

COUPLED MAGNETOELASTIC OSCILLATIONS IN ANTIFERROMAGNETICS

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A phenomenological theory of coupled magnetoelastic oscillations in antiferromagnetics is given (the coupling between elastic and magnetic waves is due to magnetostriction and spontaneous magnetization). The velocities of sound in the antiferromagnetic are determined; they are found to depend on the magnetization and the applied magnetic field. The absorption coefficient of sound is found.

1. In an elastically strained antiferromagnetic, because of magnetostriction and because of ponderomotive action due to the presence of spontaneous magnetization, there should occur a coupling between elastic and magnetic (spin) waves. When the conductivity of the medium is high, the coupled magnetoelastic waves that thus originate are similar to the waves that can propagate in metals in the presence of an external magnetic field. The interaction between magnetic and elastic waves leads to a change in the velocity of sound and to an additional sound absorption. These changes are especially large at resonance, when the frequency and wave vector of the sound wave coincide with the frequency and wave vector of the magnetic wave. The present work is devoted to a treatment of these problems for the case of an antiferromagnet; the treatment is analogous to that given earlier by Akhiezer, Bar'yakhtar, and the author for ferromagnets.<sup>1</sup>

2. The free energy of an antiferromagnet, with account taken of the coupling between acoustic and magnetic oscillations, is<sup>2</sup>

$$\begin{aligned} \mathcal{H} = \int dV \left\{ \frac{1}{2} \alpha \left[ \left( \frac{\partial M_1}{\partial x_k} \right)^2 + \left( \frac{\partial M_2}{\partial x_k} \right)^2 \right] + \alpha' \frac{\partial M_1}{\partial x_k} \frac{\partial M_2}{\partial x_k} + \beta(M_1, M_2) \right. \\ \left. - (M_1 + M_2) H_0 + \frac{h^2 + ed}{8\pi} + \frac{1}{2} \rho \dot{\mathbf{u}}^2 \right. \\ \left. + \frac{1}{2} \lambda_{iklm} u_{ik} u_{lm} + F_{lm}(M_1, M_2) u_{lm} \right\}, \end{aligned} \quad (1)$$

where the last term in curly brackets is the magnetostrictive energy;  $\beta(M_1, M_2)$  includes the anisotropy energy and the part of the exchange energy not related to nonuniformities of the magnetic moments  $M_1$  and  $M_2$  of the sublattices;  $\alpha$  and  $\alpha'$  are exchange integrals. The remaining symbols are the same as in reference 1. Just as in the theory of ferromagnetism, the constants  $\alpha$  and  $\alpha'$  can be determined from the specific heat

or the temperature dependence of the magnetic susceptibility of the antiferromagnet. In order of magnitude, they are determined by the Curie temperature  $\Theta_C$  in the following manner:  $\alpha' \sim \alpha \sim \Theta_C a^2 / g M_0 \hbar$ .

The equations of motion of the magnetic moments  $M_1$  and  $M_2$  have the form

$$\begin{aligned} \frac{\partial M_1}{\partial t} + \frac{\partial}{\partial x_k} (M_1 \dot{u}_k) = g [M_1 \times H_1^{(e)}] - \frac{\lambda}{M_1^2} [M_1 \times [M_1 \times H_1^{(e)}]], \\ \frac{\partial M_2}{\partial t} + \frac{\partial}{\partial x_k} (M_2 \dot{u}_k) = g [M_2 \times H_2^{(e)}] - \frac{\lambda}{M_2^2} [M_2 \times [M_2 \times H_2^{(e)}]], \end{aligned} \quad (2)$$

where  $H_1^{(e)}$  and  $H_2^{(e)}$  are the effective magnetic fields acting on the magnetic moments of the first and second sublattices, respectively. To Eqs. (2) must be added Maxwell's equations and the equations of elasticity,

