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ON THE CONNECTION OF ISOTOPIC SPIN AND STRANGENESS WITH THE BEHAVIOR OF SPINORS UNDER INVERSION

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THE usual treatments of isotopic and strangeness properties involve an isotopic space of two, three, or four dimensions, sometimes with the possibility of transitions to a pseudo-Euclidean space. One can, however, try to describe these properties within the framework of ordinary space, by bringing in hitherto unused possibilities of different behaviors of spinors under inversions and taking into account nonconservation of the parity P . The interpretation of isotopic properties in the framework of ordinary space that we propose here is a development of earlier more special considerations.¹⁻⁸

As has been pointed out,^{8,9} under both space and time reflections spinors can behave in different ways, with transformation matrixes that differ by factors -1 , i , γ_5 , or products of these. In addition to $\psi' = \gamma_4\psi$ (under $x'_{1,2,3} = -x_{1,2,3}$), etc., there are also the possibilities $\psi' = \gamma_5\gamma_4\psi$ or $\psi' = i\gamma_4\psi$, etc. Thus there arise different spinor representations of the Lorentz group, some of which are equivalent by unitary transformations (but differ from each other under charge conjugation).

A more important difference between spinors, not having the property of unitary equivalence, appears when the additional factors that have been mentioned occur under space reflections only or under time reflection only. We shall characterize spinors by two pairs of indices a, b and α, β . The index a takes one of the two values 1 or 2, depending on whether or not the additional factor γ_5 is used for space reflection. Similarly, the

index $b = 1, 2$ characterizes the geometrical time reflection T^0 , which can be replaced by the Schwinger reflection $T^S = T^0 \times (\sim) = TC$, where (\sim) denotes transposition in Hilbert space and T the Wigner time reversal. The indices α, β run through the four values $(0, 1, 2, 3)$ corresponding to the appearance of the additional factors i^α for space inversion and i^β for time inversion. The essential difference between two spinors is characterized by the differences $(a-b)$ and $(\alpha-\beta)$, or, more precisely, by their absolute values. In particular, the "mixed" spinors with $(a-b) \neq 0$ that we introduced earlier⁸ provide a realization, without doubling of the number of components, of the "anomalous" representation, for which $T^0P = +PT^0$, in contrast to the usual anticommutation. For the "mixed" spinors the construction of the Dirac equation with a mass is possible only with violation of invariance with respect to P , together with preservation of the invariance with respect to the strong (combined) inversion $P^S = PC$.⁸

When there is invariance only with respect to P^S and T^S we have the question of the characteristics of spinors of distinct types. To solve it we introduce the self-adjoint ("large") spinors

$$\begin{aligned} \Psi(1) &= \frac{1}{2} [(1 + i\gamma_5)\psi + (1 - i\gamma_5)\psi^c], \\ \Psi(2) &= \frac{1}{2} [(1 - i\gamma_5)\psi + (1 + i\gamma_5)\psi^c], \\ \Psi^c(1, 2) &= C\Psi^*(1, 2) = \Psi(1, 2), \quad \gamma_5^2 = -1. \end{aligned} \quad (1)$$

Under the strong inversions of the small ψ the quantities $\Psi(1, 2)$ transform linearly, each one by itself, in complete analogy with the transformation of the ordinary ψ under geometrical inversions. Corresponding to the phase transformation $\psi' = e^{i\alpha}\psi$ we have $\Psi'(1, 2) = \exp(\pm\gamma_5\alpha) \cdot \Psi(1, 2)$. For self-adjoint small ψ (neutrino), $\Psi(1)$ and $\Psi(2)$ coincide. An additional difference between $\Psi(1)$ and $\Psi(2)$ is due to the possibility of different or equal relative signs under inversions. Self-adjoint ψ 's are possible only for those $\Psi(1, 2)$ for which these signs are the same.

For characterizing the behavior of spinors under the strong inversions P^S, T^S we need only the pairs of indices $J = a + \alpha, K = b + \beta$, and accordingly the one difference

$$N = J - K = (a - b) + (\alpha - \beta) \pmod{2}. \quad (2)$$

Here a, b, α, β relate to the original small spinors ψ from which the $\Psi(1, 2)$ are constructed. $\Psi(1)$ and $\Psi(2)$ form a doublet, whose components go over into each other under geometrical inversions or charge conjugation of the original ψ . These transformations, together with the Salam-Touschek transformation, can be put in basic correspondence with three-dimensional isotopic rotations.¹⁰ The

construction of bilinear combinations (bosons) having the usual covariance properties (scalar, pseudoscalar, vector, etc.) from spinors of which the first has $N = 1$ ("mixed") and the second $N = 0$ ("normal") is impossible even when we use the strong inversions. It is also impossible to construct bosons that behave in the usual way under geometrical inversions from spinors that differ in the values of $(a - b)$ or $(\alpha - \beta)$ (modulo 2). This provides a basis for identifying, for example, N with the baryon number, $(\alpha - \beta)$ with the strangeness S , and $(a - b)$ with the hypercharge Y .

We then have the usual relation $N = Y - S$. Corresponding numbers can be assigned to bosons; then bosons constructed from spinors that differ in N , S , Y form doublets, whose components transform into each other under inversions, i.e., particles of the type of the K mesons. We then get new conservation laws, in which N is conserved strictly (modulo 4), and S and Y are conserved when there is invariance only under the geometrical inversions. Therefore the fact of simultaneous violation of the conservation of P , S , Y , which has seemed accidental, can now receive a legitimate explanation. If we take as a universal invariance condition the conservation of P^S and T^S , then when there is violation of the isotopic group the ordinary parity P is also not conserved.¹¹

We point out that it is convenient to carry out the construction of the interaction Lagrangian by means of the $\Psi(1, 2)$, since then the invariance with respect to P^S and T^S is most explicitly manifest, and, for example, it can be seen why for two-component spinors one is confined (Feynman) to the vector and pseudovector terms, which are invariant under the Salam-Touschek transformations for the large $\Psi(1, 2)$.

In conclusion we remark that it is most natural to characterize the leptons by "normal" spinors, assigning different factors ± 1 , i , γ_5 to the particles e , ν , μ , and the baryons by spinors that are "mixed" under strong inversions ($N \neq 0$); bosons are assigned bilinear combinations of spinors. In view of the absence of absolute conservation of the number of baryons in this formalism, they can in principle be converted into mesons and leptons; owing, however, to the existing conservation laws and the necessity of contact interaction of several particles, the probability for this conversion will be extremely small.

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BETA DECAY OF THE NEGATIVE MUON

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ALL experiments on pion beta decay performed to date are devoted to β decay of the stopped positive mesons.⁴⁻⁸ In two recent papers, the following result is obtained for the relative probability of this process:

$$(\pi^+ \rightarrow e^+ + \nu) / (\pi^+ \rightarrow \mu^+ + \nu) \approx 1 \cdot 10^{-4} \pm (20-40\%).$$

This agrees with the value 1.3×10^{-4} which follows from the universal V-A theory of β decay.⁷ It follows from relativistic invariance (CPT theorem)⁸ that this process should have the same relative probability for negative pions as for the positive ones. However, we deemed it important to determine the relative probability of β decay of negative pions by direct experiment.

Unlike $\pi^+ \rightarrow e^+$ decay, $\pi^- \rightarrow e^-$ decay can be observed only in flight. We therefore sought for