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OPTICAL ANISOTROPY OF ATOMIC NUCLEI

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Concepts of molecular optics are extended to photonuclear reactions. The consequences of the existence of tensor polarizability of atomic nuclei, not yet discovered experimentally, are discussed. Various models of tensor polarizability are examined and estimates are given for the magnitudes of the effects that could be observed with present experimental capabilities.

1. INTRODUCTION

IN preceding studies[1,2] it has been shown that the cross section for the interaction of γ-ray quanta with nuclei can depend substantially upon the spin orientation of the nucleus with respect to the wave vector of the photon and that the detection of such dependence would furnish proof of the existence of a tensor component in the electric dipole polarizability of atomic nuclei. As already pointed out,[2] a possible model for such polarizability is contained in the successful interpretation of the giant resonance widths, given by Okamoto[3] and Danos;[4] in addition, other models of tensor polarizability are also possible.

The study of this new property of asymmetry in the electromagnetic radiation of atomic nuclei possesses indubitable interest. The object of the present work is a systematic formulation of the theory of the electric dipole polarizability of atomic nuclei, a discussion of experiments by means of which it might be possible to obtain information about the tensor component of the electric polarizability, and an estimate of the magnitude of the effects with the aid of two different nuclear models — the independent-particle model and the collective model.[3,4]

A theory of the polarizability of atomic nuclei can be constructed by analogy with the theory of molecular polarizability.[5] It should be remarked that, in general, many of the methods of molecular optics can be applied to photonuclear reactions. Thus, for example, it would undoubtedly be interesting to study the combination scattering of γ-ray quanta by nuclei or the effects of the depolarization of scattered radiation for the purpose of securing a series of important nuclear parameters. The scope of this article does not comprise the broad extension of the theory of the interaction of light with molecules to the interaction of γ-ray quanta with nuclei. We shall consider only the elastic scattering of γ-ray quanta by nuclei and the photonuclear reactions (total cross section for the absorption of dipole γ-ray quanta by nuclei).

2. GENERAL CONSIDERATIONS

As is well known, electric dipole absorption plays a principal role in the interaction of γ-ray quanta with nuclei in an energy range up to ~ 20 Mev. We shall therefore confine ourselves to this part of the interaction.

The possible existence of tensor polarization in nuclei signifies that the scattering amplitude may depend upon the spin of the system, i.e., the induced dipole moment of the nucleus can depend upon the orientation of the nucleus with respect to the electric field. The most general formula for the scattering amplitude, as a function of the nuclear spin operator $\hat{\mathbf{J}}$ and of the polarizations of the incident photon $\boldsymbol{\lambda}$ and the scattered photon $\boldsymbol{\lambda}'$, is written in the form

$$\hat{R} = R^{(1)}(\hat{\mathbf{J}}\boldsymbol{\lambda})(\hat{\mathbf{J}}\boldsymbol{\lambda}') + R^{(2)}(\hat{\mathbf{J}}\boldsymbol{\lambda}')(\hat{\mathbf{J}}\boldsymbol{\lambda}) + R^{(3)}(\boldsymbol{\lambda}'\boldsymbol{\lambda}) + T(\boldsymbol{\lambda}'\boldsymbol{\lambda}), \quad (1)$$

where $T = \omega Z^2 e^2/2\pi AM$ is the amplitude of Thomson scattering, which may make a substantial contribution (we assume that $\hbar = c = 1$ everywhere).* The amplitudes, $R^{(1)}$, $R^{(2)}$, $R^{(3)}$, being functions of the photon frequency, are proportional to the polarizability of the system. Thus, if only scalar polarizability of the system is present ($R^{(1)} = R^{(2)} = 0$), we have $R^{(3)} = -(\omega^3/2\pi)\,\alpha(\omega)$, where α is the electric dipole polarizability, through which

*Of course, we neglect Delbrück scattering and scattering by the magnetic moment.

the Rayleigh cross section for scattering by a small particle (nucleus) is expressed in accordance* with

$$d\sigma / d\Omega = \omega^4 |\alpha(\omega)|^2 (\boldsymbol{\lambda}'\boldsymbol{\lambda}).$$

The expression (1) can be simplified. In molecular optics the case usually considered is that for which the polarizability tensor is Hermitian, or $\hat{R}^+ = \hat{R}$. This case is of little interest for the nucleus, since its energy levels have large widths, owing to the strong interaction between the constituent particles. Therefore, $R^{(1)}$, $R^{(2)}$, $R^{(3)}$ have real and imaginary parts over a broad frequency range.

