

## KNOCK-ON ALPHA PARTICLES PRODUCED BY FAST NUCLEONS

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We investigate the cross section for knock-out of  $\alpha$  particles with energies  $> 30$  Mev from photographic emulsion nuclei bombarded by 100, 140, 200, 360, and 660 Mev protons. The knock-out of the particles by cascade nucleons is computed under the assumption that the intranuclear nucleons have definite momenta. It is found that there is an appreciable probability of formation of  $\alpha$  substructures in light nuclei such as  $C^{12}$  or  $O^{16}$ , as well as in the diffuse region of heavy nuclei.

## INTRODUCTION

THE prevalent opinions concerning the mechanism of splitting of nuclei by high-energy particles is based on the Serber-Goldberger idea of individual collisions between the incident particle and the intranuclear nucleons. The nucleus is considered as a Fermi gas whose particles — the neutrons and protons — interact weakly with each other. The presence of other nucleons during the instant of collision of two particles does not influence directly the results of the collision. The experimental disintegration data obtained during the past ten years, principally with photoemulsions, have demonstrated the correctness of these ideas. However, there are also certain facts that disagree with this simplified picture of the interaction. In particular, it has been established in many investigations that the cascade process which develops inside the nucleus upon incidence of a high energy particle involves not only the nucleons but also the more complex formations such as  $\alpha$  particles. Thus, one must admit that there exist inside the nucleus more or less stable complexes, capable of leaving the nucleus when the latter collides with a fast nucleon. This point of view finds confirmation in the Brueckner model.<sup>1,2</sup> The strong correlation between nucleons, which according to this model does take place in a real nucleus, should lead to the formation of a quasi-stable complex, consisting of two, three, etc. nucleons. Paired and quadruple configurations, i.e., quasi-deuterons and alpha groups, have apparently the highest probabilities.<sup>3,4</sup>

An investigation, by Meshcheryakov and his associates,<sup>5</sup> of fast deuterons emitted from light nuclei under the influence of 660-Mev protons has

shown these deuterons to be connected with the elastic collisions between the protons and the intranuclear deuteron groups. Barton and Smith,<sup>6</sup> in an analysis of the disintegration of He and Li by high energy photons, have also concluded that these nuclei contain correlated neutron-proton pairs, although their estimate of the quasi-deuteron momentum is somewhat lower than the value obtained in reference 5.

Rotenberg and Willets<sup>4</sup> note that the probability of production of nucleon complexes is considerable in the peripheral region of the nucleus, where the density of nuclear matter is lower. On the other hand, the mean free paths of the deuterons and  $\alpha$  particles are found to be less than the dimension of the heavy nucleus<sup>7-9</sup> and one can therefore expect the collision processes, which cause the complex particles to be emitted from the nucleus in the cascade stage to occur in the surface layer.

Interesting results, which also confirm the suggested knock-on origin of the  $\alpha$  particles, have been obtained by Cüer et al.<sup>10,11</sup> in photoemulsion investigations of stars produced by 180- and 340-Mev protons on  $C^{12}$ . A kinematic analysis of the reaction  $C^{12}(p, 3\alpha p')$  made it possible for Cüer et al. to conclude that in 30% of the cases this reaction proceeds via scattering of the incident proton by the  $\alpha$  group contained in the  $C^{12}$  nucleus. The kinetic energy of the intranuclear  $\alpha$  particles was found to be approximately 5–10 Mev. To reconcile these conclusions with the results of other researches, it was proposed that the  $\alpha$  substructures were short-lived and dissociated rapidly into nucleons. Serebrennikov<sup>12</sup> estimated their lifetimes and obtained a value of  $\sim 4 \times 10^{-23}$  sec. Naturally, this figure says nothing of the contribution of the  $\alpha$ -particle states to the total wave

function of the nucleus, since the probability of  $\alpha$ -group production can also be large. Hodgson<sup>13</sup> has found that the probability of finding each nucleon in bound state in the  $\alpha$  particle is approximately 0.4 for light nuclei (C, N, O) and for the peripheral region of heavy nuclei (Ag and Br). Hodgson's calculations, however, did not take into account the momentum of the  $\alpha$  particle proper, with which the proton collides. Nor did he take into consideration the fact that the scattering process occurs in the potential well of the nucleus and that the  $\alpha$  particle is in a state with negative total energy. An allowance for these factors may change considerably the results obtained in reference 13.

