investigation of states with large spins by the proposed method possesses advantages over other means [reactions with complex nuclei, ( $\alpha, \mathrm{p}$ ) reactions, and others], since the angular distribution features of the ( $d, p$ ) reactions are revealed with significantly greater clarity.

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## THE PROTON SUBSHELL $Z=100$

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INVESTIGATIONS undertaken for the purpose of finding the new 102 nd element were recently crowned with success. Groups headed by Flerov in the U.S.S.R. and by Seaborg and Ghiorso in the U.S.A. have synthesized the short-lived isotopes
$102^{253}$ and $102^{254}$, of which the first decays via emission of an $8.8-\mathrm{Mev}$ alpha particle with a period from 2 to 30 seconds, ${ }^{1,2}$ and second decays both via fission ( $30 \%$ ) and via emission of alpha particles with energy close to 8.3 Mev with a period of approximately 3 seconds. ${ }^{3}$ In addition, it was shown that the activity with a period of approximately 10 minutes, observed previously by the Swedish scientists, ${ }^{3}$ was in all appearance not connected with the element 102.

We wish to call attention to the anomalous properties of the isotopes of the 102 nd element, observed even on a simple graph showing the dependence of the alpha-decay energy on N (analogous to the graphs cited in reference 4). However, the observed slight excess of the alphadecay energy of isotopes of the 102 nd element over those of the neighboring even elements can be the consequence of the fact that these isotopes, which are quite far from the beta-stability curve ${ }^{5}$ (as are, in general, all the lighter isotopes of the heavy elements), have excessive alpha-decay energies, other conditions being equal. To exclude the extraneous effect of the increase of the alphadecay energy upon deviation from the betastability curve, we used the empirical dependence of the alpha-decay energy $\mathrm{Q}_{\alpha}$ on Z , for nuclei with identical N but different Z (see reference $5)$ :

$$
\begin{equation*}
Q_{\alpha}^{*}(N, Z)=Q_{\alpha}(N)-0.8\left(Z-Z^{*}\right) \tag{1}
\end{equation*}
$$

where $Z^{*}$ is the value of $Z$ corresponding to the most beta-stable nucleus for a given A , and $\mathrm{Q}_{\alpha}^{*}(\mathrm{~N}, \mathrm{Z})$ is the alpha-decay energy of the nucleus ( $\mathrm{N}, \mathrm{Z}^{*}$ ) in Mev . One can put (see references 5 and 6)

$$
Z^{*}=0.356 A+9.1
$$

It follows from (1) that the $Q_{\alpha}^{*}(N)$ found from the experimental values of $Q_{\alpha}$ should coincide at each value of $N$, even in the presence of neutron shells and subshells; only in the case of proton subshells will the corresponding points deviate. Figure 1 shows the dependence of $Q_{\alpha}^{*}$ on N. For each of the values of N it was found here that the values of $Q_{\alpha}^{*}$, calculated from different experimental values of $\mathrm{Q}_{\alpha}$ (taken from reference 7), were almost the same. Nevertheless, to exclude the spread (which reaches $\pm 0.15 \mathrm{Mev}$ ), we have drawn the curve $Q_{\alpha}^{*}=Q_{\alpha}^{*}(N)$ only through the averaged points. As can be seen from Fig. 1, in this region only two isotopes of the 102 nd element lie without any doubt above the curve $\mathrm{Q}_{\alpha}^{*}=\mathrm{Q}_{\alpha}^{*}(\mathrm{~N})$. Inasmuch as the isotopes of the 102 nd element are converted into Fm by alpha decay, this is evidence of a reduced binding energy past $Z=100$.


