SOVIET PHYSICS JETP VOLUME 37 (10), NUMBER 2 FEBRUARY, 1960

TRANSITION RADIATION EFFECTS IN PARTICLE ENERGY LOSSES

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Submitted to JETP editor March 19, 1959; resubmitted April 29, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 527-533 (August, 1959)

The energy lost by a particle that passes through a layer of matter of finite thickness is computed. It is found that at high energies the particle losses due to passage through the interface between two media (transition radiation) can become important.

THE energy lost by a particle (per unit path length) is usually computed under the assumption that the particle moves in an unbounded uniform medium (cf., for example, reference 1). In the present paper we calculate the energy lost by a particle in traversing a layer of finite thickness. As has been shown by Ginzburg and Frank,[2] when a charged particle passes from one medium into another a readjustment must take place in the field associated with the particle; as a result part of the field is "shaken off." This effect is the so-called transition radiation. It is of interest to consider the total energy lost by a particle in passing through the boundary separating two media.

Before solving this problem for finite slabs we consider the case in which the particle moves from one semi-infinite medium into another. The particle losses are computed by the method developed by Landau (cf. reference 1). The fields produced by the charged particle in cases of this kind have been given earlier.[3,4]

1. CASE OF A SINGLE BOUNDARY DIVIDING TWO MEDIA

Suppose that the particle moves along the z axis with velocity **v** and goes from one medium into another [with dielectric permittivities $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$ respectively]; the plane $z = 0$ is taken as the plane of separation between these media. The fields in both half-spaces will consist of two parts: one is the same as the field associated with a charge which moves in an infinite uniform medium while the other is the radiation field. We write the expressions for the Fourier components of the longitudinal (in the direction of motion of the particle) radiation fields in the first and second media:

$$E'_{1n}(\mathbf{k}) = \frac{ei}{2\pi^2}\frac{\varkappa^2}{\varepsilon_2\lambda_1+\varepsilon_1\lambda_2}\left\{\frac{\varepsilon_2/\varepsilon_1 - v\lambda_2/\omega}{k^2-\omega^2\varepsilon_1/c^2} + \frac{-1+v\lambda_2/\omega}{k^2-\omega^2\varepsilon_2/c^2}\right\}, \quad (1)$$

$$E'_{2n}(\mathbf{k}) = \frac{ei}{2\pi^2}\frac{\varkappa^2}{\varepsilon_2\lambda_1+\varepsilon_1\lambda_2}\left\{\frac{1+v\lambda_1/\omega}{k^2-\omega^2\varepsilon_1/c^2} + \frac{-\varepsilon_1/\varepsilon_2 - v\lambda_1/\omega}{k^2-\omega^2\varepsilon_2/c^2}\right\}. \quad (2)$$

Here, in contrast with the expressions given earlier[3] the real and imaginary parts of λ_1 and λ_2 are taken as positive $(\lambda_{1,2}^2 = \omega^2\epsilon_{1,2}/c^2 - \kappa^2)$.

As in all problems of this type we assume that the energy lost by the particle is small compared with the kinetic energy so that the velocity remains constant.

To calculate effects due to interactions over distances much larger than interatomic distances (in which case a macroscopic analysis can be used) it is sufficient to compute the work done on the particle by its field. The work done by the first part of the field is similar to that computed in reference 1. We compute the work associated with the fields given by Eqs. (1) and (2). In this case it is more pertinent to consider the total work done in each medium rather than the work done per unit length of path.

First we calculate the work done by the Fourier field component in Eq. (2):

$$F_2 = ev\int_0^\infty dt \int E'_{2n}(\mathbf{k})\, e^{i(\lambda_2 v-\omega)t}\, d\mathbf{k} = \frac{e^2}{\pi v^2}\left(F_2^{(1)} - F_2^{(2)}\right), \quad (3)$$

where

$$F_2^{(1)} = \int \frac{\varkappa^3\, d\varkappa}{\varepsilon_2\lambda_1+\varepsilon_1\lambda_2}\,\frac{(1+v\lambda_1/\omega)(\lambda_2 v+\omega)}{(k^2-\omega^2\varepsilon_2/c^2)(k^2-\omega^2\varepsilon_1/c^2)}\,d\omega, \quad (4)$$

$$F_2^{(2)} = \int \frac{\varkappa^3\, d\varkappa}{\varepsilon_2\lambda_1+\varepsilon_1\lambda_2}\,\frac{(\varepsilon_1/\varepsilon_2+v\lambda_1/\omega)(\lambda_2 v+\omega)}{(k^2-\omega^2\varepsilon_2/c^2)^2}\,d\omega. \quad (5)$$

