*ON THE CRITICAL MODE IN EXPERIMENTS WITH AN OSCILLATING DISK IN HELIUM II**

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The results of measurement of the damping of an oscillating disk immersed in helium II are given. The onset of the critical mode and the motion of the disk at supercritical velocities were investigated, and also the dependence of critical velocity on temperature and period of oscillation. The critical velocity was found to be influenced by the cleanliness of the surface. In this connection, the dependence of critical velocity on the number and size of small particles deposited on the disk surface, and also on the radius of the region covered by the particles, was studied. The dependence of the damping in the supercritical regime on temperature, particle concentration, and radius of the contaminated region was also examined.

IT was established by Andronikashvili and Kaverkin,[1] Osborne,[2] Hollis-Hallett,[3] Benson and Hollis-Hallett,[4] and others that in the supercritical mode the superfluid fraction of helium II partakes both in rotational and in oscillatory motion.

The onset of the critical mode for rotational oscillations of a disk is characterized by a critical amplitude, Φ_k above which an extra damping appears, $\Delta\gamma = \gamma' - \gamma_n$ where γ_n is the damping of the oscillating system in the subcritical mode, determined by the action of the normal component of helium II on the disk, and γ' is the damping in the supercritical mode.

Andronikashvili[5] and Smith[6] first studied the amplitude dependence of the logarithmic decrement of a pile of disks and of a single disk immersed in helium II, and oscillating about its axis. In both experiments the damping was found to be independent of amplitude at $\Phi = 0.2$ radians (maximum velocity 0.6 cm/sec for the pile and 0.25 cm/sec for the disk).

The critical phenomena were first studied by Hollis-Hallett,[3] who found the onset of the critical mode for motion of a single disk at a velocity $v_k = 0.1$ cm/sec ($T = 1.46°$K, period 11 sec), giving rise to an increase in damping decrement. For a pile of disks the minimum value of the critical velocity was determined as $v_k = 0.05$ cm/sec and at this value both the decrement and the density of the normal component increased. In view of the discrepancy between the results of Andronikashvili[5] and Smith,[6] on the one hand, and of Hollis-Hallett[3] on the other, we thought it appropriate to investigate the conditions for the onset of the critical regime in experiments with oscillating disks.

APPARATUS

The smooth polished disk 20 (see Fig. 1) of radius $R = 1.60$ cm was hung, by means of a glass rod 17 and a clamp 12, on a phosphor bronze thread 10, of diameter 50 μ and length 150 mm (in some experiments a 100 μ thread was used). The other end of the thread was fixed by clamp 9 to the rod 6 which was joined to the head A by means of screw 4. The head was screwed to the end of tube 5 of the stuffing box 8. By turning the head 1 the screw 2 was screwed out of the plate 3 and the whole suspension system, together with the plate, was lowered. When the small rod 13 rested on the brass ring 14, fixed between the plated rods 11, the balance was freed from the tension and the system came to rest.

The upper part of the suspension system was contained in the glass cylinder 7, while the lower part (the glass rod 17 and the disk 20) was surrounded by the glass tube 18 and the beaker 19, fitted to it by the ground glass join to protect the oscillating system from external disturbing influences. The glass cylinder 7 was sealed with picein near tube 5 and cone 16.

Observation of large amplitude oscillations was made with the help of a light spot focussed onto a circular scale after reflection from the mirror 15. The scale had its center on the axis of rotation and was of 18 cm radius. Small amplitudes were meas-

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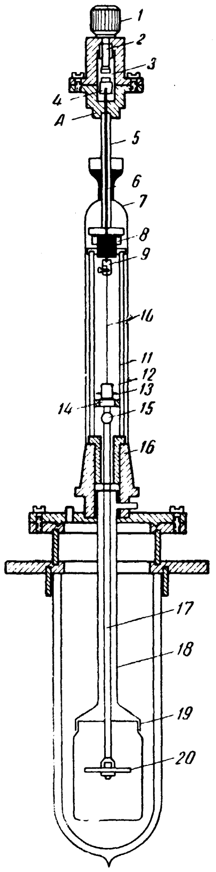

FIG. 1. Diagram of apparatus.

ured by a mirror (not shown in the figure) and light pipe PS-35. Both mirrors were fixed to the glass rod by cement BF-4 in such a way that their planes passed through the axis of the rotating system.

The system was set into oscillation by bringing a permanent magnet up to the rod 13.

The period of oscillation Θ was determined by timing a large number of swings (50 to 150).

