THEORY OF THE STABILITY OF MAGNETIC STATES OF FERROMAGNETIC MATERIALS IN THE MAGNETIZATION PROCESS

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The physical principles that determine the stability of magnetic states of a ferromagnetic monocrystal, with respect to external magnetic fields and to elastic forces, are considered. A formula is derived for the minimum value of magnetic fields and stresses at which the equilibrium of a domain wall passing near a nonmagnetic inclusion is destroyed, and at which an irreversible change of magnetization occurs. On this basis an explanation is given for the phenomenon, familiar experimentally, of strong magnetization of ferromagnets in weak magnetic fields by shocks or blows, and formulas are derived for estimating the irreversible changes of magnetization produced by elastic stresses. An explanation is given for the observed stability, with respect to elastic forces, of magnetic states corresponding to the ideal magnetization curve.

THE magnetic moment and mean magnetization of a ferromagnetic body are, as is known, not single-valued functions of the magnetic field intensity H, the temperature T, and the components σ_{ik} of the stress tensor that represents the effect of external forces. The magnetic moment can assume a value other than the original one when the values of H, T, or σ_{ik}, undergoing some kind of fluctuation, return to their original values (magnetic, thermomagnetic, and magnetoelastic hysteresis); it can also change with time though the values of these quantities remain constant (magnetic aftereffect).

The present work deals with the physical principles that determine the stability of magnetic states of a ferromagnetic monocrystal with respect to elastic forces. An explanation is given for the observed stability of magnetic states corresponding to the ideal magnetization curve.

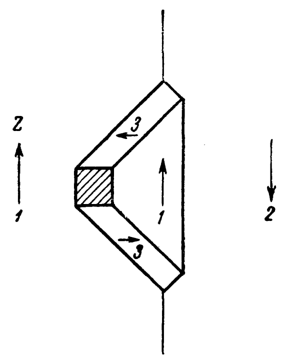
1. EFFECT OF EXTERNAL STRESSES ON THE COERCIVE FORCE AND MAGNETIZATION OF MONOCRYSTALS

The coercive force of a ferromagnetic monocrystal, magnetized along an axis of easy magnetization, depends on a number of factors and is usually a sum of several terms, for example

$$H_c = H_\sigma + H_d, \qquad (1)$$

where the first term is connected with the effect of inhomogeneous internal stresses, the second with the effect of inclusions on the movement of walls between domains. If the magnetic anisotropy constant $K \gg \lambda_s \sigma_i$ (where λ_s is the saturation magnetostriction and σ_i the mean magnitude of the internal stresses), then the second term plays the fundamental role in magnetization processes.

In works of Néel[1] and the author[2] it has been shown that 180° walls are attracted by secondary domains toward inclusions; when no external magnetic field is present, these walls pass through inclusions[3] or are located in regions of greatest inclusion concentration. The first of these deductions was confirmed experimentally by observation of powder figures on the surfaces of monocrystals of silicon iron.[4] In these investigations it was also shown that in a magnetic field the walls begin to move, and thereupon the secondary domains that are produced around an inclusion become extended in a direction at an angle of 45° to the plane of the wall, forming a connection between the wall and the inclusion (cf. the figure). H_d represents the field intensity at which the wall breaks away from the secondary domain and an irreversible jump of magnetization occurs.



The surface and volume energies of domains depend, as is known, on the components of the strain tensor. It follows that external stresses should have an effect on the energy of secondary domains and thus should affect the value of H_d. The magnetic state of a crystal in a field H will be stable with respect to the elastic forces if the change of H_d due to these forces is such that the difference $H_d - H$ does not become zero. In the contrary case, a crystal in a magnetic field, under the influence of elastic stresses, will undergo irreversible changes of magnetization.