We shall utilize the invariance of the S-matrix under time reversal, which gives $R^{(1)} = R^{(2)}$. Thus, (1) may be rewritten in the form

$$\hat{R} = R^T \frac{3}{J(2J-1)} \left\{ \frac{1}{2} [(\hat{\mathbf{J}}\boldsymbol{\lambda}')(\hat{\mathbf{J}}\boldsymbol{\lambda}) + (\hat{\mathbf{J}}\boldsymbol{\lambda})(\hat{\mathbf{J}}\boldsymbol{\lambda}')] - \frac{1}{3} \hat{\mathbf{J}}^2 (\boldsymbol{\lambda}'\boldsymbol{\lambda}) \right\} + (R^S + T)(\boldsymbol{\lambda}'\boldsymbol{\lambda}). \quad (2)$$

The expediency of introducing R^T and R^S will become clear in the sequel. In accordance with references 1 and 2, we shall call $\alpha^S = -(2\pi/\omega^3)R^S$ and $\alpha^T = -(2\pi/\omega^3)R^T$ the scalar and tensor polarizabilities. Obviously, they are complex parameters. The imaginary and real parts of these parameters are not independent, but connected by dispersion relationships.

In order to express the imaginary part of the amplitude, we make use of the unitarity of the S-matrix:

$$i(\hat{R}^+ - \hat{R}) = \hat{R}\hat{R}^+. \quad (3)$$

We shall take the matrix element of matrix equation (3) for the state $(\alpha m j)\, 2\pi/\omega$, where m is the projection of the nuclear spin on the z axis and j is the vector projection of the photon polarization, while α represents all the remaining quantum numbers of the nuclear ground state; we get

$$i(2\pi/\omega)^2 [(m'j'\alpha | R^+ | \alpha m j) - (m'j'\alpha | R | \alpha m j)] = (2\pi/\omega)^2 \sum_N (m'j'\alpha | R | N)(N | R^+ | \alpha j m). \quad (4)$$

The summation includes all the states permissible from the viewpoint of the law of the conservation of energy. We define the cross-section operator $\hat{\sigma}$ as that for which the mean value corresponding to the polarized state of the target nuclei and the incident photons gives the cross section

$$\sigma = \mathrm{Sp}\, \hat{\rho}\hat{\sigma}$$

(here ρ is the density matrix). Substituting (2) into (4), we obtain

$$2\frac{4\pi^2}{\omega^2} \left\{ \frac{3}{J(2J-1)} \left(m' \left| \frac{1}{2}(\hat{J}_{j'}\hat{J}_j + \hat{J}_j\hat{J}_{j'}) - \frac{1}{3}\hat{J}^2 \delta_{jj'} \right| m\right) \mathrm{Im}\, R^T + \delta_{jj'} \mathrm{Im} R^S \right\} = (m'j' | \hat{\sigma} | mj). \quad (5)$$

Thus, the imaginary parts of R^T and R^S are simply related to the absorption cross section. At the same time Eq. (5) leads to a more general dependence of the absorption cross section upon the nuclear spin. For instance, let the polarization of the photon be given rigorously, i.e., the density matrix expressing the polarization state of the photon has the form $\rho = \delta_{jz}\delta_{j'z}$ (the z axis is oriented along the photon polarization). Then the cross section of the photon absorption is expressed in the form

$$\sigma = \mathrm{Sp}\, \hat{\rho}\hat{\sigma} = \frac{4\pi^2}{\omega^2} 2 \left\{ \frac{3(J+1)}{(J-1)} \left[\frac{\overline{J_z^2}}{J(J+1)} - \frac{1}{3} \right] \mathrm{Im}\, R^T + \mathrm{Im}\, R^S \right\}; \quad (6)$$

and for the unpolarized photon beam we get

$$\bar{\sigma} = \frac{8\pi^2}{\omega^2} \left\{ \frac{3(J+1)}{(2J-1)} \left[\frac{1}{6} - \frac{\overline{J_\varkappa^2}}{2J(J+1)} \right] \mathrm{Im}\, R^T + \mathrm{Im}\, R^S \right\}. \quad (7)$$

Here $\overline{J_\kappa^2}$ is the mean square of the spin projection on the wave vector of the photon.

For unoriented nuclei $[\rho_{m'm} = \delta_{m'm}/(2J+1)]$ the terms with R^T in formulas (6) and (7) vanish. Therefore, as evident from (6) and (7), to prove the existence of the tensor part of the polarizability of atomic nuclei it is necessary to demonstrate the dependence of the cross section for photon absorption upon the orientation of the nucleus.

From formula (7) it is easy to derive that

$$\mathrm{Im}\, R^T = \frac{2}{3} (\omega / 2\pi)^2 (\sigma_\perp - \sigma_\parallel),$$

where $\sigma_\parallel$ and $\sigma_\perp$ are the total absorption cross sections for γ-ray quanta when the nuclei are completely oriented parallel to the wave vector of the photons and perpendicular thereto, respectively. Having determined the imaginary part of R^T, it is possible to find its real part by using the dispersion relationships.

As noted earlier,[1] the influence of the R^T effect must be especially strong both on the absolute magnitude of the cross section for scattering by unoriented nuclei and on the azimuthal asymmetry of scattering by oriented nuclei. We present here more detailed estimates.