$$\begin{aligned} \text{curl } \mathbf{h} = \frac{1}{c} \frac{\partial \mathbf{d}}{\partial t} + \frac{4\pi}{c} \mathbf{j}, \quad \text{curl } \mathbf{e} = -\frac{1}{c} \frac{\partial}{\partial t} (\mathbf{h} + 4\pi \mathbf{M}), \\ \mathbf{j} = \sigma (\mathbf{e} + \frac{1}{c} [\dot{\mathbf{u}} \times \mathbf{B}]), \quad \rho \ddot{\mathbf{u}} = \mathbf{f}, \end{aligned} \quad (3)$$

where  $\mathbf{f}$  is the force per unit volume of the medium,  $\mathbf{M} = M_1 + M_2$ , and  $\mathbf{B} = \mathbf{H} + 4\pi \mathbf{M}$ . We shall find the values of  $H_1^{(e)}$ ,  $H_2^{(e)}$ , and  $\mathbf{f}$  from the requirement that the free energy (1) be conserved by virtue of Eqs. (2) and (3).

On carrying out calculations analogous to those carried out in reference 1, we get

$$\begin{aligned} H_1^{(e)} = H_0 - \partial \beta / \partial M_1 + \mathbf{h} - G_1 + \alpha \Delta M_1 + \alpha' \Delta M_2, \\ H_2^{(e)} = H_0 - \partial \beta / \partial M_2 + \mathbf{h} - G_2 + \alpha \Delta M_2 + \alpha' \Delta M_1, \\ f_i = \frac{\partial \sigma_{ik}}{\partial x_k} + \frac{1}{c} [\mathbf{j} \times \mathbf{B}]_i + M_1 \frac{\partial}{\partial x_i} H_1^{(e)} + M_2 \frac{\partial}{\partial x_i} H_2^{(e)}, \end{aligned} \quad (4)$$

where

$$\begin{aligned} G_1 = u_{lm} \partial F_{lm} / \partial M_1, \quad G_2 = u_{lm} \partial F_{lm} / \partial M_2, \\ \sigma_{ik} = \lambda_{iklm} u_{lm} + F_{ik}(M_1, M_2). \end{aligned}$$

An expression for  $d\mathcal{H}/dt$  is given by the formula

$$\begin{aligned} \frac{d\mathcal{K}}{dt} = & - \int \left\{ \frac{j^2}{\sigma} + \frac{\lambda}{M^2} [\mathbf{M}_1 \times \mathbf{H}_1^{(e)}]^2 + \frac{\lambda}{M^2} [\mathbf{M}_2 \times \mathbf{H}_2^{(e)}]^2 \right\} dV \\ & + \oint dS_k \left\{ \frac{c}{4\pi} [\mathbf{h} \times \mathbf{e}]_k + \alpha \left( \dot{\mathbf{M}}_1 \frac{\partial M_1}{\partial x_k} + \dot{\mathbf{M}}_2 \frac{\partial M_2}{\partial x_k} \right) \right. \\ & + \alpha' \left( \dot{\mathbf{M}}_1 \frac{\partial M_2}{\partial x_k} + \dot{\mathbf{M}}_2 \frac{\partial M_1}{\partial x_k} \right) \\ & \left. + \dot{u}_i [\sigma_{ik} + \delta_{ik} (M_1 \mathbf{H}_1^{(e)} + M_2 \mathbf{H}_2^{(e)})] \right\}. \end{aligned} \quad (5)$$

From formula (5) can be found the coefficient of absorption of the magnetoelastic oscillations,

$$\Gamma = - \frac{1}{\mathcal{K}} \frac{d\overline{\mathcal{K}}}{dt},$$

where  $\overline{d\mathcal{K}/dt}$  is the time-average value of the volume integral in (5).