Calculations of H_d for various cases have already been made,[1-3] but the effect of elastic stresses on H_d has so far not been considered. Furthermore, in the calculation of H_d no attention has been paid to the elastic energy of secondary domains, an energy due to magnetostriction. In the present work, in calculating the H_d of a strained crystal, we shall restrict ourselves to consideration of the case in which a tensile or compressive force acts along the direction of a cube edge parallel to the magnetization of the primary domains of the crystal, such as domains 1 and 2 in the figure. Furthermore, following the method used by E. M. Lifshitz[5] in calculating the domain structure of an iron crystal, we shall suppose that the strain tensor is constant within the crystal, since the volume occupied by the secondary domains 3, formed around the inclusion, is small in comparison with the volume of the primary domains magnetized parallel or antiparallel to the z axis. In this case the difference of the strain tensor components is

$$\tau_{zz} - \tfrac{1}{2}(\tau_{xx} + \tau_{yy}) = \tau_{zz} - \tau_{xx} = \tfrac{3}{2}\lambda_{100} + \sigma / 2C_2, \quad (2)$$

where λ_{100} is a magnetostriction constant, C_2 is an elastic modulus of the crystal, and σ is the value of the external tensile stress; hence the free-energy density of the secondary domains is

$$F = 3\lambda_{100}C_2(\tau_{zz} - \tau_{xx}) = \tfrac{9}{2}\lambda_{100}^2 C_2 + \tfrac{3}{2}\lambda_{100}\sigma. \quad (3)$$

The change $\Delta\Phi$ in the thermodynamic potential $\Phi = F - HI$, referred to unit area of the wall, upon displacement of the latter through Δx in a constant field H, is

$$\Delta\Phi = \left[-2HI_s\left(1 - \tfrac{1}{2}a_2 v^{2/3}\right) + a_1\gamma_\sigma v^{2/3}/d + \tfrac{3}{2}a_2\lambda_{100}(3\lambda_{100}C_2 + \sigma)v^{2/3}\right]\Delta x, \quad (4)$$

$$\gamma_\sigma = \gamma\left[1 + 3\lambda_{100}\sigma / 2(K + 3\lambda_{100}^2 C_2)\right]^{1/2}, \quad (4')$$

where γ is the surface energy of the wall at $\sigma = 0$, d is the mean diameter of the inclusions, v is their volume concentration, K is the anisotropy constant, and a_1 and a_2 are dimensionless coefficients of order unity, dependent on the form of the inclusions. The mean number of inclusions per unit area of the wall is $v^{2/3}/d^2$. Formula (3) is valid under the conditions $v \ll 1$ and $d > \delta$, where $\delta = \pi\sqrt{J/Ka_0}$ is the so-called wall thickness, J is the exchange integral, and a_0 is the lattice parameter; for an iron crystal, $\delta \approx 10^{-5}$ cm.

The first term in square brackets in (4) is the change of magnetic energy of the volume in the crystal that reverses its magnetization on displacement of the wall through Δx (the term $a_2 v^{2/3}/2$ is a small correction for the volume of the secondary domains); the second term gives the change of surface energy, and the third the change of volume energy, of the secondary domains. The minimum values of field, $H = H_d$, and of external stress σ for which the wall can break away from the inclusion correspond to $\Delta\Phi = 0$. Hence, discarding the small term $a_2 v^{2/3}/2$, we get*

$$H_d = a_1\gamma_\sigma v^{2/3} / 2I_s d + (3a_2\lambda_{100} / 4I_s)(3\lambda_{100}C_2 + \sigma)v^{2/3}. \quad (5)$$

For $|\lambda_{100}\sigma| << |K|$, the value of γ_σ is $\approx \gamma$, and (5) can be rewritten in the following form:

$$H_d = H_{d0}\left(1 + \frac{3a_2}{2a_1}\frac{\lambda_{100}}{\gamma / d + (9a_2 / 2a_1)\lambda_{100}^2 C_2}\sigma\right), \quad (6)$$

where

$$H_{d0} = a_1\frac{\gamma v^{2/3}}{2I_s d}\left(1 + \frac{9}{2}\frac{a_2}{a_1}\frac{\lambda_{100}^2 C_2}{\gamma / d}\right). \quad (7)$$