From these expressions it is apparent that by computing the integral

$$J = \int \frac{\varkappa^3\, d\varkappa}{\varepsilon_2\lambda_1+\varepsilon_1\lambda_2}\,\frac{([\varepsilon_1/\varepsilon_2]+v\lambda_1/\omega)(\lambda_2 v+\omega)}{(k^2-\omega^2\varepsilon_2/c^2)(k^2-\omega^2\varepsilon_3/c^2)}\,d\omega, \quad (6)$$

and then letting $\epsilon_1/\epsilon_2 \to 1$ (in the places denoted by the square brackets) and $\epsilon_3 \to \epsilon_1$, we can obtain $F_2^{(1)}$; then, by taking the limiting case $\epsilon_3 \to \epsilon_2$, we obtain $F_2^{(2)}$. In Eqs. (4) – (6) the integration over κ is carried out from zero to some value κ_0, corresponding to the minimum distance from the particle trajectory at which the macroscopic analysis is still valid.

First we carry out the integration over ω from $-\infty$ to $+\infty$. For this purpose we close the path of integration by a half-circle in the upper half of the plane. Since the integral along this half-circle vanishes the expression being sought will be equal to the sum of the residues in the upper half of the plane. The residues are taken at the zeros of the functions $(k^2 - \omega^2\epsilon_2/c^2)$ and $(k^2 - \omega^2\epsilon_3/c^2)$. In the upper half of the plane these functions have one zero, at the imaginary axis (cf. reference 1).* Then, we integrate with respect to $\omega(\kappa)$ instead of κ (cf. reference 1). For an ultrarelativistic particle the limits of integration are

$$\sqrt{\sigma_{2,3}/(1-\beta^2)} \quad \text{and} \quad c\sqrt{(\varkappa_0^2 + \sigma_{2,3}/c^2)/(1-\beta^2)},$$

$$\sigma_{2,3} = 4\pi N_{2,3}e^2/m$$

(N is the number of electrons per unit volume and m is the mass of the electron). Thus, we have in the ultrarelativistic case

$$J = \frac{2\pi c}{\sqrt{1-\beta^2}}\left\{\frac{c\varkappa_0}{2} - \frac{2}{3}\frac{\sigma_2^{3/2} - \sigma_3^{3/2}}{\sigma_2 - \sigma_3} + \frac{3}{8\varkappa_0 c}(\sigma_2 + \sigma_3)\right\}. \quad (7)$$

Whence it is not difficult to obtain the values of $F_2^{(1)}$ and $F_2^{(2)}$; finally, from Eq. (3) we have

$$F_2 = \frac{2e^2}{c\sqrt{1-\beta^2}}\left\{\sqrt{\sigma_2} - \frac{2}{3}\frac{\sigma_2^{3/2} - \sigma_1^{3/2}}{\sigma_2 - \sigma_1} - \frac{3}{8\varkappa_0 c}(\sigma_2 - \sigma_1)\right\}. \quad (8)$$

It is apparent from this formula that reasonable accuracy can be obtained if we take $\kappa_0 = \infty$, i.e., the effect is macroscopic and vanishes at small distances. We may note that $F_2 > 0$ when $\sigma_2 > \sigma_1$ and $F_2 < 0$ when $\sigma_2 < \sigma_1$.

By a similar calculation we can show that in contrast with F_2, F_1, the work of the radiation field on the particle in the first medium, falls off with increasing particle energy; we shall not consider it further.

Our analysis is not complete until we take account of the fact that a particle with the same velocity will have different energies in different media; this might be called macroscopic "renormalization" of the particle mass. We compute the amount of electromagnetic energy which the particle carries through a plane perpendicular to its trajectory for motion in an unbounded uniform medium:

$$W = \frac{c}{4\pi}\int_{-\infty}^{\infty}[\mathbf{E}\times\mathbf{H}]_z\,dx\,dy\,dt$$

$$= \frac{e^2}{\pi v}\int\frac{\varkappa^3\,d\varkappa\,d\omega}{\varepsilon(\omega)(k^2 - \omega^2\varepsilon(\omega)/c^2)(k^2 - \omega^2\varepsilon(-\omega)/c^2)}. \quad (9)$$