\* According to Eq. (2), in the linear approximation in the quantities  $\mathbf{u}$ ,  $\boldsymbol{\mu}_1 = \mathbf{M}_1 - \mathbf{M}_{10}$ , and  $\boldsymbol{\mu}_2 = \mathbf{M}_2 - \mathbf{M}_{20}$ , and for small  $\lambda$ ,

$$g[\mathbf{M}_1 \times \mathbf{H}_1^{(e)}] = \partial \boldsymbol{\mu}_{1\perp} / dt, \quad g[\mathbf{M}_2 \times \mathbf{H}_2^{(e)}] = \partial \boldsymbol{\mu}_{2\perp} / dt,$$

therefore,

$$\frac{d\overline{\mathcal{K}}}{dt} = - \int \left\{ \frac{j^2}{\sigma} + \frac{\lambda}{(gM)^2} \overline{\boldsymbol{\mu}_{1\perp}^2} + \frac{\lambda}{(gM)^2} \overline{\boldsymbol{\mu}_{2\perp}^2} \right\} dV. \quad (6)$$

3. We now consider the system of equations (2) – (3). First of all, this system must be linearized. For simplicity we shall suppose that the medium is isotropic in its magnetostrictive properties. This condition means that  $F_{ik}(\mathbf{M}_1, \mathbf{M}_2)$  has the form

$$\begin{aligned} F_{ik}(\mathbf{M}_1, \mathbf{M}_2) = & \delta_1 (M_{1i} M_{1k} + M_{2i} M_{2k}) \\ & + \delta_2 (M_{1i} M_{2k} + M_{1k} M_{2i}) \\ & + \delta_{ik} (\delta_3 M_1 M_2 + \delta_4 (M_1^2 + M_2^2)). \end{aligned} \quad (7)$$

We further suppose that the quantities,  $\delta_1$ ,  $\delta_2$ ,  $\delta_3$ , and  $\delta_4$  are independent of  $M_1^2$ ,  $M_2^2$ , and  $M_1 M_2$ . From (2), the equations of motion of the magnetic moments, in terms of the variables  $\boldsymbol{\eta} = \boldsymbol{\mu}_1 - \boldsymbol{\mu}_2$  and  $\boldsymbol{\mu} = \boldsymbol{\mu}_1 + \boldsymbol{\mu}_2$ , have the form

$$\begin{aligned} \partial \boldsymbol{\eta} / \partial t + 2M_0 \mathbf{n} \operatorname{div} \dot{\mathbf{u}} \\ = gM_0 \mathbf{n} \times [\mathbf{H}_1^{(e)} + \mathbf{H}_2^{(e)}] - \lambda \mathbf{n} \times [\mathbf{n} \times (\mathbf{H}_1^{(e)} - \mathbf{H}_2^{(e)})], \\ \partial \boldsymbol{\mu} / \partial t = gM_0 \mathbf{n} \times [\mathbf{H}_1^{(e)} - \mathbf{H}_2^{(e)}] \\ - \lambda \mathbf{n} \times [\mathbf{n} \times (\mathbf{H}_1^{(e)} + \mathbf{H}_2^{(e)})]. \end{aligned} \quad (8)$$

From Eq. (8) it follows that

$$\boldsymbol{\mu} \mathbf{n} = 0, \quad \boldsymbol{\eta} \mathbf{n} = -2M_0 \operatorname{div} \dot{\mathbf{u}}.$$

If we treat the body as isotropic in its elastic properties, and if we introduce the notation

$$\tilde{c}_i^2 = c_i^2 + (M_0^2/\rho) \{2\beta + a + 4(\delta_3 - 2\delta_4) + 2(\alpha' - \alpha) \Delta\}$$

(where  $a$  and  $\beta$  are constants related to exchange and to anisotropy), we get the following equation for the displacement vector  $\mathbf{u}$ :

$$\begin{aligned} \ddot{\mathbf{u}} = & c_i^2 \Delta \mathbf{u} + (\tilde{c}_i^2 - c_i^2) \nabla \operatorname{div} \mathbf{u} + \frac{1}{\rho c} [\mathbf{j} \times \mathbf{B}] \\ & - \frac{4M_0^2}{\rho} (\delta_1 - \delta_2) \nabla n_i u_{ik} n_k \\ & + \frac{M_0}{\rho} (\delta_1 - \delta_2) (\mathbf{n} \operatorname{div} \boldsymbol{\eta} + n_k \frac{\partial \eta}{\partial x_k}). \end{aligned} \quad (9)$$

