

$$\frac{\partial}{\partial t} (\rho_n v_{ni} + \rho_s v_{si}) + \frac{\partial}{\partial x_k} \left\{ \rho_n v_{ni} v_{nk} + \rho_s v_{si} v_{sk} + p \delta_{ik} - \eta \left( \frac{\partial v_{ni}}{\partial x_k} + \frac{\partial v_{nk}}{\partial x_i} - \frac{2}{3} \delta_{ik} \operatorname{div} \mathbf{v}_n \right) \right\} = 0, \quad (9)$$

$$\frac{\partial S}{\partial t} + \operatorname{div} \left[ S \mathbf{v}_n + \frac{1}{T} \left( \mathbf{q} - \frac{\mathbf{g}Z}{\rho} \right) \right] = \frac{R}{T}, \quad (10)$$

$$\frac{\partial \rho_s}{\partial t} + \operatorname{div} \rho_s \mathbf{v}_s = - \frac{\Lambda m}{2\hbar} \left\{ \frac{(\mathbf{v}_n - \mathbf{v}_s)^2}{2} + \left( \frac{\partial \epsilon}{\partial \rho_s} \right)_{\rho, S, c} \right\} \rho_s. \quad (11)$$

The dissipative function of the liquid is

$$R = \frac{\hbar \Lambda}{2m\rho_s} [\operatorname{div} \rho_s (\mathbf{v}_s - \mathbf{v}_n)]^2 + \frac{2\Lambda m}{\hbar} \left[ \frac{1}{2} (\mathbf{v}_s - \mathbf{v}_n)^2 + \left( \frac{\partial \epsilon}{\partial \rho_s} \right)_{\rho, S, c} \right]^2 \rho_s + \frac{1}{2} \eta \left( \frac{\partial v_{ni}}{\partial x_k} + \frac{\partial v_{nk}}{\partial x_i} - \frac{2}{3} \delta_{ik} \frac{\partial v_{kl}}{\partial x_l} \right)^2 - \mathbf{q} \frac{\nabla T}{T} - \mathbf{g} T \nabla \frac{Z}{\rho T}.$$

Formally Eqs. (6) to (10) are the same as the usual set of equations for the hydrodynamics of solutions of extraneous particles in helium II except, however, that  $\rho_s$  is not given in them but is an independent quantity for which the approach to its equilibrium value is described by the additional Eq. (11). In the given equations the quantity

$$(\partial \epsilon / \partial \rho_s)_{\rho, S, c} + (\partial \epsilon / \partial \rho)_{\rho, S, c} - Zc/\rho,$$

where  $Z = (\partial \epsilon / \partial c)_{\rho, S, \rho_s}$ , plays the role of the chemical potential of  $\text{He}^4$  in the solution.

The parameter  $\Lambda$  entering into the equations could be estimated from a comparison of the absorption coefficient for first sound evaluated from Eqs. (6) to (11) with the measured value of the absorption coefficient in  $\text{He}^3$ - $\text{He}^4$  solutions near the  $\lambda$  point. There are not, however, at the present time any such experimental data. An estimate made by Pitaevskii<sup>2</sup> for pure helium II gives  $\Lambda \approx 15$ .

For definite applications of the theory it is necessary to know the function  $\epsilon(\rho, S, \rho_s, c)$ , which can be determined from experimental data on the dependence of the superfluid component of  $\text{He}^3$ - $\text{He}^4$  solutions near the  $\lambda$  point on  $p$ ,  $T$ ,  $c$ , and on its density.

In conclusion the author expresses his sincere gratitude to I. M. Khalatnikov and L. P. Pitaevskii for suggesting this topic and for valuable discussions.

<sup>1</sup>V. L. Ginzburg and L. P. Pitaevskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 1240 (1958), Soviet Phys. JETP **7**, 858 (1958).

<sup>2</sup>L. P. Pitaevskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 408 (1958), Soviet Phys. JETP **8**, 282 (1959).

<sup>3</sup>I. M. Khalatnikov, Usp. Fiz. Nauk **60**, 69 (1956), Fortschr. Phys. **5**, 287 (1957).