The second term in parentheses in (7) represents a correction connected with the magnetostrictive energy of the secondary domains. For iron crystals, $\frac{9}{2}\lambda_{100}^2 C_2 \approx 10^3$ erg/cm^3, $\gamma = 1.8$ erg/cm^2. Thus for $d \approx 10^{-4}$ cm, this correction amounts to about 10%, and for $d \approx 10^{-5}$ it is practically negligible. The size of the second term in parentheses in expression (6) depends on σ. For $\sigma = 10^9$ cgs units = 10 kg/mm^2, we get $\frac{3}{2}\lambda_{100}\sigma \approx 3 \times 10^4$ erg/cm^3. In this case, for d $\approx 10^{-4}$ to 10^{-5} cm we get $3\lambda_{100}\sigma d/2\gamma \approx 1$. It follows that H_d can, in accordance with the sign of $\lambda_{100}\sigma$, be several times larger or smaller than H_{d0}. On passage through the crystal of an elastic wave with amplitude of order 10^9 cgs units or more, H_d will periodically become zero; this will cause an increase of magnetization even for insignificantly small external fields H. Thus it is evidently possible to explain the phenomenon, well known from experiment, of strong magnetization of ferromagnetic bodies in very weak magnetic

*The first term of formula (5) was obtained by the author earlier[2] by a similar method.

fields under the influence of shocks or blows. It must be emphasized that, as follows from formula (6), the application of external stresses substantially affects the stability of 180° walls, despite the still prevalent opinion that elastic stresses for $H \neq 0$ can produce only motion of walls between domains magnetized at an angle different from 180° (for example, 90° walls).

In polycrystalline ferromagnets, besides the reasons indicated above for an effect of external stresses on coercive force, there are a number of others, connected with conditions on grain boundaries and with the effect of external stress on the orientation of axes of easy magnetization. We shall not be concerned with these topics here; to some extent they have already been treated in the literature.[6-8]

If, as is usual, irreversible changes of magnetization occur in some field interval, then the increase of the irreversible part of the magnetization takes the form $\Delta I_{ir} = \kappa_{ir}\Delta H$, where ΔH is the increase in the effective field. From (6) it follows that application of a stress σ is equivalent to change of the field by $\Delta H_\sigma = H_{d0} - H_d$. For $H \neq 0$ and $\Delta H_\sigma > 0$, this leads to an irreversible change of magnetization

$$\Delta I_{ir} = \varkappa_{ir}(H_{d0} - H_d). \tag{8}$$

If $H \approx H_d$ and if $\lambda\sigma \ll K$, then according to (2) – (5)

$$\Delta I_{ir} = \varkappa_{ir} v^{2/3} \lambda\sigma / I_s \approx \varkappa_{ir} H_d \sigma\lambda d / \gamma. \tag{9}$$

Formula (9) permits approximate estimation of the irreversible changes of magnetization that occur under the influence of external elastic stresses.

In the most general case, with an arbitrary mechanism for the effect of external stresses on irreversible changes of magnetization (we considered above only one of the possible mechanisms for such an effect), the condition for stability of 180° walls with respect to a change of stress is the vanishing of the field H_i that acts on the wall. This field is connected with the external field H and with the magnetization by the known relation

$$H_i = H - \nu_e I, \tag{10}$$

where ν_e is the local demagnetizing factor, dependent on the shape of the body and on its structure, in particular on the shape and concentration of the inclusions; in the general case ν_e is a function of the coordinates of points within the body.

Apart from the trivial case in which $I = 0$ at $H = 0$ (the demagnetized state), vanishing of the effective field H_i occurs in states for which $H = \nu_e I$. Thus the states that have greatest stability with respect to σ are those with mean magnetization

$$I = H / \nu, \tag{11}$$

where ν is the mean demagnetizing factor of the body.

Experiment shows that in bodies with a sufficiently large shape demagnetizing factor N_0, so that $\nu \approx N_0$, the stable states described by Eq. (11) can be obtained by means of the magnetic treatment that leads to "ideal" magnetization. It will be shown below that this result follows from modern theoretical ideas.

2. INITIAL MAGNETIC SUSCEPTIBILITY OF THE IDEAL CURVE, AND THE DEMAGNETIZING FACTOR

We consider the processes that take place in a ferromagnetic monocrystal during ideal magnetization. Let the values of the coercive forces in different regions of the monocrystal be included within narrow limits $H_{c\,min} \leq H_c \leq H_{c\,max}$. Upon the monocrystal, with an internal demagnetizing factor, let there act a constant field $H \leq \nu I_s$ and a slowly decreasing alternating field with amplitude h_0, directed along an axis of easy magnetization; and let the frequency of the alternating field be so low that its amplitude and phase are approximately the same at all points of the monocrystal.