Finally, mention should be made of the paper by Schiff,<sup>14</sup> in which are calculated the probabilities of the electric monopole transitions in C<sup>12</sup> and O<sup>16</sup> on the basis of the independent-particle model and the  $\alpha$ -particle model. The experimental value of the transition matrix element was found to lie between the two calculated ones.

Our own investigation is devoted to a further study of the existence inside the nucleus of the  $\alpha$  particles capable of participating in the cascade process as a whole. To exclude the influence of impurity particles produced by evaporation, we investigated only the disintegration of emulsion nuclei accompanied by an emission of  $\alpha$  particles with energies greater than 30 Mev. The experimental values of the cross sections for their emission were compared with calculated cross sections of elastic collisions between nucleons and He<sub>2</sub><sup>4</sup>, obtained with allowance for the motion of the  $\alpha$  particle proper.

## EXPERIMENT

We used in the experiment photographic plates with fine-grain nuclear emulsion type P-9, in which it is possible to distinguish, with good accuracy, the tracks of doubly-charged particles from singly-charged particle tracks. The plates were irradiated in the synchrocyclotron of the Joint Institute for Nuclear Research in an external collimated beam of 660, 360, 200, 140, and 100 Mev protons. The proton energy was reduced by a copper block of suitable thickness. Magnetic lenses were used to clear the extraneous particles from the beam after passage through this block.

The procedure used to pick the stars for analysis, and the criteria used for classifying these stars according to the nature of the nucleus on which the star was produced, were described elsewhere.<sup>15</sup> The intensity of the beam was determined from the number of stars per unit volume

of the investigated emulsion. The mean free path of protons of respective energies for star production was taken from various sources.<sup>16</sup>

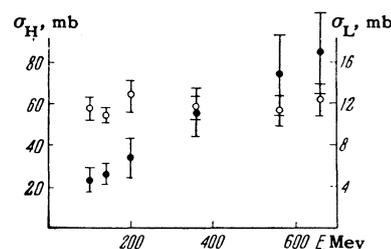


FIG. 1. Cross section for the emission of  $\alpha$  particles with energies greater than 30 Mev from light nuclei (open circles - right-hand scale) and heavy nuclei (full circles - left-hand scale) of the emulsion at different energies of bombarding protons. The point for the 560-Mev energy was taken from reference 17.

The experimentally-obtained cross sections for the emission of  $\alpha$  particles with energies greater than 30 Mev, at various bombarding proton energies, are shown in Fig. 1 separately for the light and heavy nuclei of the emulsion.\* The errors in the experimental values, indicated on the diagram, include the statistical errors in the  $\alpha$ -particle count and in the density of stars in the emulsion, and also the error due to the geometric corrections for the exit of the  $\alpha$ -particle tracks from the selected emulsion layer. The same diagram shows the cross sections as determined indirectly from the  $\alpha$ -energy spectra given by Vaganov et al.<sup>17</sup>

## CALCULATION OF THE CROSS SECTIONS

To determine the probability of elastic collision between a nucleon and an  $\alpha$  particle, in which the latter acquires a large momentum (corresponding to more than 30 Mev energy outside the nucleus), it is necessary to have experimental data on the differential cross sections over a wide range of angles and energies. We used the results summarized in the review of Hodgson<sup>18</sup> on the cross sections of elastic scattering of neutrons and protons, with energies up to 70 Mev, by He<sub>2</sub><sup>4</sup>. The experimental curves were extrapolated to higher energies (up to  $\sim 300$  Mev). We note that the experimental data available on the differential cross sections for the elastic scattering of nucleons by helium fit satisfactorily the following formula

$$d\sigma(\theta, E)/d\omega = C(\theta) \exp\{-k(\theta)\sqrt{E}\}, \quad (1)$$

\*The cross sections obtained previously<sup>15</sup> for the production of fast  $\alpha$  particles at 360 and 660 Mev were found to be overestimated, owing to an error that crept into the determination of the particle current.