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### THE PROPAGATION OF OSCILLATIONS ALONG VORTEX LINES IN ROTATING HELIUM II\*

É. L. ANDRONIKASHVILI and D. S. TSAKADZE

Tbilisi State University

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ACCORDING to the theory developed by Feynman<sup>1</sup> on the basis of Onsager's hypothesis, there should appear in rotating helium II vortex lines parallel to the axis of rotation and running through the whole of the liquid. Experiments performed by us to confirm this hypothesis<sup>2</sup> showed that rotating helium II possesses, when twirled around, a quite appreciable elasticity. The presence of such an elasticity is also confirmed by the experiments of Hall.<sup>3</sup>

In the interpretation of these experiments we assumed that transverse elastic waves were propagated along the vortex lines. This point of view was confirmed by Hall (private communication), who observed a periodic change in the frequency of the oscillation of a light disc suspended in rotating helium II under such conditions that the liquid level above it was changing continuously. At the same time the length of the vortices, which on the one side were fastened to the surface of the disc and on the other side to the free surface of the liquid, was also changing, as assumed by Hall. The periodic changes in the frequency of the oscillations were within a range of one per cent.

In contradistinction to Hall, we measured the magnitude of the logarithmic decrement of the damping  $\delta$  of the oscillations of an elastically suspended disc, which were performed at the same time as the rotation together with the helium II. The damping decrement was measured by a method described earlier.<sup>4</sup> The time dependence of the distance between the disc and the liquid surface was studied by a periscope system of mirrors and a cathetometer.

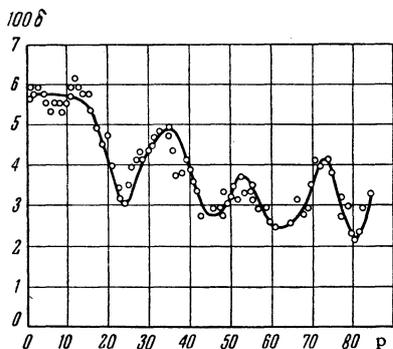


FIG. 1

We show in Fig. 1 the results of our measurement performed using one disc covered on both sides with grains of linear dimensions  $l \approx 50 \mu$  (the number of periods is along the abscissa). The measurement took place at  $1.38^\circ \text{K}$  and the rotational frequency was  $\omega = 55 \times 10^{-3} \text{sec}^{-1}$ . The level of the helium above the surface of the disc changed rapidly thanks to intensive illumination (rate of evaporation  $3.6 \times 10^{-2} \text{mm/min}$ ). The initial part of the curve corresponds to the induced swinging of the oscillating system. In Fig. 2 we show the results of an experiment performed under the same conditions as the previous one, except for the fact that the rate of change of the liquid level above the disc was much slower in this case ( $0.5 \text{mm/sec}$ ) because the illumination was switched off.

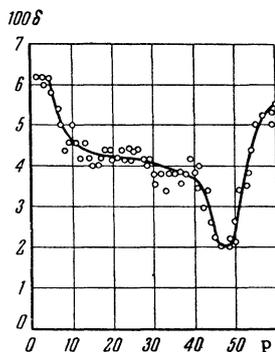


FIG. 2

From the curves obtained it is clear that when the rate of evaporation of the helium above the disc is changed the character of the periodicity of the damping is also changed. This result may be explained by assuming that a standing transverse wave appears in the vortex lines when the disc oscillates. The distance between two adjacent resonances corresponded to a lowering of the level by  $\sim 0.065 \text{cm}$ .

The authors express their thanks to Yu. G. Mamaladze for his active participation in the discussion of the experimental results in all

stages of their development, and to T. M. Shul'ts, K. B. Mesoed, and I. M. Chkheidze for their help in performing the experiment.

\*Reported at the Fifth All-Union Conference on Low Temperature Physics at Tbilisi, October 1958.

<sup>1</sup>R. P. Feynman, *Progress in Low Temperature Physics*, North Holland Publishing Company, Amsterdam, 1955, Vol. 1, p. 17.

<sup>2</sup>D. S. Tsakadze and E. L. Andronikashvili, *Сообщения АН ГрузССР*, (Reports, Acad. Sci. Georgian S.S.R.) **20**, 667 (1958).

<sup>3</sup>H. E. Hall, *Proc. Roy. Soc. (London)* **A245**, 546 (1958).

<sup>4</sup>Andronikashvili, Mamaladze, and Tsakadze, *Тр. Ин-та Физики АН ГрузССР* (Trans. Phys. Inst. Acad. Sci. Georgian S.S.R.) **7**, No. 1 (1959).

