is the function that determines the projection of the Coulomb scattering on the plane of the film, $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$; σ_1 is the mean-squared error in the measurement of the angle, σ_2 is the mean square of the projection of the angle of multiple scattering for a given energy interval. The integration was carried out numerically.

d) Calculation of the error in the measurement of the energy. The correction for the additional number of events within a given energy interval was allowed for in plotting the energy dependence of the scattering cross section by using a method similar to that described in case c). This correction was found to be negligibly small.

e) Scattering by hydrogen. The correction for Coulomb and nuclear scattering by the hydrogen contained in the propane was taken into account by the usual method used to determine the total cross section of scattering by a complex substance.

f) Inelastic scattering. In the investigation of scattering of positive pions by nuclei at low energies in a bubble chamber it is difficult to separate the elastic scattering from the inelastic one, since the positive pion has an increased track density long before stopping and a small (4 — 7 Mev) decrease in energy cannot be noticed. However, from experiments on scattering of positive pions by emulsion nuclei at our energies[1-4] and by carbon nuclei at energies greater than 30 Mev[11] it is known that the inelastic-scattering cross section diminishes rapidly with decreasing energy and is practically zero at energies less than 30 Mev. We therefore assume that the scattering measured by us is fully elastic.

ANALYSIS OF THE EXPERIMENTAL DATA

To plot the angular distributions, the energy interval investigated by us (5 — 22 Mev) was broken down into three smaller intervals: 5 — 8, 8 — 15, and 15 — 22 Mev.

The results of making all the corrections for the angular distributions are presented in Table I. We also list there the experimental cross sections by angle intervals (Coulomb plus nuclear).

It is known that the differential angular cross section for scattering by a nucleus of charge Z of the number of cases observed in two scannings. The third scanning was to detect scattering of the previously found stopped positive pions. In this scanning, too, additional stoppings were observed, but their number was small compared with the total number of cases (less than 3% of the number found in the first two scannings). We believe that such a scanning of the material excludes the possibility of predominant observation of stopping plus scattering cases.

In the measurement of scattering we excluded from consideration stopped mesons with ranges less than 6 mm in the chamber prior to decay (the decay positive pion had a range of 3.15 mm), cases when the positive pion entered the chamber from the glass or at a large angle from the upper or lower walls of the chamber, and finally cases when the angle between the projection of the positive-pion momentum prior to stopping and the direction of the momentum of the decay positive pion was less than 15° — in these cases the admixture of positive pions produced outside the chamber is large.

In the third scanning we counted all single scatterings by more than 10° (projected on the plane of the film). Measurement of the projections of the angles allowed us to accelerate the data reduction and to decrease the error in the determination of the angle. The angles were measured accurate to ± 1°.

All scattering cases were scanned two more times. In these scannings we selected carefully all cases of single scattering by angles greater than 15°. Such a selection was necessitated by the large value of the Coulomb scattering at our energies, and by the difficulty of separating single scatterings from multiple scatterings at smaller angles.

The second part of the statistical material was scanned carefully once more after the final reduction of the first part of the material (the two parts were approximately equal statistically). The results obtained with the second half of the material agreed, within the limits of statistical errors, with the results obtained in the reduction of the first half of the material. The statistics were then combined. Thus we selected 8,727 photographs of positive-pion tracks stopped in the chamber for our investigation of scattering.

The particle energy at the point of scattering was determined from the residual range. The errors in the measurements of the energy did not exceed 10% for the selected energy interval from 5 to 22 Mev. A total of 137 particles were scattered within the angle and energy intervals.