where  $d\sigma(\theta, E)/d\omega$  is the differential cross section for scattering of a nucleon of energy  $E$  by the helium nucleus through an angle  $\theta$  (in the center-of-mass system, calculated per unit solid angle), and  $C$  and  $k$  are functions that depend only on the scattering angle. These coefficients are listed in Table I. For  $E = 90$  Mev, Eq. (1) is in satisfactory agreement with the Heidman formula,<sup>19</sup> up to  $\sim 75^\circ$ . We assume that (1) remains in force up to 300 Mev nucleon energy. It should be noted, incidentally, that extrapolation to such energies was carried out only for angles less than  $60^\circ$ , since the contribution of larger angles was found to be negligible.

TABLE I

$\theta$ , deg	$C$ , mb	$k$ , Mev <sup>-1/2</sup>
30	4.1	0.12
45	3.3	0.28
60	2.1	0.35
75	1.4	0.40
90	0.4	0.37
105	0.3	0.32
120	0.2	0.28
135	0.4	0.31
150	0.7	0.33
165	1.5	0.37

The cross sections for the knock-out of fast  $\alpha$  particles by nucleons was carried out for He<sup>4</sup> nuclei with kinetic energies  $W = 0, 5, 10,$  and  $20$  Mev. The binding energy of the  $\alpha$  particle in the nucleus ( $B = 4$  Mev) was also taken into account. According to Igo and Thaler<sup>8</sup> the depth of the potential well of the Ag nucleus for  $\alpha$  particles is approximately 35 Mev and one can therefore assume that in the heavy nuclei the  $\alpha$  particles will have energies up to 30 Mev. This figure should probably be reduced for the lighter nuclei.

We are interested in those cases of elastic collisions between a nucleon of energy  $E$  and an  $\alpha$  group of kinetic energy  $W$ , which cause the latter to leave the nucleus with energy  $T > 30$  Mev. Obviously  $T$  depends not only on  $E, W,$  and the scattering angle  $\theta$ , but also on the relative directions of the initial momenta of the nucleon and of the target particle. We denote the angle between these two vectors by  $\varphi$  ( $0 \leq \varphi \leq \pi$ ).

The resultant velocity of the recoil particle depends, in the laboratory system, also on the angle of rotation of the scattering plane relative to the plane that is determined by the initial momenta of the colliding particles (we shall denote this angle by  $\beta$ ). Figure 2 illustrates the geometry of the collision. Using the laws of elastic collision between two moving particles in the nonrelativistic

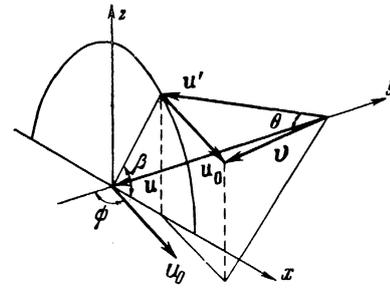


FIG. 2. Velocity vector diagram:  $xy$ -plane defined by the vectors of the momenta of the incident nucleon and of the  $\alpha$  particle,  $u$  - velocity of the  $\alpha$  particle in the c.m.s. prior to colliding with the nucleon,  $u'$  - the same after colliding with the nucleon,  $u_0$  - velocity of the center of mass,  $v$  - velocity of the recoil  $\alpha$  particle in the laboratory system;  $\theta$  - scattering angle in the c.m.s.

form, and using the geometric relations derived from Fig. 2, we obtain the condition that must be satisfied by the quantities,  $E, W, \theta, \varphi,$  and  $\beta$  if the  $\alpha$  particle is to leave the nucleus with energy greater than 30 Mev:

$$1.5 \sqrt{EW} \cos \varphi + (\cos \theta \cos \phi + \sin \theta \sin \phi \cos \beta) \times \left\{ E^2 + W^2 + \frac{17}{4} EW - 4EW \cos^2 \varphi + 3(E - W) \sqrt{EW} \cos \varphi \right\}^{1/2} \geq 3.1B + 94 + W - E. \quad (2)$$