In the time interval during which the amplitude of the alternating field, h_0, exceeds $H_{c\,max} + H + \nu I_s$, domain walls move back and forth past an inclusion, executing oscillations at frequency ω. In this process the mean magnetization of the monocrystal is zero, and the resultant field H_i changes, with cyclic magnetization reversal, between the limits $h_0 + H - \nu I_s$ and $-h_0 + H + \nu I_s$. At the instant when h_0 becomes less than $H_{c\,max} + H + \nu I_s$, and consequently the absolute value of the lower limit of H_i becomes less than $H_{c\,max}$, a part of the walls can no longer move past inclusions; there comes into existence a constant component I_0 of magnetization, directed along H, and there appears an additional constant component of field, equal to $-\nu I_0$. From this instant on, as long as the upper limit remains equal to I_s, the magnetization changes from I_s to $-I_s + 2I_0$ and the resultant field H_i accordingly from H_{i1} to $-H_{i2}$, where

$$-H_{i2} = -h_0 + H + \nu(I_s - 2I_0) \leqslant H_i \leqslant h_0 + H - \nu I_s = H_{i1}. \tag{12}$$

Under these conditions the various regions of the monocrystal can be divided into two groups: 1) regions in which H_c lies within the limits

$$H_{c\,min} \leqslant H_c \leqslant H_{i2}; \tag{13}$$

2) regions in which

$$H_{i2} < H_c \leqslant H_{c\,max}. \tag{14}$$

In regions of the first group, walls move past inclusions twice during each period of alternation of h, i.e., even when h is directed opposite to the constant field H. These regions obviously make no contribution to I_0. In regions of the second group, whose coercive force is larger than the lower limit H_{i2} of the resultant field, walls cannot move past inclusions when the alternating field is directed opposite to the constant field. The magnetization of these regions remains parallel to the constant field, since, under the conditions assumed for the upper limit of the resultant field, saturation is attained (the upper limit of the magnetization is equal to I_s).

From (12) it is evident that as long as the upper limit of the magnetization is equal to I_s, the difference between the absolute values of the upper and lower limits of the resultant field is

$$H_{i1} - H_{i2} = 2(H - \nu I_0). \tag{15}$$

It is easy to demonstrate that (15) remains valid as long as $H > \nu I_0$, i.e., $H_{i1} > H_{i2}$. In fact, while the last inequality is satisfied, H_{i1} according to (13) is always larger than the largest of the coercive forces of regions of the first group; this guarantees attainment of saturation at field H_{i1}, i.e., attainment of the upper limit of the magnetization, equal to I_s.

With decrease of h_0, the number of regions falling in the second group increases, the number of regions remaining in the first decreases, and the constant component I_0 increases. Since, in the case we are considering, $H < \nu I_s$, there comes in the process of increase of I_0 an instant when $H - \nu I_0$ vanishes or changes sign, and when the upper limit of H_i becomes equal to the absolute value of the lower limit (or becomes less than the absolute value of the lower limit). Let this instant come at $H_{i2} = H_{c0}$. From this instant on, in the process of extinction of h, each successive value of the amplitude of H_i will be less in absolute value than the preceding (regardless of the sign of H_i). Therefore in only about half of the regions with coercive forces $H_c < H_{c0}$ will the magnetization, on transition from the first group to the second, remain parallel to the field H. In the other half of the regions, which pass from the first group to the second when $|H_{i2}| > H_{i1}$, the magnetization will remain antiparallel to the field. Thanks to this, at the instant when H_{i1} and H_{i2} become less than H_{c0}, the increase of the constant component I_0 will cease, and the upper limit of the magnetization will become less than I_s.

Thus in the process of decrease of the alternating field, the value of I_0 reaches a limiting value I_{0m}, which according to (15) is equal to

$$I_{0m} = H/\nu. \tag{16}$$

Comparison of (16) and (11) shows that states corresponding to points on the ideal curve actually are the most stable with respect to the effects of external stresses.

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