To proceed with the calculation of the cross sections and to determine the angular distributions of the scattering, it is necessary to introduce corrections for many factors, which we shall now discuss.

a) Geometric correction. The finite dimensions of the chamber cause some of the scattered particles to go into the walls of the chamber and become unobservable. The calculation of the correction for such omissions, owing to the rectangular form of the chamber, is similar to the problem of finding the probability that a segment of length l will fall in a random test on one of two parallel lines, the distance between which is a (see, for example, reference 10). Taking into account the features of the chamber geometry and of the particle scattering it is possible, using the solution to the above problem, to obtain the following value for the correction coefficient:

$$K_{\text{corr}} = \left[1 - \frac{2l\sin\theta}{\pi}\left(\frac{1}{a}+\frac{1}{b}\right) + \frac{l^2\sin^2\theta}{\pi}\right]^{-1}, \quad (1)$$

where θ is the three-dimensional angle of particle scattering, l the true length of the particle track after scattering, while a and b are the transverse dimensions of the chamber (in our case the height was equal to the depth). We used a correction coefficient of somewhat simpler form

$$K_{\text{corr}} = (1 - 2l\sin\varphi/\pi a)^{-1}, \quad (2)$$

where φ is the projection of the scattering angle on the plane of the film. This results in a slightly overvalued correction, but by an amount which is much smaller than the statistical error, and is therefore insignificant.

Corrections based on (1) and (2) are introduced for each individual scattering case and thus take automatically into account the spectrum of the scattered particles; (1) takes into account the distribution in space, while (2) takes into account the distribution in the projection on the emulsion plane.

The correction coefficients (1) and (2) disregard a certain inhomogeneity in the positive-pion current over the height and depth of the chamber, and also the dip angle of the entering positive-pion beam. Estimates show that these can be neglected in our case.

b) Corrections for positive-pion in-flight decay identified as scattering of positive pions. A positive pion can decay in flight and the positive decay muon, which experiences scattering shortly before stopping, may imitate a $\pi^+ \to \mu^+$ decay. Such cases can be mistaken for a positive-pion scattering.

The overall probability of registering a positive-pion decay as a scattering is

$$W = W_1 W_2 W_3, \quad (3)$$

where W_1 is the probability that the positive pion is given, at small energies, by the following formula (see, for example, reference 12):

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{4k^2}\left|\frac{\exp\{-i\alpha\ln\sin^2(\theta/2)\}}{\sin^2(\theta/2)}+\frac{i}{\alpha}(e^{2i\eta_0^*}-1)\right.$$
$$\left.+\frac{3i}{\alpha}\frac{1+i\alpha}{1-i\alpha}(e^{2i\eta_1}-1)\cos\theta\right|^2, \qquad (5)$$

where $\alpha = Ze^2/\hbar v$, v the velocity of the incident particle, k the wave number, e the electron charge, θ the scattering angle, $\eta^* = \eta_0 + i\delta_0$ the complex phase shift of the S wave, and η_1 the phase shift of the P wave. It is assumed here that in the investigated energy region the principal contribution to the scattering is made by the S and P waves and that only the S wave is absorbed (see, for example, the paper by Byers[13]). Formula (5) is correct in the case of a target nucleus of infinite mass. In our case this condition is satisfied with high accuracy.

We reduced the data by projection, i.e., we measured the projections of the scattering angles of the positive pions on the plane of the emulsion. To compare the experimental angular distributions with the theoretical formula (5), the latter had to be projected on the plane passing through the primary direction of the particle track. In machine computation, the formula was projected exactly. Here we give an approximate value of the projected formula.

Omitting the intermediate computations, we can write for small phases η_0, δ_0, and η_1 (i.e., using the approximation $\sin\eta_1 = \eta_1$ and $\sin\delta_0 = \delta_0$)

$$\frac{d\sigma}{d\varphi}=\left[\frac{\alpha^2 r_0^2}{4\beta^2}F_1(\varphi)+24\frac{r_0^2}{\beta^2}\eta_1^2\cos^2\varphi\right.$$
$$\left.-\frac{\alpha r_0^2}{\beta^2}F_2(\varphi)-\frac{3\pi r_0^2}{\beta^2}\eta_1 F_3(\varphi)+A\right]\text{rad}^{-1},$$

where $r_0 = \hbar/\mu c$, μ is the reduced mass, φ the projection of the scattering angle θ on the plane passing through the initial direction of the scattered particles,

$$F_1 = 16(\sin\varphi + (\pi-\varphi)\cos\varphi)/\sin^3\varphi.$$

is the projection of the Coulomb scattering,

$$F_2 = 4\frac{2(\pi-\varphi)-\pi\sin\varphi}{\cos\varphi\sin\varphi}\,[(1-C_1)(\eta_0-2\delta_0\eta_0+3\eta_1)$$
$$-C_2(\delta_0+\eta_0^2+6\alpha\eta_1)],$$