The angle  $\psi$  is determined from the relation

$$\cot \psi = \frac{-4\sqrt{EW} \cos \varphi + \sqrt{EW} - 2(E - W)}{5\sqrt{EW} \sin \varphi}. \quad (3)$$

Inequality (2) defines the region of permissible values of  $\varphi$  and  $\beta$  for specified  $E, W,$  and  $\theta$ . The cross section of interest to us, that for differential scattering with production of a fast  $\alpha$  particle, is

$$\frac{d\sigma(E, W, \theta)}{d\omega} = \iint \frac{d\sigma_0(E, W, \theta, \varphi)}{d\omega} P(E, W, \theta, \varphi, \beta) d\varphi d\beta, \quad (4)$$

where  $d\sigma_0/d\omega$  is the cross section for scattering the free  $\alpha$  particles, taken from Eq. (1) with allowance for the dependence of the particle energy on the angle  $\varphi$ ;  $P$  is the statistical weight of the states in which  $\varphi$  and  $\beta$  satisfy condition (2).

It is natural to assume that the distribution of the intranuclear  $\alpha$  particles is isotropic with respect to the directions of their momenta, and also that the scattering is actually symmetrical (i.e., the cross section is independent of  $\beta$ ). Then

$$\frac{d\sigma}{d\omega} = \frac{1}{2\pi} \int_0^\pi \frac{d\sigma_0}{d\omega} \beta_{\max}(\varphi) \sin \varphi d\varphi. \quad (5)$$

The value of  $\beta_{\max}$  is determined as a function of  $\varphi, \theta, E$  and  $W$  from Eq. (2).

The cross section curve represented by Eq. (5)

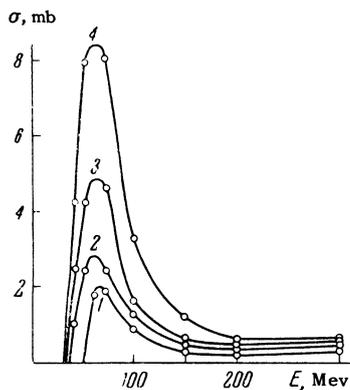


FIG. 3. Cross section for the knock-out of an  $\alpha$  particle with energy greater than 30 Mev by a nucleon of energy  $E$ , at various energies of motion of the  $\alpha$  particle in the nucleus. 1) for  $W = 0$ ; 2)  $W = 5$ ; 3)  $W = 10$ ; 4)  $W = 20$  Mev.

can be integrated over all possible angles  $\theta$ , starting with a certain  $\theta_{\min}$ , to find the total cross section for the knock-out of a fast alpha particle by a nucleon of energy  $E$ . We note that in our case the nucleon energy must be reckoned from the bottom of the potential well, since it will exceed the energy of the nucleon outside the well by approximately 30 Mev.

Figure 3 shows points corresponding to the calculated cross sections for various choices of the initial  $\alpha$ -particle kinetic energy.

#### PROBABILITY OF EXISTENCE OF ALPHA PARTICLES IN THE NUCLEUS

The curves shown in Fig. 3 indicate that the most effective region for the knock-out of alpha particles with  $T > 30$  Mev is the nucleon-energy region from 50 – 70 Mev (corresponding to nucleon energies from 30 – 50 Mev outside the nucleus), and that the cross section decreases with further increase in energy. The dependence  $\sigma_{\alpha}(E)$  shown in Fig. 4 is evidence, however, of the opposite: as the energy of the bombarding protons increases, the probability of emission of fast  $\alpha$  particles increases monotonically. This contradiction can be eliminated by assuming that the emission of the  $\alpha$  particle is due to secondary nucleons produced in the nuclear cascade process, the number of which

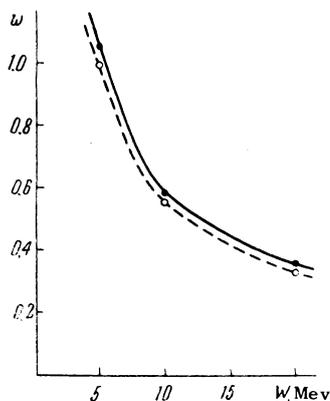


FIG. 4. Dependence of the calculated value of  $w$  on the assumed value of the energy of internal motion of the  $\alpha$  particles,  $W$ , in light (open circles) and heavy (full circles) emulsion nuclei.

