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ON THE ROTATIONAL LEVELS OF Li^7

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THE conjecture that structural subgroups consisting of two, three, and four nucleons can be formed within light nuclei has been made by many authors.[1-4] In references 5 and 6, the disintegration of Li^7 into an α particle and a triton as a result of Coulomb excitation and of scattering of a heavy nucleus has been treated in terms of the α-triton model. One can easily see that such a model will lead to rotational levels in Li^7. The axis of symmetry will be given by the line connecting the centers of mass of the α particle and of the triton while the axis of rotation will be perpendicular to the symmetry axis and will go through the center of mass of the system.

Recently Blair and Henley[7] have shown that several levels of Be can be interpreted as rotational states if this nucleus is visualized as consisting of two separate α particles oscillating along an axis connecting their centers of gravity. In the present paper it will be shown that one can also interpret some levels of Li^7 as having rotational character if one assumes the α-triton model.

As is well known, the ground-state spin of Li^7 differs from zero ($J_0 = 3/2$). Taking further into account that the present model has just an axis of symmetry (not a center of symmetry) one deduces that the rotational spectrum will have angular momenta $J = J_0$, $J_0 + 1$, $J_0 + 2, \ldots$ while the parities will coincide with the ground-state parity ($3/2^-$). The energies of the levels are given by the expression

$$E_J = (\hbar^2/2I)] J(J+1) - J_0(J_0+1)], \qquad I = \mu\overline{r^2}, \qquad (1)$$

where μ is the reduced mass of the $(\alpha+t)$ system, and r is the distance between α and t. It follows from (1) that the ratios of the excitation energies of the rotational levels are

$$E_{5/2} : E_{7/2} : E_{9/2} \ldots = 1 : 2.40 : 4.20 \ldots$$

Amongst the levels of Li^7 there exists[8] one 7.46-Mev level with spin $5/2^-$. Taking this to be the first rotational level, we see that the 17.5-Mev level can be assumed to be the next rotational level with spin $7/2^-$ since the experimental ratio 2.35 of the energies is close enough to the theoretical ratio 2.40.

To verify our treatment we must obtain the right value for the energy of the first level, viz. 7.46 Mev. To that end we utilize the rms value 2.71×10^{-13} cm obtained by Hofstadter[9] for the charge radius of the Li^7 nucleus. Assuming that the mean distance between the α particle and the triton equals roughly the charge radius, we obtain from (1) a value 8.22 Mev, which is close enough to the experimental value of 7.46 Mev. If we require that the energy of the first level coincide exactly with the experimental value, we obtain for the rms distance a value 2.85×10^{-13} cm. This value is somewhat larger than the charge radius. However, as is known the nuclear radius always turns out larger than the charge radius.

The value obtained for $\overline{r^2}$ allows also the evaluation of the quadrupole moment of the Li^7 nucleus. Taking it into account that the quadrupole moments of He^4 and He^3 vanish, we obtain, in a coordinate system in which the origin coincides with the center of mass of the $(\alpha+t)$ system and where the z axis is oriented along the axis of symmetry of the nucleus, the following expression for the quadrupole moment operator;

$$\hat{Q} = (68/49)\sqrt{4\pi/5}\, r^2 Y_{20}(\vartheta). \qquad (2)$$

In our coordinate system the wave function of the $(\alpha+t)$ system will have the form

$$\psi = [\delta(\mathbf{r} - \mathbf{R}_0)]^{1/2}, \qquad \mathbf{R}_0 = (\sqrt{\overline{r^2}}, 0, 0). \qquad (3)$$

Using this expression, we obtain for the intrinsic quadrupole moment of Li^7

$$Q_0 = 68\overline{r^2}/49 = 11 \cdot 10^{-26}\ \text{cm}^2.$$

This value is several times larger than the experimental value, 2×10^{-26} cm^2. However we have to consider the obtained value to be more or less acceptable when we recall that even the unified model which describes the nuclear states rather satisfactorily leads to too large a value for the quadrupole moment. Also, the hydrodynamic model (assuming that Li^7 is deformed in the sense of the unified model and utilizing the energy of the first rotational level) yields a value for the quadrupole moment which is an order of magnitude larger than the experimental value. As has been shown earlier[5] the present model of Li^7 leads to a good agreement also for the magnetic moment ($\mu_{\text{theoret}} = 3.56$; $\mu_{\text{exp}} = 3.25$).

We finally point out that the value for the distance between the α particle and the triton (2.8×10^{-13} cm) is larger than the particle size, $\sim 1.5 \times 10^{-13}$ cm. This indicates that the employed model is not self-contradictory.

In conclusion we express our gratitude to I. Sh. Vashakidze and G. A. Chilashvili for discussions.

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ON THE THEORY OF THE NUCLEAR MOMENT OF INERTIA

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THE aim of the present note is to establish a connection between the two ways of determining the nuclear moments of inertia which have been proposed by Inglis[1] and Bohr and Mottelson[2] on the one hand and by Villars[3] and Hayakawa and Marumori[4] on the other hand. To begin with we have to consider the formulation of the second of these approaches. Further, we are not interested in the original abstract formulation but in the one to which we must turn when actually performing a computation.

Let φ be the collective angular variable, given by the angle of rotation of the main axes of the nucleus in a plane perpendicular to the axis of rotation, Z:*

$$\varphi = {}^1\!/_2 \tan^{-1}\left[\sum 2mxy \Big/ \sum m\,(x^2 - y^2)\right]. \qquad (1)$$

The summation in (1) is over all nucleons; the indices showing the nucleon number have been omitted; m is the nucleon mass.

We note the important commutation relation: $i\,[M_Z, \varphi] = \hbar$ where M_Z is the projection of the angular momentum operator of the nucleus on the Z axis.

Let H_0 be a model Hamiltonian of the nucleus oriented in a given manner in the XY plane. The kinetic energy operator of such a Hamiltonian commutes with M_Z while the potential energy operator does not. We now define the quantities N_Z and I_0 by means of the relations

$$i\hbar^{-1}\,[H_0, \varphi] = -N_Z/I_0, \qquad -\hbar^{-2}\,[[H_0, \varphi], \varphi] = 1/I_0. \quad (2)$$

The quantity $L_Z = M_Z + N_Z$ is the projection of the angular momentum on the Z axis in a coordinate system fixed with respect to the nuclear axes. It commutes both with φ and M_Z, while $i\,[N_Z, \varphi] = -\hbar$. The quantity I_0 is the so-called hydrodynamic moment of inertia. It is a continuous function of the coordinates and commutes with φ as well as with M_Z and N_Z. As a simplification we shall take I_0 to be a c-number, but as one can easily convince oneself the final result does not depend on this assumption.

According to the references 3 and 4 the nuclear moment of inertia is roughly given by

$$I = I_0 + 2\sum_{n \neq 0} |\langle \Phi_n, L_Z \Phi_0\rangle|^2/(E_n - E_0). \qquad (3)$$

Here Φ_n and E_n are the eigenfunctions and eigenvalues of the Hamiltonian H_0 respectively.

On the other hand, according to references 1 and 2 the moment of inertia is given by

$$I = 2\sum_{n \neq 0} |\langle \Phi_n, M_Z \Phi_0\rangle|^2/(E_n - E_0). \qquad (4)$$

We now compare these two expressions. First we note that in deriving (3) it is implicitly assumed that in a deformed nucleus the orientation of the main axes cannot deviate appreciably from the orientation of the self-consistent field. This implies in particular that the first of the relations (2) can be replaced by