$$F_3 = \cos\varphi\,(C_1\alpha + 2\alpha^2 C_2 - 2\eta_0),$$

$$A = \frac{4r_0^2}{\beta^2}[\eta_0^2+\delta_0^2-C_1\alpha(\eta_0-2\delta_0\eta_0+6\eta_1)$$
$$-C_2\alpha(\delta_0+\eta_0^2+12\alpha\eta_0)+6\alpha\eta_1].$$

The constants C_1 and C_2 are determined from the conditions of approximate equality of the function used in the projection

$$\cos\left(\alpha\ln\sin^2\frac{\theta}{2}\right)=1-C_1\cos^2\frac{\theta}{2},\qquad C_1=C_1(\alpha),$$
$$\sin\left(\alpha\ln\sin^2\frac{\theta}{2}\right)=C_2\cos^2\frac{\theta}{2},\qquad C_2=C_2(\alpha).$$

A better agreement between the exact and approximate functions was obtained for C_1 at $\theta = 20°$ and, for C_2 at $\theta = 60°$ — the discrepancy in the remaining points did not exceed 10% in this case.

It can be shown that in the geometry of our experiment the conical projections of the scattering angles on the plane of the film differ very little from the orthogonal projections.[14]

From the plotted experimental angular distributions we found the phase shifts η_0 and η_1. To determine the shifts we used a somewhat modified least-squared method, which took into account the errors in the experimental values. The method was used in this form by Fermi[15] and consisted essentially of a method of estimating parameters by the minimum of χ^2 (reference 16). The phases were determined with a "Ural" digital computer, which evaluated the sum

$$\sum_i\left(\frac{\sigma_{ti}-\sigma_{ei}}{\varepsilon_i}\right)^2 = N(\eta_0,\ \eta_1). \qquad (6)$$

The value of N was calculated in the phase-shift range from +50 to −50°. This interval was specified on the basis of the values of the phase shifts calculated in the paper by Byers[13] for the scattering of negative pions by carbon and on the basis of analysis of mesic-atom data.

In Eq. (6), σ_{ei} is the experimental scattering cross section in the i-th angle interval, ϵ_i is the error in the determination of the experimental value of the cross section, and σ_{ti} is the theoretical value of the cross section in the given angle interval. The cross section σ_{ti} was calculated from the projected formula (5), neglecting also the absorption of the S wave. This assumption can be justified by the fact that the data on the absorption of positive pions in beryllium at 20 and 39 Mev,[17] and also the data on the measured width of the 1S level in the π-mesic atom from beryllium,[18] would lead us to expect that, in the investigated energy interval, the phase shifts corresponding to absorption δ_0 are one-third the scattering phase shifts η_0.[13] Allowance for absorption phase shifts of this order of magnitude changes the differential scattering cross section by not more than 5%, which is much less than the error in the cross-section measurements. This allows us to neglect absorption of the S-wave. Unfortunately, it is very difficult to determine experimentally in a bubble chamber the absorption cross section at energies 5 — 22 Mev.

In the indicated phase-shift interval we obtained for N three relative minima at 8 — 15 and 15 — 22

TABLE II. Relative minima of the values of N and the corresponding values of the phase shifts

ΔE, Mev	5–8		8–15			15–22		
Solution	I	II	I	II	III	I	II	III
η_0	−8°	33°	−7°30′	5°	20°	−4°	12°	20°
η_1	−1°45′	11°30′	−1°	−5°	14°	+2°	−0°45′	0°45′
N	5.6	9.2	0.6	2.5	6.9	2.0	2.3	2.1
P, %	80.0	0.05	99.0	90.0	3	87.0	4	50

Mev, and two minima at 5 — 8 Mev. The values of the phase shifts and the corresponding values of N are given in Table II. The sixth, seventh, and eighth columns of Table I give the cross sections corresponding to these solutions, calculated by formula (5).