The equations of motion of the magnetic moments, in expanded form, are

$$\begin{aligned} \frac{\partial \boldsymbol{\eta}}{\partial t} + 2M_0 \mathbf{n} \operatorname{div} \dot{\mathbf{u}} \\ = gM_0 \mathbf{n} \times [2\mathbf{h} - (2\gamma + \beta) \boldsymbol{\mu} - \frac{H_0}{M_0} \boldsymbol{\eta} + (\alpha + \alpha') \Delta \boldsymbol{\mu}] \\ - \lambda \mathbf{n} \times \left\{ \mathbf{n} \times \left[ -\beta \boldsymbol{\eta} - \frac{H_0}{M_0} \boldsymbol{\mu} + (\alpha - \alpha') \Delta \boldsymbol{\eta} \right. \right. \\ \left. \left. - 4M_0 (\delta_1 - \delta_2) \hat{\mathbf{u}} \mathbf{n} \right] \right\}, \\ \frac{\partial \boldsymbol{\mu}}{\partial t} = gM_0 \mathbf{n} \times \left[ -\beta \boldsymbol{\eta} - \frac{H_0}{M_0} \boldsymbol{\mu} + (\alpha - \alpha') \Delta \boldsymbol{\eta} - 4M_0 (\delta_1 - \delta_2) \hat{\mathbf{u}} \mathbf{n} \right] \\ - \lambda \mathbf{n} \times \left\{ \mathbf{n} \times \left[ 2\mathbf{h} - (2\gamma + \beta) \boldsymbol{\mu} - \frac{H_0}{M_0} \boldsymbol{\eta} + (\alpha + \alpha') \Delta \boldsymbol{\mu} \right] \right\}, \end{aligned} \quad (10)$$

where  $\hat{\mathbf{u}} \mathbf{n}$  is a vector with the components  $u_{ik} n_k$ , and where  $\gamma$  is an exchange constant. The magnetic field  $\mathbf{H}_0$  is assumed to be directed along an axis of easiest magnetization  $\mathbf{n}$ .

Henceforth we shall treat in detail only the case in which the wave is propagated along the axis of easiest magnetization ( $\mathbf{k} \parallel \mathbf{n}$ ). As usual, we seek a solution of the form  $\exp \{-i(\omega t - \mathbf{k} \cdot \mathbf{x})\}$ .

Taking the component of Eqs. (9) and (10) parallel to the vector  $\mathbf{n}$ , we get

$$\begin{aligned} \mu_{\parallel} = 0, \quad \eta_{\parallel} + 2iM_0 k u_{\parallel} = 0, \\ (\omega^2 - k^2 \tilde{c}_i^2) u_{\parallel} + (2iM_0/\rho) (\delta_1 - \delta_2) k \eta_{\parallel} \\ + (4M_0^2/\rho) (\delta_1 - \delta_2) k^2 u_{\parallel} = 0. \end{aligned}$$

Hence we find the frequency and the phase velocity of longitudinal sound:

$$\omega^2 = v^2 k^2 = k^2 \{ \tilde{c}_i^2 - (8M_0^2/\rho) (\delta_1 - \delta_2) \}, \quad (11)$$

where

$$\tilde{c}_i^2 = c_i^2 + (M_0^2/\rho) \{2\beta + a + 4(\delta_3 - 2\delta_4) - 2(\alpha' - \alpha)k^2\}. \quad (11')$$

The dependence of the phase velocity on the wave vector  $\mathbf{k}$  through the term  $2(M_0^2/\rho)(\alpha' - \alpha)k^2$  in formula (11') can be neglected in the acoustic frequency range:

$$\omega \ll c_l/\sqrt{\alpha} \sim (\Theta_l/\hbar)(g\hbar M_0/\Theta_c)^{1/2} \sim 4 \cdot 10^{11} \text{ sec}^{-1}$$

for  $\Theta_l = c_l \hbar/a \approx 3 \times 10^{-14}$  ( $\Theta_l \sim 300^\circ\text{K}$ ),  $M_0 \sim 10^3 \text{ G}$ , and  $\Theta_c \sim 500^\circ\text{K}$ .

