ON THE THEORY OF THE PASSAGE OF THE NUCLEAR CASCADE THROUGH THE ATMOSPHERE

I. P. IVANENKO

Nuclear Physics Institute, Moscow State University

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A method for the solution of the equations describing the passage of the nuclear cascade through the atmosphere is proposed. The boundary conditions can be prescribed at any arbitrary depth. The proposed method makes it possible to obtain the solution in a form similar to the one obtained by the usual method of successive generations with boundary conditions prescribed at the top of the atmosphere. The form of the solution is discussed for various boundary conditions.

A number of methods for calculating the characteristics of the nuclear cascade process in the atmosphere have been developed in recent years.[1-3] Of these, the method of successive generations[3] has been found to be a convenient and efficient one for the calculation of the required characteristics. In all calculations, it was assumed that the boundary conditions are given at the top of the atmosphere, at $x = 0$.

In experiments on extensive air showers (EAS), the energy of the primary particle initiating the shower is usually not known, and neither is the depth of the initiation of the shower. In general, various characteristics of different EAS components are known at a certain depth of the atmosphere. Therefore, for an estimate of the influence of the decay processes on the development of EAS in the depth of the atmosphere, and also for the calculation of the various development schemes of the shower,[4] it is necessary to find a solution of the nuclear cascade equations, the boundary conditions being given at a certain arbitrary depth x_n in the atmosphere.

Zatsepin, Nikol'skiĭ, and Pomanskiĭ[5] proposed to solve the problem by the method of successive approximations. It was shown that, for a certain specific choice of the zero generation, the i-th term of the series gives the depth and energy distribution of the particles of the i-th generation. However, the form of the solution given in reference 5 needs to be integrated over x and E for the determination of each consecutive generation. This makes it difficult to use this method of solution in practice.

In the present article, a method is presented for the solution of the nuclear-cascade equation with boundary conditions prescribed at a given depth of the atmosphere, which is analogous to the general method of successive generations with boundary conditions prescribed at the top of the atmosphere, at $x_n = 0$.

We shall solve the one-dimensional problem neglecting, for the time being, the ionization loss, and assuming an isothermal atmosphere. We shall use a system of notation similar to that used by Zatsepin and Rozental'.[3]

A system of nuclear-cascade equations, describing the atmospheric-depth dependence of the number of particles of type η having an energy E can be written in the form

$$\frac{dP^{(\eta)}(E,x)}{dx} = -P^{(\eta)}(E,x) + \sum_{\zeta}\int_{E}^{\infty} P^{(\zeta)}(E',x)\,W_{\eta}^{(\zeta)}(E',E)\,dE' - \frac{K_{\eta}(E)}{x}P^{(\eta)}(E,x) + \frac{1}{x}\sum_{\varkappa}\int_{E_1}^{E_2} P^{(\varkappa)}(E',x)\,K_{\varkappa}(E')\,D_{\eta}^{(\varkappa)}(E',E)\,dE. \quad (1)$$

We assume that, in the energy range under consideration, $\sigma_{\eta}^{(K)}(E) = \sigma_0$.

The boundary conditions at the atmospheric depth x_n are given by the values of the function $P^{(\eta)}(x_n, E)$, $p^{(\zeta)}(x_n, E)$, $p^{(K)}(x_n, E)$ etc. We are looking for the solution of Eqs. (1) in the form

$$P^{(\eta)}(x,E) = e^{-(x-x_n)}\sum_{i=0}^{\infty}\frac{(x-x_n)^i}{i!}P_i^{(\eta)}(x,E,x_n). \quad (2)$$

Substituting (2) into (1), and equating the coefficients of identical powers of $(x - x_n)$, we obtain the following system of equations for the determination of the functions $P_i^{(\eta)}$:

$$P_{i+1}^{(\eta)}(x, E, x_n) = -\frac{\partial P^{(\eta)}(x, E, x_n)}{\partial x} + \sum_{\zeta}\int_{E}^{\infty} P_i^{(\zeta)}(x, E', x_n) W_{\eta}^{(\zeta)}(E', E)\, dE' - \frac{K_{\eta}(E)}{x} P_i^{(\eta)}(x, E, x_n) + \frac{1}{x}\sum_{\varkappa}\int_{E_1}^{E_2} P_i^{(\varkappa)}(x, E', x_n) K_{\varkappa}(E') D_{\eta}^{(\varkappa)}(E', E)\, dE'. \quad (3)$$

$P_0(x, E, x_n)$ has to be suitably defined to insure a rapid convergence of the series (2). From Eq. (2), for $x = x_n$, we have

$$P(x_n, E) = P_0(x_n, E, x_n).$$

Hence it follows that $P_0(x, E, x_n)$ should be defined as the zero generation (i.e., primary particles).