increases continuously with increasing beam energy. One can then assume approximately that the yield of  $\alpha$  particles per single nuclear disintegration induced by a proton of energy  $E_0$  will be

$$\sigma_{\alpha}(E_0) = \int_0^{E_0} \sigma(E) n(E) N_{\text{eff}} dE. \quad (6)$$

Here  $\sigma(E)$  is the cross section shown in Fig. 3,  $n$  the number (per disintegration) of cascade nucleons of energy  $E$  passing through a nuclear surface layer that contains  $N_{\text{eff}}$  alpha particles.

We introduce a quantity  $w$ , defined by the relation

$$N_{\text{eff}} = Nw, \quad (7)$$

where  $N$  is the maximum possible number of  $\alpha$  groups in the surface layer. The size of this layer is determined, as already indicated, by the free path of the  $\alpha$  particle in the nuclear matter. In accordance with experiment<sup>7-9</sup>, we assume that the latter is  $2.5 \times 10^{-13}$  cm. If we take the density distribution functions of the nucleons in the nucleus in the following form<sup>20</sup>

$$\rho(r) = \rho_0 \frac{1 + \exp(-c/a)}{1 + \exp[(r-c)/a]},$$

we obtain  $N = 12$  for Ag and Br. For the light nuclei, we can assume  $N = 3$ .

The unknown quantity in (6), which can be written in the following form

$$\sigma_{\alpha}(E_0) = Nw\bar{n}(E_0) \int_0^{E_0} f(E) \sigma(E) dE, \quad (6a)$$

is  $w$ , since  $\sigma_{\alpha}$  is measured experimentally, and the average number of cascade nucleons  $\bar{n}$  and their energy distribution  $f(E)$  can be calculated by the Monte Carlo method.<sup>21</sup> The probabilities  $w$  for the existence of  $\alpha$  groups, calculated from (6a), are listed in Table II.

Figure 4 shows the mean value of  $w$  vs. the energy  $W$ . If it is assumed that the kinetic energy of the  $\alpha$  particles in the nucleus amounts to 10 Mev, it follows from our calculations that in light nuclei such as  $C^{12}$  and  $O^{16}$ , and also in the diffusion region of the heavy nuclei, the nucleons spend half their lifetimes in the form of alpha complexes.

#### DISCUSSION OF THE RESULTS

The values of  $w$ , found in our experiments, remain approximately constant as the energy of the incident protons changes. This circumstance serves as an indication of the correctness of the chosen model for the production of fast  $\alpha$  particles. There is no assurance, however, that these calculations are not subject to systematic errors

TABLE II

$E_0$ , Mev	Nuclei Ag and Br				Nuclei C, O, N			
	$w$ , Mev							
	0	5	10	20	0	5	10	20
100	2.19	1.00	0.56	0.34	2.40	1.05	0.62	0.37
140	1.99	0.91	0.51	0.32	2.01	0.93	0.52	0.31
200	2.04	0.93	0.54	0.32	2.19	0.99	0.57	0.34
360	2.60	1.07	0.58	0.36	2.15	1.02	0.54	0.32
560	2.61	1.21	0.66	0.41	2.01	0.95	0.55	0.33
660	2.58	1.17	0.65	0.40	2.14	1.00	0.58	0.34
average	2.33	1.05	0.58	0.36	2.15	0.99	0.56	0.33

which may either overestimate or underestimate  $w$  by the same amount for all beam energies. This pertains primarily to the cross sections used for the elastic scattering of the nucleons by the free  $\alpha$  particles, and also to the extrapolation of Eq. (1). In addition to this source of error, there exist others, for example, corrections for the probability of emission of a recoil  $\alpha$  particle from the surface layer of the nucleus, for the effect of the Pauli principle (which forbids certain collisions between nucleons and  $\alpha$  particles) etc., none of which are accounted for here. These factors result in an increase of  $w$ . Factors that contribute to a decrease of  $w$  include: the presence among the observed fast  $\alpha$  particles of a certain fraction of particles due to some other mechanism (for example, according to Ostroumov et al.,<sup>15</sup> the admixture of evaporated  $\alpha$  particles with energies  $> 30$  Mev amounts to 5–10%), the presence in the nucleus of  $\alpha$  particles with greater velocities than assumed by us, the knock-out of alpha particles by cascade nucleons, which for some reason or another do not leave the nucleus, and consequently are not taken into account in the calculations by the Monte Carlo method, etc.