Equations (9) and (10), with Maxwell's equations taken into account, lead to the following dispersion equation for magnetoelastic oscillations that correspond to transverse sound:

$$\begin{aligned} & (\omega^2 - k^2 \tilde{c}_t^2) \{ (\omega - gH_0)^2 - \Omega\Omega_1 + i \frac{\lambda}{gM_0} \omega (\Omega + \Omega_1) \\ & - \left( \frac{\lambda}{gM_0} \right)^2 (\Omega\Omega_1 - g^2 H_0^2) \} + \frac{2M_0^2}{\rho} (\delta_1 - \delta_2) k^2 gM_0 \\ & \times \left\{ \left[ (\delta_2 - \delta_1) \Omega + gH_0 \frac{H_0}{M_0} \zeta \right] \left[ 1 + \left( \frac{\lambda}{gM_0} \right)^2 \right] \right. \\ & \left. + \omega \left[ \frac{i\lambda}{gM_0} (\delta_1 - \delta_2) - \frac{H_0}{M_0} \zeta \right] \right\} \\ & + \frac{2M_0^2}{\rho} \zeta k^2 gH_0 \left\{ \left[ gH_0 (\delta_1 - \delta_2) - \Omega_1 \frac{H_0}{M_0} \zeta \right] \left[ 1 + \left( \frac{\lambda}{gM_0} \right)^2 \right] \right. \\ & \left. + \omega \left[ \frac{i\lambda}{gM_0} \frac{H_0}{M_0} \zeta + \delta_2 - \delta_1 \right] \right\} = 0, \end{aligned} \quad (12)$$

$$\Omega = gM_0 \{ \beta + 2\gamma - 8\pi\zeta + (\alpha + \alpha') k^2 \},$$

$$\Omega_1 = gM_0 \{ \beta + (\alpha - \alpha') k^2 \},$$

$$\tilde{c}_t^2 = c_t^2 - H_0^2 \zeta / 4\pi\rho, \quad \zeta = \{ c^2 k^2 / 4\pi i \omega \sigma - 1 \}^{-1}. \quad (13)$$

In the absence of coupling between acoustic and magnetic oscillations ( $M_0^2/\rho c_t^2 \rightarrow 0$ ), we arrive at a dispersion equation for a magnetic wave with account taken of absorption through relaxation processes described by the constant  $\lambda$  and the conductivity  $\sigma$ :

$$v^2 = \frac{\omega^2}{k^2} = \tilde{c}_t^2 \left\{ 1 - \frac{2M_0^2}{\rho c_t^2} \frac{gH_0 [2(\delta_1 - \delta_2) (\omega - gH_0) - \Omega_1 H_0 / M_0 - (\Omega M_0 / H_0) (\delta_1 - \delta_2)^2]}{(\omega - gH_0)^2 - \Omega\Omega_1} \right\}. \quad (16')$$

When  $M_0 = \delta_1 = \delta_2 = 0$ , formulas (11) and (16') reduce to the formulas for the velocity of sound in a metal in the presence of an external magnetic field.

Formulas (16) and (16') are valid when the sound frequency  $\omega$  is not close to  $gH_0 \pm \sqrt{\Omega\Omega_1}$ ; in the contrary case, "entanglement" of the acoustic and magnetic oscillations occurs, just as in the case of a ferromagnet.

Inclusion of the terms containing  $\lambda$  and  $\sigma$  leads, first, to an additional change of the phase velocity of the waves; and second, to damping of the oscillations. We consider first the change of the phase velocity of transverse sound.