STATEMENT OF THE PROBLEM

It follows from the definition of the critical amplitude that above it a dependence of damping on amplitude must appear, i.e., the linear relation between $\ln \Phi$ and the number of oscillations, n, must break down. By finding the amplitudes corresponding to this departure on $\ln \Phi = f(n)$ plots at different temperatures, the temperature dependence of the critical amplitude can be found.

In our experiments the amplitudes of oscillation of the disk, Φ_n, were determined from the swing, A, of the light spot along the circular scale of radius r, from the equation

$$\Phi_n = A / 4r = (a_n + a_{n+1}) / 4r, \tag{1}$$

where a_n and a_{n+1} are the values of the extreme positions of the spot on the scale. (The dependence of log A on the number of oscillations, n, at different temperatures is shown in Fig. 2). The damping, γ, is derived from the relation

$$\gamma = (2.302 / \Theta)\, d\,(\lg A_n) / dn. \tag{2}$$

The well-known relation between v and Φ:

$$v_k = (2\pi / \Theta)\, \Phi_k R, \tag{3}$$

was used to calculate the critical velocity, where R is the radius of the oscillating disk.

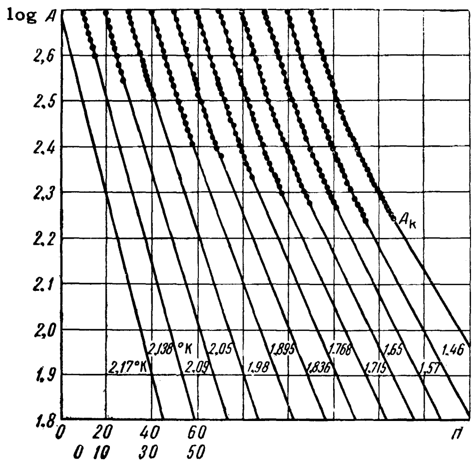

FIG. 2. Dependence of the logarithm of the full amplitude of oscillation of the light spot on the number of oscillations, at the different temperatures shown on the curves in °K (disk radius R = 1.605 cm and Θ = 8.95 sec). The points show the transition to the supercritical mode. The origin of the abscissa scale moves by n = 10 to the right for each succeeding curve relative to the previous one.

DESCRIPTION OF THE EXPERIMENTS

Determinations of critical velocities were made with several disks of different thicknesses and the period of oscillation varied between 3.4 and 14.5 sec. For disks with the cleanest surfaces we obtained the temperature dependence of critical velocity shown in Fig. 3. Bearing in mind the experimental errors, which reached 6 to 7% in these experiments, we can conclude that the critical velocity is independent of the period of oscillation for periods between 6.85 and 14.5 sec (lower curve of Fig. 3).

The data of Hollis-Hallett are shown in the figure for comparison. Our values of critical velocity are 1.5 to 3 times larger, and at some temperatures even 4 times larger than his corresponding results. Our data also differ from those of Hollis-Hallett in that the critical velocities measured on a 3.42-sec period disk (the upper curve of Fig. 3), lie above the curve for the larger periods, and almost join it. In Hollis-Hallett's work the 3.78 and 3.15 sec oscillations give twice the value obtained with a period of 11 sec. We should remark that

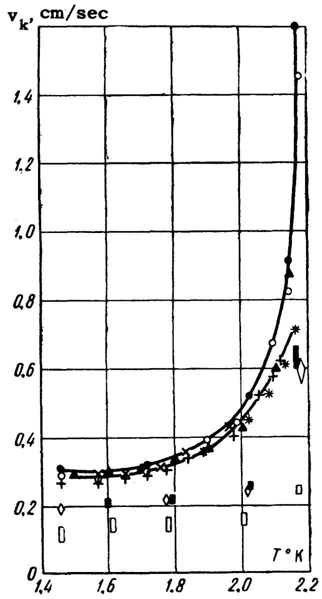

FIG. 3. Temperature dependence of critical velocity for smooth disks with radius R = 1.605 cm for periods of oscillation: o, ● – 3.42 sec, * – 6.85 sec, + – 8.95 sec, × – 14.45 sec, ▲ – values derived from Eq. (4). Hollis-Hallett's data[3] for a single disk of radius R = 1.572 cm and period: ■ – 3.15 sec, ◇ – 3.78 sec, □ – 11 sec: The height of the points corresponds to the experimental uncertainty.