The integration over κ is taken from 0 to κ_0 so that the numerical expression gives the flux through the entire plane except for a circle of radius $1/\kappa_0$ whose center coincides with the particle trajectory. Assuming for simplicity that $\epsilon(-\omega) = \epsilon(\omega)$, we compute the following integral (the zeros in the denominator are traversed from above):

$$W' = \frac{e^2}{\pi v}\int\frac{\varkappa^3\,d\varkappa\,d\omega}{\varepsilon(\omega)(k^2 - \omega^2\varepsilon(\omega)/c^2)(k^2 - \omega^2\varepsilon'(\omega)/c^2)}.$$

By taking $\epsilon' = \epsilon$ in the final result we obtain an expression for W. Thus,

$$W = \frac{e^2}{c\sqrt{1-\beta^2}}\left\{\frac{\varkappa_0 c}{2} - \sqrt{\sigma}\right\}, \quad (10)$$

where $\sigma = 4\pi Ne^2/m$.

It can be shown that the field at distances up to $1/\kappa_0$ is the same in all media. Thus, if a particle has an energy $\mu c^2\sqrt{1-\beta^2}$ in the first medium, a particle of the same velocity will have in the second medium an energy

$$\mu' c^2/\sqrt{1-\beta^2},$$

where $\mu' = \mu + (W_2 - W_1)/c^2 = \mu + e^2(\sqrt{\sigma_1} - \sqrt{\sigma_2})/c^2$.

Returning to our problem, we now take account of the force F_2 which also acts on the particle; the energy of the particle is

$$\frac{1}{\sqrt{1-\beta^2}}\left\{\mu c^2 - \left[\frac{4}{3}\frac{e^2}{c}\frac{\sigma_2^{3/2} - \sigma_1^{3/2}}{\sigma_2 - \sigma_1} - \frac{e^2}{c}(\sqrt{\sigma_1} + \sqrt{\sigma_2})\right]\right\}.$$

It is easy to show that the quantity in the square brackets is always positive, i.e., in passing through the boundary between the two media the particle always loses an amount of energy given by

$$I = \frac{2e^2}{c\sqrt{1-\beta^2}}\left\{\frac{2}{3}\frac{\sigma_2^{3/2} - \sigma_1^{3/2}}{\sigma_2 - \sigma_1} - \frac{1}{2}(\sqrt{\sigma_1} + \sqrt{\sigma_2})\right\} \quad (11)$$

For vacuum-medium and medium-vacuum cases this energy loss is $(e^2/3c)\sqrt{\sigma/(1-\beta^2)}$.

We now show that the indicated energy losses are completely due to the transition radiation.* We compute the electromagnetic radiation flux through some plane perpendicular to the z axis (in the second medium) for the entire particle time of flight:

$$S_{2z} = \frac{c}{4\pi}\int_{-\infty}^{+\infty}[\mathbf{E}_2\times\mathbf{H}_2]_z\,dt\,dx\,dy. \quad (12)$$

*In a similar manner it can be shown that λ_1 and λ_2 do not have zeros in the upper half of the plane. Furthermore, since ε_1 and ε_2 do not have zeros in the upper half of the plane and are positive along the real axis it is easy to show that the sum $\varepsilon_2\lambda_1 + \varepsilon_1\lambda_2$ has no zeros in the upper half of the plane.

*Attention has been directed to this fact by K. A. Barsukov in a similar calculation for a waveguide.

We use the formulas for $\mathbf{E}_2$ and $\mathbf{H}_2$ which have been derived in reference 3, and make use of the δ-function in the integration over t, x, and y. Then, converting from integration over κ to ω, as in the preceding cases, we find that this flux consists of three parts: one is due to the field of the charged particle itself and is given by Eq. (10) (with $\sigma = \sigma_2$); the second is the radiation field (given by I); the third is due to interference between these fields and vanishes when $(\omega/v - \lambda_2)z \gg 1$.

We now find the angular distribution and frequency dependence of the transition radiation field in the second medium. To find the conditions under which the interference term vanishes (i.e., the transition radiation zone), after integrating over t, x, and y in Eq. (12) we make the substitution $\kappa = (\omega/c)\sqrt{\epsilon_2}\sin\theta$, where θ is the angle between the wave vector of the radiation field and the z axis. Assuming for simplicity that $\epsilon_2 = 1$ and $\epsilon_1 = \epsilon$, we find the energy flux in the second medium due to transition radiation

$$dS'_{2z} = \frac{2e^2\beta^2}{\pi c}\frac{\sin^3\theta\cos^2\theta\,d\theta}{(1-\beta^2\cos^2\theta)^2} \times \int_0^\infty \left(\frac{(\varepsilon-1)(1-\beta^2-\beta\sqrt{\varepsilon-\sin^2\theta})}{(\varepsilon\cos\theta+\sqrt{\varepsilon-\sin^2\theta})(1-\beta\sqrt{\varepsilon-\sin^2\theta})}\right)^2 d\omega \qquad (13)$$