We shall consider the following types of primary particles: a) stable nuclear-active particles which cannot be produced in the decay of any other particles, b) stable particles which can be produced in the decay of other particles, and c) and d) unstable particles analogous to a) and b). It is easy to obtain the dependence of $P_0(x, E, x_n)$ on all arguments. In case a),

$$P_0(x, E, x_n) = P(x_n, E). \quad (4a)$$

In case b),

$$P_0(x, E, x_n) = P(x_n, E) + \sum_{\varkappa}\int_{x_n}^{x} \frac{e^{x-x_n}}{x}\, dx \int_{E_1}^{E_2} P_0^{(\varkappa)}(E', x, x_n) K_{\varkappa}(E') D_{\eta}^{(\varkappa)}(E', E)\, dE' \quad (4b)$$

In case c),

$$P_0(x, E, x_n) = P(x_n, E)(x_n / x)^{E_\eta / E} \quad (4c)$$

In case d),

$$P_0(x, E, x_n) = P(x_n, E)(x_n / x)^{E_\eta / E} + (x_n / x)^{E_\eta / E} \sum_{\varkappa}\int_{x_n}^{x} \frac{e^{x-x_n}}{x}(x / x_n)^{E_\eta / E}\, dE \times \int_{E_1}^{E_2} P_0^{(\varkappa)}(E', x, x_n) K_{\varkappa}(E') D_{\eta}^{(\varkappa)}(E', E)\, dE'. \quad (4d)$$

Thus, Eqs. (2), (3), and (4) fully determine the solution of Eq. (1) with boundary conditions prescribed at the depth x_n. The solution is in the form of an infinite power series in $(x - x_n)$. It can be shown, however, that, in cases most interesting in practice, the series converges rapidly, and the sum of the terms from $i = 0$ to $i \leq (x - x_n)$ represents the solution with an acceptable accuracy.

We shall determine the depth and energy dependence of nuclear-passive mesons of type ρ. For these particles, Eq. (1) can be written in the form

$$\frac{dP^{(\eta)}(E, x)}{dx} = -\frac{K_{\eta}(E)}{x} P^{(\eta)}(E, x) + \frac{1}{x}\sum_{\varkappa}\int_{E_1}^{E_2} P^{(\varkappa)}(E', x) K_{\varkappa}(E') D_{\eta}^{(\varkappa)}(E', E)\, dE'. \quad (1')$$

Its solution is easy to find:

$$P^{(\eta)}(E, x) = \left(\frac{x_n}{x}\right)^{E_\eta / E} P(E, x_n) + x^{-E_\eta / E} \sum_{\varkappa}\int_{x_n}^{x} x^{E_\eta / E}\, dx \times \int_{E_1}^{E_2} P^{(\varkappa)}(E', x) \frac{E_{\varkappa}}{E'} D_{\eta}^{(\varkappa)}(E', E)\, dE'. \quad (5)$$

Thus, to calculate the number of nuclear-passive mesons it is first necessary to find, according to Eqs. (2) – (4), the depth and energy distributions of the mesons whose decay results in the mesons under consideration.

The above discussion can easily be generalized to take the ionization loss into account. The ionization-loss term can be written in the form $\beta\partial P^{(\eta)}(E, x)/\partial E$, where β is the ionization loss per mean free path, assumed to be independent of the energy. As before, we shall look for a solution in the form of Eq. (2). It should be noted that the series represented by Eq. (2) will converge rapidly only if the ionization loss is not the main process that determines the passage of nuclear-active particles through the matter. Consequently, the particle energy E should be markedly greater than βx. In analogy to Eq. (3), we obtain the following system of equations for the function $P_i^{(\eta)}$:

$$P_{i+1}^{(\eta)}(x, E, x_n) = -\frac{\partial P_i^{(\eta)}(x, E, x_n)}{\partial x} + \sum_{\zeta}\int_{E}^{\infty} P_i^{(\zeta)}(x, E', x_n) W_{\eta}^{(\zeta)}(E', E)\, dE' - \frac{E_\eta}{Ex} P_i^{(\eta)}(x, E, x_n) + \frac{1}{x}\sum_{\varkappa}\int_{E_1}^{E_2} P_i^{(\varkappa)}(x, E', x_n) \frac{E_{\varkappa}}{E'} D_{\eta}^{(\varkappa)}(E', E)\, dE' + \beta\frac{\partial P_i^{(\eta)}(x, E, x_n)}{\partial E}; \; i \geqslant 1. \quad (3')$$

In this case, the dependence of $P_0(x, E, x_n)$ on all arguments is also given by Eqs. (4a) – (4d), where, on the right-hand side, we should substitute for E the value $E + \beta_x - \beta x_0$. In this way, the above formulae give the most complete solution of nuclear cascade equations by the method of successive generations. Formulae for the remaining components of the nuclear cascade do not differ from the corresponding formulae given by Zatsepin and Rozental'.[3]

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203