Inasmuch as the differential elastic-scattering cross sections of zero-spin particles permit a unique determination of the scattering amplitude,[19] only one of the found solutions should be retained. The choice was made on the basis of an examination of the values of N, of the agreement levels for the Kolmogorov criteria, and of the stability of solution under variation of energy.

To avoid errors in the choice of a solution, which may be caused by insufficient statistics (which can lead in turn to a change in the profile of the function $N(\eta_0, \eta_1)$ in a repetition of the experiment), we estimated all pairs of solutions by the Kolmogorov agreement criterion. The agreement levels P for the Kolmogorov criteria are listed in Table II. Specifying, as usual, a 5% value level, we can discard solution II in the 5 — 8 Mev and 15 — 22 Mev intervals and solution III in the 8 — 15 Mev interval. Among the remaining solutions, solution I has in each energy interval a definite stability under a small change of energy of scattering particles, while the other solutions have no such stability. Obviously these solutions should be chosen as the true ones.

From the integrated cross sections for scattering at angles greater than 15° we estimated the statistical errors of the obtained phase shifts. Final data are summarized in Table III.

TABLE III

ΔE, Mev	5–8	8–15	15–22
η_0	−8°±4°	−7°30′±2°	−4°±3°
η_1	−1°45′±4°	−1°±3°	2°±1°

From the values of the phase shifts it is seen that in the scattering of 5 — 22 Mev positive pions the principal role is played by scattering in the S state, and that the phase shift of the S phase is negative, corresponding to the presence of repulsion in the S state. Since the P phase is small, it can be stated that in the investigated energy interval the nuclear forces between the carbon nucleus and the positive pion are repulsive in character, in other words, there is a positive effective potential between the nucleus and the positive pion.

This positive potential is in apparent contradiction with the results of an entire group of investigations of elastic scattering of pions by carbon nuclei in the energy range from 30 to 125 Mev.[20–24] The results obtained in these investigations (based on the optical model) indicate an attractive potential. However, there is no contradiction here since in the 30 — 125 Mev region P scattering predominates, with positive phase, and this yields an effective negative potential.*

TABLE IV. Phase shifts calculated with allowance for the Coulomb interaction, based on data for π-mesic atoms[13]

ΔE, Mev	5–8	8–15	15–22
η_0	−3°	−4°30′	−6°20′
η_1	+1°	+2°30′	+6°

Our result is qualitatively in agreement with data on the measurement of energy level shifts of negative pions in π-mesic atoms,[25] where the rise in the levels is evidence of a repulsive potential acting on the pion. Byers[13] calculated, from an analysis of the level shift in π-mesic atoms, the possible phase shifts in scattering of negative pions by carbon at 5 and 10 Mev. Using the results of this paper, we calculated (with allowance for the Coulomb interaction) the expected phase shifts for the scattering of positive pions by carbon at our energies. These phase shifts are listed in Table IV, and the corresponding cross sections are given in the last column of Table I. The cal-

*We can note in this connection that in the optical model the graph of the real part of the potential becomes positive at small energies, and does not vanish from the side of negative values, as shown by Frank et al.[26] (Fig. 2b).

culated phase shifts deviate somewhat from those we obtained experimentally, but we do not attach too great a significance to this since, firstly, the statistical errors in our experiments are large, and secondly the data on π-mesic atoms for the P wave are not reliable.

Assuming additivity of the pion-nucleus interaction potential, we can obtain from our phase shifts the connection between the phase shifts of the S waves in meson-nucleon scattering. Calculations yield $\delta_1 + 2\delta_3 < 0$ (where δ_1 and δ_3 are the phase shifts of the S wave at $T = 1/2$ and $T = 3/2$) in the entire energy interval under investigation. This agrees both with conclusions drawn from mesic-atom data and with direct measurements of meson-nucleon scattering.[27]

We note that an analysis of the experimental data on nuclear scattering at low energies is difficult because of the presence of a large Coulomb background, particularly at small angles, and the presence of interference between the Coulomb and nuclear scattering. The first difficulty can be eliminated by correct allowance for Coulomb scattering and by increasing the statistical material. Correct computation of the interference terms makes it desirable to obtain data on scattering of negative pions at the same energies.

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