(we assume that the medium is transparent).* It is apparent from this formula that the radiation vanishes at small angles $(\theta \sim \sqrt{1-\beta^2})$ with respect to the direction of motion of the particle. Taking account of this fact, it is easy to show that the main contribution in the integral in Eq. (13) is at frequencies larger than the optical frequencies [because of the small factor $(1-\beta\sqrt{\epsilon - \sin^2\theta})$ which appears in the denominator of Eq. (13)]. Then, substituting the expression $\epsilon(\omega) = 1 - \sigma/\omega^2$ in Eq. (13) and replacing $\sin\theta$ by θ, we integrate first over the frequencies, and then over the angles from 0 to ∞. We find $S'_{2z} = e^2\sqrt{\sigma}/3c\sqrt{1-\beta^2}$.

As to the frequency distribution of the transition radiation spectrum, we find from Eq. (13) that the radiation intensity is almost constant from optical frequencies to the limiting frequency $\omega_{\lim} = \sqrt{\sigma}/2\sqrt{1-\beta^2}$. At this frequency the radiation intensity falls off by a factor of 2 as compared with the intensity at lower frequencies. The main contribution in the integral is due to frequencies which are not too small compared with the limiting frequency.

The transition radiation in the first medium can be obtained in essentially the same way. The well-known formula given by Ginzburg and Frank is used (cf. also references 3, 5, and 6); this formula differs from Eq. (13) in that β is replaced by $-\beta$. As a result, the small factor in the denominator vanishes and the backward transition radiation encompasses only the optical part of the spectrum, being given by an expression which diverges logarithmically with energy.

If we now assume that the first medium is a vacuum and that the second is a dielectric, Eq. (13) undergoes changes which are unimportant at high frequencies, so that the foregoing results still apply.

As has been noted, the transition radiation zone is determined by the inequality $z \gg |\omega/v - \lambda_2|^{-1}$. If the second medium is a vacuum the transition radiation zone is determined, for all frequencies, by the inequality $z \gg \lambdabar/(1-\beta^2)$ ($\lambdabar$ is the wavelength of the radiation divided by 2π). However, if the second medium is not vacuum, the transition radiation zone is of the order of the optical wavelength for optical frequencies so that $z \gg \lambdabar/(1-\beta^2+\sigma\lambdabar^2/2c^2)$ for frequencies close to the limiting frequency. At the limiting frequency $z_{\lim} \gg c/\sqrt{\sigma(1-\beta^2)}$, i.e., the transition radiation zone grows much larger.

Finally we consider the number of transition radiation photons. Again limiting ourselves to the medium–vacuum or the vacuum–medium case, the number of photons with frequencies $\omega' \gtrsim \omega_{\text{opt}}$, up to the hardest, is given by the expression

$$N'_2 = \frac{1}{137}\frac{2}{\pi}\left[\ln\left(\frac{1}{\omega'}\sqrt{\frac{\sigma}{2(1-\beta^2)}}\right) - \frac{1}{2}\right]. \qquad (14)$$

Whence it is apparent for example that if $E/\mu \sim 10^{10}$ and $\omega' \sim \sqrt{\sigma/2}$, $N'_2 \sim 0.1$, i.e., out of ten particles on the average only one emits a transition photon. If the frequency of the transition photon is close to the limiting frequency its energy is approximately $\sim\sqrt{\sigma/(1-\beta^2)}$, i.e., this energy is approximately 137 times greater than S'_{2z}. Thus, as follows from Eq. (14), a photon of this kind appears approximately once out of 137 particles).

In spite of the classical nature of the effect, the radiation of transition photons from singly charged particles is a rare phenomenon, sub-

*We may note that this formula coincides with Eq. (28) of reference 3, bearing in mind that the significance of the angle θ in both formulas is the same for transition radiation, but not for Cerenkov radiation. The infinity which appears in Eq. (13) when $1 - \beta\sqrt{\varepsilon - \sin^2\theta} = 0$ is due to the fact that this formula also gives the intensity of the Cerenkov radiation emitted with a semi-infinite trajectory in the first medium, which is assumed to be transparent. The intensity of the transition radiation itself, however, is always finite (cf. reference 3).

ject to large fluctuations. This effect can be made classical if the number of emitted photons is increased. This can be achieved, for example, if the particle is multiply charged or if it passes through a large number of boundaries. In the first case the number of emitted photons and the total energy increases quadratically with the charge of the particle; in the second case these quantities increase in proportion to the number of boundaries traversed. In both cases the limiting frequency of the photon remains the same. However in the second case the transition photon zone must be considered.