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### THE USE OF A SUPERCONDUCTING RING FOR REGISTERING THE PHASE TRANSITION IN LIQUID HELIUM

B. N. ESEL'SON and A. D. SHVETS

Physico-Technical Institute, Academy of Sciences, Ukrainian S.S.R.

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IT is often desirable, when investigating the properties of  $\text{He}^3$ - $\text{He}^4$  mixtures, especially the properties of rotating helium, to have a thermometer which does not need external leads, is sufficiently sensitive, and dissipates a negligible amount of heat. These requirements are satisfied by a superconducting ring,\* in which a current is induced by switching on and off an external magnetic field higher than the critical value.

When the temperature dependence of the current in the ring is known, the determination of the temperature of the liquid helium reduces to a measurement of the magnetic field of the ring by some instrument located outside the apparatus.

In experiments where there is a steady increase in temperature and, consequently, a continuous decrease of current in the ring, a stationary induction coil connected to a galvanometer can

Vacuum Tubes (see Methods and Instruments)

Viscosity (see Liquids)

Wave Mechanics (see Quantum Mechanics)

Work Function (see Electrical Properties)

#### X-rays

Anomalous Heat Capacity and Nuclear Resonance in Crystalline Hydrogen in Connection with New Data

on Its Structure. S. S. Dukhin — 1054L.

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### ERRATA TO VOLUME 9

On page 868, column 1, item (e) should read:

(e). Ferromagnetic weak solid solutions. By way of an example, we consider the system Fe-Me with A2 lattice, where Me = Ti, V, Cr, Mn, Co, and Ni. For these the variation of the moment  $m$  with concentration  $c$  is

$$dm/dc = (Nd)_{Me} \mp 0.642 \{ 8 (2.478 - R_{Me}) + 6 |2.861 - R_{Me}| \mp [ 8(2.478 - R_{Fe}) + 6(2.861 - R_{Fe}) ] \},$$

where the signs  $-$  and  $+$  pertain respectively to ferromagnetic and paramagnetic Me when in front of the curly brackets, and to metals of class 1 and 2 when in front of the square brackets. The first term and the square brackets are considered only for ferromagnetic Me. We then have  $dm/dc = -3$  ( $-3.3$ ) for Ti,  $-2.6$  ( $-2.2$ ) for V,  $-2.2$  ( $-2.2$ ) for Cr,  $-2$  ( $-2$ ) for Mn,  $0.7$  ( $0.6$ ) for Ni, and  $1.2$  ( $1.2$ ) for Co; the parentheses contain the experimental values.

### ERRATA TO VOLUME 10

Page	Reads	Should Read
224, Ordinate of figure	$10^{23}$	$10^{29}$
228, Column 1, line 9 from top	$3.6 \times 10^{-2}$ mm/min	0.36 mm/min
228, Column 1, line 16 from top	0.5 mm/sec	0.05 mm/min
329, Third line of Eq. (23a)	$+ (1/4 \cosh r + \dots$	$+ 1/4 (\cosh r + \dots$
413, Table II, line 2 from bottom	$-0.0924 \pm$	$-1.0924 \pm$
413, Table II, line 3 from bottom	$+1.8730 \pm$	$+0.8370 \pm$
479, Fig. 7, right, 1st line	92 hr	9.2 hr
499, Second line of Eq. (1.8)	$+\tilde{k} \sin^2 \alpha / \omega_N^2 + \langle c^2 \tilde{k}^2 \dots$	$+\left(\tilde{k}/\omega_H\right)^2 \sin^2 \alpha \langle c^2 \tilde{k}^2 \dots$
648, Column 1, line 18 from top	$18 \times 80$ mm	$180 \times 80$ mm
804, First line of Eq. (17)	$-1/3 (\alpha_x^2 \alpha_y^2 + \dots$	$\dots - 3 (\alpha_x^2 \alpha_y^2 + \dots$
967, Column 1, line 11 from top	$\sigma(N', \pi) \approx 46(N', N')$	$\sigma(N', \pi) > \sigma(N', N')$
976, First line of Eq. (10)	$= \frac{e^2}{3r^2c^4}$	$= \frac{e^2}{3\hbar^2c^2}$
978, First line of Eq. (23)	$\left[ \frac{(2\gamma^2 - 1)^2}{(\gamma^2 - 1) \sin^4(\theta/2)} \right]$	$\left[ \frac{(2\gamma^2 - 1)^2}{(\gamma^2 - 1)^2 \sin^4(\theta/2)} \right]$