As can be seen from Fig. 2, the principal contribution to the process of impact production of fast  $\alpha$  particles is from nucleons with energies 60–70 Mev, scattered at angles of approximately 120–150°. In the laboratory system, the  $\alpha$  particle moves after the collision in a direction close to the initial direction of the nucleon. But since we take  $\bar{n}$  in Eq. (6a) to mean the number of observed nucleons, i.e., the cascade nucleons that leave the nucleus, the direction of motion of the  $\alpha$  particle will also be such that the latter is capable of leaving the nucleus. Nor is allowance for the Pauli principle likely to cause great changes in the calculations. In fact, a 70-Mev nucleon colliding with an  $\alpha$  particle loses approximately 35 Mev and remains with an energy greater than the Fermi level. Furthermore, one can assume that the for-

mation of nucleon complexes will contribute to a weakening of the effect of the Pauli principle.

As regards the correctness of the choice of the  $w$ , nothing more specific can be added to what has already been said in the introduction. In all probability there is a certain distribution of  $\alpha$  particles by momenta in the nucleus, and perhaps, as shown by the results of Cüer,<sup>11</sup> this distribution is quite broad; without knowing its form, it is difficult to calculate the contribution of the high-energy  $\alpha$  particles to the computed cross section. As to the correctness of the extrapolation of Eq. (1), it is seen from Fig. 1 that nucleons with energies 200–300 Mev contribute little to the total cross section and possible errors in the extrapolation will not affect greatly the calculated values of  $w$ . Summarizing, we can assume that the foregoing factors, not accounted for in our calculation, act in opposite directions and cancel each other out to a considerable extent. It should be added that the value of  $w$  is quite sensitive to the form of the energy spectrum of the cascade nucleons, particularly in the region of small energies, and that a modification of the form of the function  $f(E)$ , which changes with changing energy of the bombarding proton, may lead to a certain modification of  $w$ .

The value of  $w$  can also be estimated for the Ag and Br nuclei by using the data of Ostroumov,<sup>22</sup> in which the relative contribution of cascade  $\alpha$  particles of all energies is determined. This estimate leads to  $w \sim 0.5–0.6$ , which does not disagree greatly with our results and the data of Hodgson.<sup>13</sup> It should be noted that Hodgson assumed  $N = 15$  for heavy nuclei and a  $N = 3.3$  for light nuclei, while in our calculations these values are respectively 12 and 3.

It can be assumed that a certain fraction of the tracks of doubly-charged particles in the stars belongs to the  $\text{He}_2^3$  nuclei. The emission of nuclei of the light helium isotope is connected, apparently, with the same processes as the emission of  $\alpha$  particles. An additional source may be the reaction

produced when an  $\alpha$  particle is struck by a fast nucleon [for example, of the type  $\text{He}_2^4(p, d)\text{He}_2^3$ ]. It would be interesting to determine experimentally the relations between the yields of the isotopes of He and H, due to the impact mechanism, and to compare them with the cross sections of the corresponding processes. This would throw light on the probability of realization of various groupings of nucleons in the nuclei.

A considerable similarity was established in reference 15 between the emission of fast  $\alpha$  particles and that of fragments. This similarity, in light of the results of the present paper, can serve as a certain basis for stating that multiply-charged particles are emitted from a nucleus, to a considerable extent, via elastic or quasi-elastic collisions between nucleons or their complexes and corresponding instantaneous nuclear substructures. It appears to us therefore that further research in this direction is of importance, not only to clarify the nature of fragmentation but also from the point of view of understanding the internal structure of the nucleus.

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