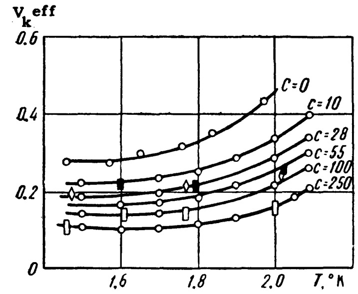

FIG. 5. Temperature dependence of effective critical velocity for different particle concentrations (c is the number per cm²). ■, ◇, □ – data of Hollis-Hallett (see Fig. 3).

our critical velocities are close to those found by Andronikashvili in his experiments on a pile of oscillating disks when laminar flow was not destroyed.

An empirical relation between v_k and temperature leads to the conclusion that

$$v_k = 0{,}105 / \sqrt{\rho_s} \text{ cm/sec}, \qquad (4)$$

while from Hollis-Hallett's work it follows that $v_k \sim \rho_s^{-1/3}$. The triangles in Fig. 3 show the critical velocities calculated from (4).

During observations of torsional oscillations of a disk we noticed that the critical mode is reached at considerably lower amplitudes if the surface of the disk is contaminated, even very little, by granules of solid air. We made some measurements with contaminated surfaces to investigate this phenomenon, and deposited particles 0.05, 0.1, and 0.2 mm in size on the surface. Figure 4 shows the relation $\Phi_k = f(T)$ for various sizes of contaminating grains. The value of the critical velocities for rough surfaces also depends on the concentration, c, of particles (Fig. 5) and it can be seen that the critical velocity falls by a factor of at least 3 as the concentration of particles increases from 0 to 250 particles per cm². The dimensions of all the grains we used were considerably smaller than the penetration of viscous waves in the normal component of the liquid, so that they did not affect the damping until the critical mode was reached.

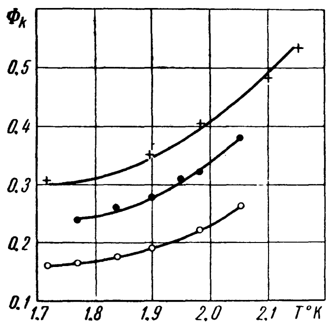

FIG. 4. Temperature dependence of critical amplitude for different contaminating grain sizes: + – 0.05 mm, ● – 0.1 mm, o – 0.2 mm. in all cases there were 250 particles per cm².

We show Hollis-Hallett's data (rectangles) together with our own (points) on the curves of temperature dependence of critical velocity for rough surfaces (Fig. 5). It can be seen that his results agree with ours for rough surfaces so that there was evidently contamination present in his experiments.

As the radius of the contaminated region, R_c, decreases, the critical amplitude increases so that the product $\Phi_k R_c$ stays constant (Fig. 6). In our experiments R_c varied from 16 to 6 mm.

An analysis of the temperature dependence of the extra damping $\Delta\gamma$ confirms Hollis-Hallett's conclusion that $\Delta\gamma \sim \rho_s$. All the increase in damping is therefore, in fact, connected with the superfluid component taking part in the motion of the disk.

Besides the magnitude of the critical amplitude, the decrement, γ', in the supercritical mode also depends on the degree of contamination. The dependence of $\Delta\gamma$ in the supercritical region on particle concentration is shown in Fig. 7. It follows from Fig. 8 that $\Delta\gamma$ is proportional to the area of contaminated surface.

Figure 9 shows the amplitude dependence of the total damping, at different temperatures, for a disk with period Θ = 8.95 sec. It can be seen that the transition region, where the superfluid component does not take full part, gradually narrows as the λ-point is approached (see also Fig. 2).

DISCUSSION OF THE RESULTS

1. The temperature dependence of critical velocity [Eq. (4)] is apparently connected with the quantization of energy of a vortex, and indicates that the critical mode is attained with the expenditure of a given amount of energy.

2. The reduction of critical velocity for contaminated surfaces seems apparent. This reduction can be explained by the fact that one is measuring

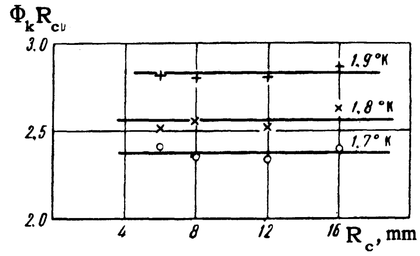

FIG. 6. Dependence of critical amplitude on the radius of the contaminated region, R_c, at different temperatures.