FIG. 1. Dependence of the reduced energies of alpha decay $Q_{\alpha}^{*}$ on the number of neutrons: - - experimental and calculated values (Refs. 1, 2, 7), 0 estimated values. ${ }^{7}$

We note that the rise of the curve $\mathrm{Q}_{\alpha}^{*}=\mathrm{Q}_{\alpha}^{*}(\mathrm{~N})$ itself indicates the existence of subshells at $\mathrm{N}=152$ and $\mathrm{N}=144$, as has already been noted earlier. ${ }^{6}$ The change in the properties of the nuclei past $Z=100$ manifests itself also in a sharp decrease in the period of spontaneous fission at $102^{254}$, which is observed when the spontaneousfission period $\log \tau_{\mathrm{f}}$ is plotted vs. $\mathrm{Z}^{2} / \mathrm{A}$ (such a graph was plotted earlier ${ }^{4}$ ). At the same time one observes, in both isotopes of the 102nd element on the graph $\log \tau_{\alpha}=\mathrm{f}\left(\mathrm{E}_{\alpha}\right)$ (see Fig. 2), an increase in the forbiddenness in alpha decay.


FIG. 2. Dependence of the period of alpha decay ( $\tau_{\alpha}$, sec) on the energy of the alpha particles. ${ }^{1,2,7}$

Thus, the latest data in our possession are evidence that at $Z=100$, apparently, there occurs either a filling of one of the subshells (which would be in accordance with the Mayer-Jensen or Nilsson scheme) or else a jump-like change in the deformations. In the former case, assuming the Nilsson scheme to be correct, one can assume, for example, a deformation parameter $\delta \approx 0.18$ to obtain jumps in the binding energies past $\mathrm{Z}=92,96$, and 100 (see reference 8 ). The increase in the probability of spontaneous fission
for $102^{254}$ is possibly connected in part with the appearance of spontaneous fission into three fragments, ${ }^{9}$ as observed in the case of $\mathrm{Cf}^{252}$ (reference 10 ).

In connection with the reduced stability of the nuclei past $N=152$ (references 4 and 6) and $Z=100$, one must review all possible limits of the existence of nuclei ${ }^{11,12}$ on the downward side, even compared with the pessimistic estimates of Ivanenko. ${ }^{11}$ A preliminary estimate based on the spontaneous-fission curve shows now that the limit of nuclei with a lifetime of approximately $10^{-12}-10^{-16}$ seconds should lie most likely between $Z=108$ and 112, i.e., very far from the "optimistic" estimates of Wheeler. ${ }^{11}$

In conclusion, we express our gratitude to Prof. D. D. Ivanenko, Prof. A. Ghiorso, S. Thompson, and Prof. G. N. Flerov for valuable discussions and remarks, and also to S. I. Larin who graciously reported to us that the number $Z=100$ stood out also on a curve he plotted for the dependence of the energy differences of the alpha decay of the neighboring isotopes on $Z$.

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## ON THE DISTRIBUTION FUNCTION FOR DISSIPATIVE PROCESSES IN A RAREFIED RELATIVISTIC GAS

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THe motion of a rarefied gas or of a gas with a large flow gradient, cannot, generally speaking, be treated as the motion of a continuous medium, and additional consequences of kinetic theory must be used.

As is known, Grad ${ }^{1}$ considered such a flow by adding to the independent parameters (in addition to the mean velocity, density, and temperature) also the heat flow and the tensor of viscous stresses. Using a specific form of nonrelativistic Maxwellian distribution (which can be considered as a weighting function for three-dimensional Hermite polynomials in velocity space), Grad expanded the distribution function in Hermite polynomials. Retaining only the first three terms of the series, he obtained a distribution function over the coordinates and velocities, describing the processes of viscosity and heat conduction in the nonrelativistic approximation.

It is easy to obtain a distribution function for rarefied relativistic gas, with allowance for the phenomena of viscosity and heat conduction, by introducing orthogonal polynomials with a weight $\exp \left(-\sigma \sqrt{1+\mathrm{u}^{2}}\right)$. Here $\sigma=\mathrm{mc}^{2} / \mathrm{kT}$, where T is the temperature in the proper reference system of the given gas element; $u^{2}=u_{\alpha}^{2}$, where $u_{\alpha}$ are the spatial components of the four-velocity of the gas particles.* By way of an example we cite the first two polynomials of this type

$$
\begin{gather*}
g^{(0)}=1 / 2 \sqrt{\pi} \sigma^{1 / 4} K_{2}^{1 / 2}(\sigma), \quad g_{\alpha}^{(1)}=\xi_{\alpha} / 2 \sqrt{\pi} \sigma^{1 / 4} K_{3}^{1 / 2}(\sigma), \\
\xi_{\alpha}=\sigma^{1 / 2} u_{\alpha} . \tag{1}
\end{gather*}
$$