All the results cited above refer to the vacuum–medium or medium–vacuum case. However, as is apparent from Eq. (11), these results apply qualitatively for the medium–medium case, which differs from the preceding cases only in certain small numerical factors.

2. CASE OF A FINITE SLAB

Suppose now that a particle which moves in vacuum enters a slab whose boundaries are the planes[4] $z = 0$ and $z = a$ (cf. also reference 7).

Because the particle moves in the same medium (vacuum) on both sides of the slab it is not necessary to "renormalize" the particle mass.

As in the preceding case we consider the work done on the particle by the radiation field and obtain the following expressions for the region in front of the slab (W_0), in the slab (w_1) and behind the slab (W_1):

$$W_0 = -\frac{e^2}{\pi}\int\frac{\varkappa^3\xi\,d\varkappa\,d\omega}{\lambda_0 F(\lambda_0 v+\omega)}, \tag{15}$$

$$w_1 = -\frac{e^2}{\pi}\int\frac{\varkappa^3\eta'\,d\varkappa\,d\omega}{\lambda F(\lambda v-\omega)}\left(e^{i(\lambda v-\omega)a/v}-1\right) - \frac{e^2}{\pi}\int\frac{\varkappa^3\eta''\,d\varkappa\,d\omega}{\lambda F(\lambda v+\omega)}\left(e^{-i(\lambda v+\omega)a/v}-1\right), \tag{16}$$

$$W_1 = -\frac{e^2}{\pi}\int\frac{\varkappa^3\zeta\,d\varkappa\,d\omega}{\lambda_0 F(\lambda_0 v-\omega)}, \tag{17}$$

where

$$\left.\begin{matrix}\xi\\ \zeta\end{matrix}\right\} = \left(\frac{\varepsilon}{\lambda}\pm\frac{1}{\lambda_0}\right)\alpha e^{\mp i\lambda a} + \left(\frac{\varepsilon}{\lambda}\mp\frac{1}{\lambda_0}\right)\beta e^{\pm i\lambda a} + \frac{2\varepsilon}{\lambda}\left\{\begin{matrix}\gamma\\ \delta\end{matrix}\right\} e^{\pm i\omega a/v},$$

$$\left.\begin{matrix}\eta'\\ \eta''\end{matrix}\right\} = -\left(\frac{\varepsilon}{\lambda}\pm\frac{1}{\lambda_0}\right)\delta e^{\mp i\lambda a} + \left(\frac{\varepsilon}{\lambda}\mp\frac{1}{\lambda_0}\right)\gamma e^{+i\omega a/v},$$

$$\left.\begin{matrix}\alpha\\ \beta\end{matrix}\right\} = \frac{\pm\varepsilon/\lambda - v/\omega}{k^2-\omega^2/c^2} + \frac{\mp 1/\lambda + v/\omega}{k^2-\omega^2\varepsilon/c^2},$$

$$\left.\begin{matrix}\gamma\\ \delta\end{matrix}\right\} = \frac{\mp 1/\lambda_0 + v/\omega}{k^2-\omega^2/c^2} + \frac{\pm 1/\lambda_0\varepsilon - v/\omega}{k^2-\omega^2\varepsilon/c^2},$$

$$F = (\varepsilon/\lambda + 1/\lambda_0)^2 e^{-i\lambda a} - (\varepsilon/\lambda - 1/\lambda_0)^2 e_{i\lambda a}.$$

It is easy to show that F has no zeros in the upper half of the complex ω plane. Furthermore, in the expression for F only the first term need be used. As in the preceding case, we find that W_0 falls off with increasing particle energy; hence this term will not be considered here.

In integrating the expressions for w_1 and W_1 we limit ourselves to "thick" and "thin" slabs. We note that w_1 and W_1 contain exponential terms with the characteristic argument $i(\lambda - \omega/v)a$. A slab will be considered thick or thin depending on whether this argument is large or small compared with unity. In the first case we neglect terms containing this exponential; in the second case we expand the exponential and limit ourselves to the first nonvanishing term. We then analyze the manner in which ω and κ appear in the integrals to determine whether a slab is to be considered "thick" or "thin."