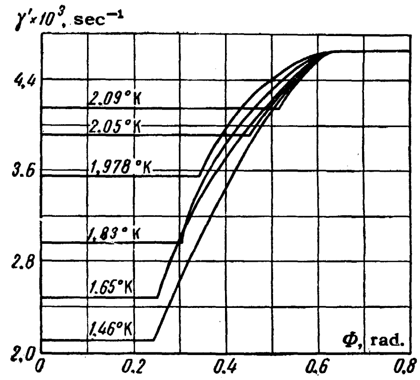

FIG. 9. The amplitude dependence of total damping γ' at different temperatures. Disk radius R = 1.605 cm and Θ = 8.95 sec.

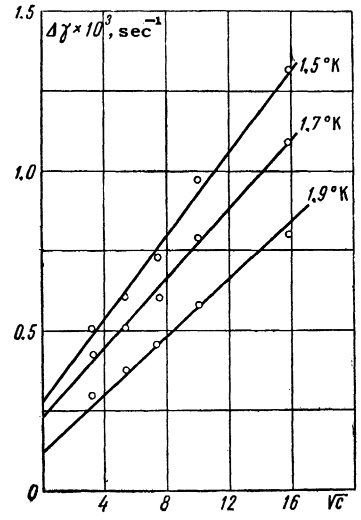

FIG. 7. Dependence of extra damping, $\Delta\gamma$, on particle concentration, c, at different temperatures.

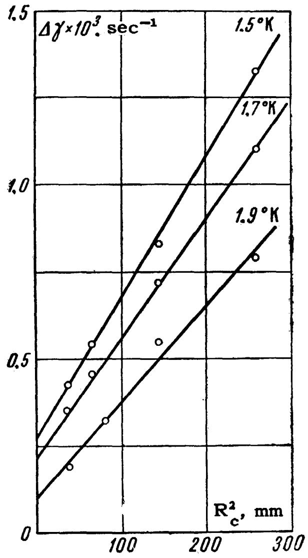

FIG. 8. Dependence of extra damping on the radius of the contaminated region.

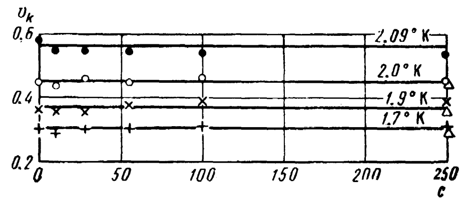

FIG. 10. Independence of the true critical velocity of the concentration at different temperatures and for different grain sizes: ●, ○, + – 0.2 mm, △ – 0.1 mm.

the speed of motion of the disk, and when contamination is present this does not coincide with the speed of the motion of the liquid relative to the oscillating surface. The difference between these is due to the increase in effective path of the superfluid component while the time for the motion, determined by the period of oscillation, remains the same.

The increase in path is determined by the concentration and size of the particles. Making the correction for the increase in flow velocity we obtain the following relation for the critical velocity:

$$v_k = v_k^{\mathrm{eff}}(1 + k\pi d\sqrt{c}), \tag{5}$$

where v_k^{eff} is the effective critical velocity, i.e., the velocity of the disk, $v_k^{\mathrm{eff}} = (2\pi/\Theta)\,\Phi_k^{\mathrm{eff}} R$, d is the mean linear dimension of the grains, c is their concentration and k = 1.75 is a constant. Figure 10 shows the values of v_k calculated from (5). These values are independent both of concentration and of size of particles and agree well with the results obtained with smooth disks.

3. The constancy of the product $\Phi_k R_c$ found for radii of the particle-covered region from 16 to 6 mm, shows that the critical velocity is independent of radius. Apparently the critical mode is always reached on the periphery of the contaminated region at the same value of the velocity.

4. The dependence of the extra damping on the degree of contamination (Fig. 7) can also be explained by an increase in the mean velocity of the liquid, which must lead to an increase in the number of vortices.

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[1] É. L. Andronikashvili and I. P. Kaverkin, JETP **28**, 126 (1955); Soviet Phys. JETP **1**, 174 (1955).
[2] D. V. Osborne, Proc. Phys. Soc. **A63**, 909 (1950).
[3] A. C. Hollis-Hallett, Proc. Roy. Soc. **A210**, 404 (1952).
[4] C. B. Benson and A. C. Hollis-Hallett, Canad. J. Phys. **34**, 668 (1956).
[5] É. L. Andronikashvili, Thesis, Institute for Physical Problems, Moscow (1948).
[6] P. L. Smith, Physica **16**, 808 (1950).

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