Here $K_{\nu}(\sigma)$ is the MacDonald function.
As is known, ${ }^{2}$ the scalar distribution can be written in the form

$$
\begin{equation*}
F=i c f(x, p) \delta(H+m c) . \tag{2}
\end{equation*}
$$

Here $H$ is the invariant Hamiltonian function, while x and p are the 4-coordinates and 4momentum of the particle. The scalar $f(x, p)$ coincides with the ordinary distribution function and its expression in the proper coordinate system of the gas in equilibrium differs from $\exp \left(-\sigma \sqrt{1+u^{2}}\right)$ only by a multiplicative factor. If we now expand $\mathrm{f}\left[\exp \left(-\sigma \sqrt{1+\mathrm{u}^{2}}\right)\right]^{-1 / 2}$ in terms of the functions $\left[\exp \left(-\sigma \sqrt{1+\mathrm{u}^{2}}\right)\right]^{1 / 2} \mathrm{~g}_{\alpha}^{(\mathrm{n})}$ (the expansion is valid in the sense of convergence in the mean) and confine ourselves to the first three terms of the expansion, we obtain after simple calculations an expression for $f(x, p)$ in the proper system of reference of the given element of gas

$$
\begin{align*}
& f(x, p)=\exp \left(-\sigma \sqrt{1+u^{2}}\right)\left\{\frac{n \sigma}{4 \pi(m c)^{3} K_{2}(\sigma)}+\frac{\sigma^{2} \tau_{\alpha \beta} \xi_{a} \xi_{\beta}}{8 \pi m^{4} c^{5} K_{3}(\sigma)}\right. \\
& \left.\quad+\frac{\sigma^{1 / 2}}{24 \pi m^{6} c^{6} K_{4}(\sigma)}\left[T_{\alpha \beta \gamma} \xi_{\alpha} \xi_{\beta} \xi_{\gamma}-3 \frac{K_{4}(\sigma)}{K_{3}(\sigma)} T_{\alpha \beta \beta} \xi_{\alpha}\right]\right\} . \tag{3}
\end{align*}
$$

Here $n$ is the density of the particles in a proper system of the given element of gas, $\tau_{\alpha \beta}$ is the additional term in the three-dimensional portion of the energy-momentum tensor due to the dissipative processes, and $\mathrm{T}_{\alpha \beta \gamma}$ are the spatial components of the tensor $T_{i k l}=\int p_{i} p_{k} F_{l} d^{4} p$.

The expression for $\mathrm{T}_{\mathrm{ik} l}$ in any system of reference can be obtained from its components in the proper system of reference. As a result we obtain

$$
\begin{align*}
T_{i k l} & =\frac{n m^{2} c^{3}}{\sigma} \frac{K_{3}(\sigma)}{K_{2}(\sigma)}\left(U_{i} \delta_{k l}+U_{k} \delta_{i l}+U_{l} \delta_{i k}\right)+\frac{n m^{2} c^{8} K_{4}(\sigma)}{K_{2}(\sigma)} U_{i} U_{k} U_{l} \\
& +\frac{m c K_{4}(\sigma)}{K_{3}(\sigma)}\left(U_{i} \tau_{k l}+U_{k} \tau_{i l}+U_{l} \tau_{i k}\right)+R_{i k l}, \tag{4}
\end{align*}
$$

where $U_{i}$ is the 4-velocity corresponding to the average motion, and $R_{i k l}$ is a tensor, whose components $\mathrm{R}_{\alpha \beta 4}$ and $\mathrm{R}_{444}$ vanish in the proper system and whose remaining components coincide with the components of $\mathrm{T}_{\mathrm{ik} l}$ in the same system. The tensor $\mathrm{T}_{\mathrm{ik}} l$ can be used in the study of transport phenomena, and also to investigate the structure of a shock wave. We note that for large values of $\sigma$, Eqs. (3) and (4) go into the corresponding nonrelativistic expressions.

Inasmuch as the function $f$ is a scalar, its form for any system of reference should be


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