The dispersion equation (12) leads in the case  $\sigma = 0$  to the following formula instead of formula (16):

$$\begin{aligned} & (\omega - gH_0)^2 - \Omega\Omega_1 + i(\lambda/gM_0)\omega(\Omega + \Omega_1) \\ & - (\lambda/gM_0)^2(\Omega\Omega_1 - g^2 H_0^2) = 0. \end{aligned} \quad (14)$$

Hence we find for  $\lambda \ll gM_0$

$$\begin{aligned} \omega & = gH_0 \pm \sqrt{\Omega\Omega_1} \\ & - i(\lambda/2gM_0)(\Omega + \Omega_1) \{ 1 \pm gH_0/\sqrt{\Omega\Omega_1} \}. \end{aligned} \quad (14')$$

Formula (14') shows that the coefficient of absorption of a magnetic wave by virtue of relaxation processes when  $\sigma = 0$  is

$$\Gamma = -2\text{Im} \omega|_{\sigma=0} = (\lambda/gM_0)(\Omega + \Omega_1)(1 \pm gH_0/\sqrt{\Omega\Omega_1}).$$

Absorption of the energy of the magnetic wave will take place if  $\Gamma > 0$ , i. e. if

$$gH_0 < \sqrt{\Omega\Omega_1}. \quad (15)$$

This relation is the condition for existence of a ground state of an antiferromagnetic with  $\mathbf{M}_{10} = -\mathbf{M}_{20}$  along  $\mathbf{H}_0$ . At fields  $gH_0 > \sqrt{\Omega\Omega_1}$ , the ground state corresponds to a structure with  $\mathbf{M}_{10} = -\mathbf{M}_{20}$  perpendicular to  $\mathbf{H}_0$ .<sup>3</sup>

When  $\sigma = \lambda = 0$  and  $M_0^2/\rho c_t^2 \ll 1$ , formula (12) leads to the following expression for the phase velocity of transverse sound:

$$v^2 = \frac{\omega^2}{k^2} = c_t^2 \left\{ 1 + \frac{2M_0^2}{\rho c_t^2} (\delta_1 - \delta_2)^2 \frac{\Omega gM_0}{(\omega - gH_0)^2 - \Omega\Omega_1} \right\}. \quad (16)$$

When  $\sigma = \infty$ ,  $\lambda = 0$ , and  $M_0^2/\rho c_t^2 \ll 1$ , the same formula (12) leads to the following expression for the phase velocity of transverse sound:

$$\begin{aligned} v^2 & = c_t^2 \left\{ 1 + \frac{2M_0^2}{\rho c_t^2} (\delta_1 - \delta_2)^2 \Omega gM_0 \right. \\ & \left. \times \frac{(\omega - gH_0)^2 - \Omega\Omega_1}{[(\omega - gH_0)^2 - \Omega\Omega_1]^2 + (\lambda/gM_0)^2 \omega^2 (\Omega + \Omega_1)^2} \right\}. \end{aligned} \quad (17)$$

The relative change of sound velocity at resonance,  $\Delta v/v$ , for  $\lambda/gM_0 \sim 10^{-1}$ ,  $gM_0/\omega \sim 10^2$ , and  $\delta_1 \sim \delta_2 \sim 1$ , is about 1%, as can be seen from formula (17).

We note that in the case of antiferromagnets, the phenomenon of resonance (reinforcement of the coupling between acoustic and magnetic oscillations) takes place both for  $\sigma = 0$  and for  $\sigma = \infty$ , as can be seen from formula (13) in view of the large value of the exchange constant  $\gamma$ . But in contrast to ferromagnets, the resonance frequencies can be made small in comparison with  $gM_0$  ( $\sim 10^{10} \text{ sec}^{-1}$ ) only

in strong magnetic fields  $H_0$ :

$$gH_0/\sqrt{\Omega\Omega_1} - 1 \approx H_0/M_0\sqrt{2\beta\gamma} - 1 \ll 1. \quad (18)$$

From this it is clear that these fields must be on the scale of  $M_0\sqrt{2\beta\gamma} \sim 10M_0 \sim 10^4$  G. Smaller values of the field  $H_0$  may be obtained if antiferromagnets are chosen with small anisotropy and a small exchange constant  $\gamma$ .

4. The absorption coefficient  $\Gamma$  of magnetoelastic oscillations in an antiferromagnet can be determined from formula (6) in exactly the same way as for a ferromagnet or ferroelectric.