In the case of the thick slab, w_1 is given by an expression which coincides with F_2 for the motion of a particle from vacuum into a medium, while W_1 coincides with F_2 for the motion of a particle from a medium into vacuum. Thus, from Eq. (8) it follows that the total loss of energy is $-(2e^2/3c)\sqrt{\sigma/(1-\beta^2)}$. Obviously this loss approaches the transition radiation arising at each boundary of the slab. This same result can be obtained by direct calculation of the flux of electromagnetic energy which passes through a plane perpendicular to the z axis and located in the space beyond the slab.*

Assuming that the slab is thick, substituting the limiting value of the frequency, and assuming that $\kappa \sim \omega\sqrt{1-\beta^2}/c$, we obtain the following requirement for the slab thickness:

$$a \gg c/\sqrt{\sigma(1-\beta^2)}. \tag{18}$$

If the thickness of the slab is less than the limiting value (18), the transition radiation spectrum will not contain hard photons; these photons cannot be produced in the slab and it follows that they cannot be formed in the space beyond the slab.

We now consider the thin slab. The following expressions are obtained for w_1 and W_1:

$$w_1 = \frac{\sigma e^2}{c^2}a\left[\ln\frac{v\varkappa_0}{\sqrt{\sigma}}+\frac{1}{2}\right], \tag{19}$$

$$W_1 = -\frac{\sigma e^2}{c^2}a\left[\ln\frac{v\varkappa_0}{\sqrt{1-\beta^2}\,\Omega}+c_1\right],$$

$$c_1 = \frac{8}{\sigma}\int_0^{\Omega}\frac{\sqrt{\varepsilon}(\sqrt{\varepsilon}-1)}{(\sqrt{\varepsilon}+1)^2}\,\omega\,d\omega \tag{20}$$

*We may note that in references 4 and 8, in the calculation of the Poynting vector in the ultra-relativistic case, only the optical part of the transition radiation spectrum was investigated.

($\Omega \gtrsim I/\hbar$, where I is the ionization energy of the K-electron). Adding the last two expressions to the usual ionization losses in matter[1*] the energy loss of a particle in a thin slab is given by the expression

$$F = \frac{2\pi Ne^4}{mc^2} a\left[\ln\frac{c^2\varkappa_0^2}{(1-\beta^2)\,\Omega^2} + 2c_1 - 1\right]. \qquad (21)$$

It is apparent from this formula that in thin slabs there is no density effect. To obtain the conditions under which a slab may be assumed thin we proceed as follows. In the expanded formulas (16) and (17) we compute the last term smaller than the first-order term in a. As a result the following condition is obtained for a thin slab:

$$a \ll 2\,(c\Omega\,/\,\sigma)\ln(v\varkappa_0\,/\sqrt{1-\beta^2}\Omega). \qquad (22)$$

It is apparent from these formulas that, after first levelling off, the ionization losses in the slab again start increasing logarithmically at some particle energy if the slab is in a vacuum. However if the slab is in a medium which has a smaller electronic density (than the slab), the ionization losses again reach a plateau corresponding to the electron density of the medium surrounding the slab.

As the slab thickness is reduced, the particle can radiate more or less hard photons in the region outside the intervals given by Eqs. (18) and (22), and the term without the density becomes important in the ionization losses.

Dnestrovskiĭ and Kostomarev have considered the radiation formed in the flight of a charge through a circular aperture in an infinite ideally-conducting plane.[9] The results obtained by them for the case of an ultrarelativistic electron are in agreement with the present results in that the total radiation energy is proportional to the particle energy. The coefficient of proportionality obtained by these authors is the ratio of the classical electron radius to the dimensions of the aperture and is several orders of magnitude smaller than the completely natural coefficient of proportionality obtained from Eq. (11).

The author wishes to express his gratitude for valuable discussions to Prof. V. L. Ginzburg, L. D. Landau, I. Ya. Pomeranchuk and also A. Ts. Amatuni, B. M. Bolotovskiĭ, I. I. Gol'dman and G. S. Saakyan.

*In view of the fact that Eq. (19) (with the exception of an additive constant) can be reduced to the usual expression for ionization losses, it would appear that braking forces do not operate on the particle in the slab. However account must be taken of the fact that the total field carried by the particle is different in vacuum and in the slab. When this factor is considered it can be shown that the braking force acts on the particle only in the slab.

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Translated by H. Lashinsky
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