However, it is now easier to determine the absorption coefficient from the exact dispersion equation (12). The absorption coefficient  $\Gamma$  is found from the solution  $\omega$  of the dispersion equation by the formula

$$\Gamma = -2 \operatorname{Im} \omega. \quad (19)$$

Formulas (12) and (19) lead to the following expression for the coefficient of absorption of magnetoelastic oscillations by virtue of relaxation processes described by the constant  $\lambda$ :

$$\Gamma_\lambda = 2\omega^2 \frac{M_0^2}{\rho c_l^2} (\delta_1 - \delta_2)^2 \lambda \times \frac{\Omega^2 + (\omega - gH_0)^2}{[(\omega - gH_0)^2 - \Omega\Omega_1]^2 + (\lambda/gM_0)^2 \omega^2 (\Omega + \Omega_1)^2}. \quad (20)$$

The absorption is especially large at resonance, when  $\omega = gH_0 - \sqrt{\Omega\Omega_1}$  (magnetic fields of order  $M_0\sqrt{2\beta\gamma}$ ). The value of the absorption at resonance is

$$\Gamma_\lambda = gM_0 \frac{M_0^2}{\rho c_l^2} (\delta_1 - \delta_2)^2 \frac{gM_0}{\lambda} \frac{\Omega}{\Omega + \Omega_1} \approx gM_0 \frac{M_0^2}{\rho c_l^2} (\delta_1 - \delta_2)^2 \frac{gM_0}{\lambda}, \quad (20')$$

since  $\Omega_1 \ll \Omega$  in view of the large value of the exchange constant  $\gamma$  by comparison with the anisotropy constant  $\beta$ .

We shall compare the absorption coefficient at resonance with the absorption coefficient resulting from heat conductivity. The latter is determined by the formula<sup>4</sup>

$$\gamma \approx \omega^2 C^{-2} \kappa T \alpha_T^2 \rho = (gH_0 - \sqrt{\Omega\Omega_1})^2 C^{-2} \kappa T \alpha_T^2 \rho,$$

where  $\kappa$  is the coefficient of heat conductivity,  $T$  is the temperature in degrees,  $C$  is the volume specific heat, and  $\alpha_T$  is the coefficient of thermal expansion.

The ratio  $\Gamma_\lambda/\gamma$  is equal to

$$\Gamma_\lambda/\gamma \sim (M_0^2/\rho c_l^2) (\delta_1 - \delta_2)^2 C^2 / \lambda \kappa T \alpha_T^2 \rho.$$

For  $\alpha_T = 10^{-5}$ ,  $C = 10^6$ ,  $\kappa = 10^6$ ,  $T \sim 10^2$  K,  $\lambda/gM_0 \sim 10^{-1}$ ,  $\delta_1 \sim \delta_2 \sim 1$ ,  $c_l \sim 5 \times 10^5$ , and  $M_0 \sim 500$ , this ratio amounts to  $\Gamma/\gamma \sim 10^2$ .

The amount of absorption of magnetoelastic oscillations by virtue of the conductivity  $\sigma$ , for small values of  $\sigma$ , is given by the formula

$$\Gamma_\sigma = \frac{\sigma}{\rho c^2} \frac{\{H_0 [(\omega - gH_0)^2 - \Omega\Omega_1] - 8\pi g M_0^2 (\omega - gH_0) (\delta_1 - \delta_2)\}^2}{[(\omega - gH_0)^2 - \Omega\Omega_1]^2}. \quad (21)$$

The absorption of sound by virtue of conductivity  $\sigma$  in the vicinity of resonance is obtained from formula (21) by the substitution

$$[(\omega - gH_0)^2 - \Omega\Omega_1]^2 \rightarrow [(\omega - gH_0)^2 - \Omega\Omega_1]^2 + \left| \frac{\lambda}{gM_0} \omega (\Omega + \Omega_1) + gM_0 \Omega_1 \frac{32\pi^2 \omega \sigma}{c^2 k^2} \right|^2.$$

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