Let us find how the cross section for the elastic scattering of γ-ray quanta by nuclei depends upon the parameter R^T introduced by us or upon the tensor polarizability, which is proportional to R^T. We denote

*We stress the fact that we mean everywhere not the local polarizability, but the polarizability of the whole particle.

$$\hat{\varepsilon}_{ik} = \frac{3}{J(2J-1)}\left[\frac{1}{2}(\hat{J}_i\hat{J}_k + \hat{J}_k\hat{J}_i) - \frac{1}{3}\hat{J}^2\delta_{ik}\right].$$

Then the scattering cross-section operator is written

$$\hat{\sigma}_{ikjq} = (2\pi/\omega)^2 [R^{T*}\hat{\varepsilon}^*_{ik} + R'^*\delta_{ik}]\,[R^T\hat{\varepsilon}_{jq} + R'\delta_{jq}]. \quad (8)$$

Here $R' = R^S + T$.

Let us find the cross sections for scattering by unoriented nuclei. For this purpose we average (8) over the state $\rho_{m'm} = \delta_{m'm}/(2J+1)$. The mean value of the product $\hat{\epsilon}_{ik}\hat{\epsilon}_{jq}$ must have the form

$$\overline{\hat{\varepsilon}_{ik}\hat{\varepsilon}_{jq}} = A_1\delta_{ik}\delta_{jq} + A_2[\delta_{iq}\delta_{kj} + \delta_{kq}\delta_{ij}].$$

Finding the convolutions for the right-hand and left-hand parts of the tensors, first with respect to the indices $i = k$ and $j = q$, and then for $i = j$ and $k = q$, and utilizing the commutation relation for the components of the spin operator, we readily obtain

$$A_1 = -2A_2/3, \qquad A_2 = 3(J+1)(2J+3)/20J(2J-1).$$

The mean value of $\hat{\epsilon}_{ik}$ is equal to zero. The final result for the cross section, averaged over the orientation of the nuclei, but for a definite orientation of the polarization of the photon in its initial and terminal states, has the form

$$\frac{d\sigma}{d\Omega} = \frac{4\pi^2}{\omega^2}\left\{|R^T|^2\frac{3(J+1)(2J+3)}{20(2J-1)J}\left[1 + \frac{1}{3}(\boldsymbol{\lambda}'\boldsymbol{\lambda})^2\right] + |R'|^2(\boldsymbol{\lambda}'\boldsymbol{\lambda})^2\right\}. \quad (9)$$

Averaging over the photon polarizations in the initial state and summing over the final states, we get for the angular distribution of the scattered photons

$$\overline{\frac{d\sigma}{d\Omega}} = \frac{2\pi^2}{\omega^2}\left\{|R^T|^2\frac{(2J+3)(J+1)}{20J(2J-1)}(13 + \cos^2\theta) + |R'|^2(1 + \cos^2\theta)\right\}. \quad (10)$$

Thus, as already pointed out,[1] the existence of the tensor polarizability can be revealed by the magnitude of the cross section for elastic scattering and by the angular distribution of the scattering (the existence of a large isotropic component in the angular distribution).

The elastic scattering of γ-ray quanta by oriented nuclei has been discussed earlier.[1] As shown, the existence of tensor polarizability brings about azimuthal asymmetry of scattering. The greatest asymmetry occurs when the nuclei are oriented perpendicularly to the wave vector of the incident photon. The angular distribution in the plane normal to the incident beam has the form (in the notation of the present article):

$$d\sigma/d\Omega = (2\pi^2/\omega^2)\{[\tfrac{3}{4}|R^T|^2 + \tfrac{3}{2}(R^{T*}R' + R'^*R^T)]\sin^2\varphi + [\tfrac{1}{4}|R^T|^2 - \tfrac{1}{2}(R^{T*}R' + R'^*R^T) + R'^2]\}. \quad (11)$$

Formula (11) has been derived from classical considerations and is valid only for $J \to \infty$. Quantum corrections for it are very substantial. For $J = 0$ or $J = \frac{1}{2}$, the azimuthal asymmetry vanishes in general.

Apart from the effects mentioned, the existence of tensor polarizability can be disclosed by measurements of the static polarizability and of the quadratic fluctuation of the dipole moment in the ground state for oriented nuclei on the basis of the application of the summation rules. These effects have been discussed previously.[2]

Thus, there are many effects upon which tensor polarizability can exert an influence and which can be fully observed by existing experimental means. To estimate the measureability of the effects under discussion, it is necessary to calculate R^T and R^S on the basis of models.

3. MODELS FOR TENSOR POLARIZABILITY

Tensor polarizability models have been discussed in part before.[2] By analogy with molecular optics, we introduce the concept of the "internal" tensor polarizability, i.e., the polarizability in a coordinate system rotating together with the nucleus. In the coordinate system affixed to the nucleus, let the tensor part of the polarizability have the form

$$\alpha_{ik}^{0T} = \frac{3}{2}\alpha_0^T\left(n_in_k - \frac{1}{3}\delta_{ik}\right). \quad (12)$$

We have assumed that this tensor has axial symmetry; n_i is the component of the unit vector along the axis of symmetry. Generalization to the axially unsymmetric case presents no particular difficulties. In addition, we assume that $\mathbf{n}$ coincides with the direction of the nuclear-surface symmetry axis, along which the nuclear spin is directed in the ground state. Under these assumptions there is a direct analogy, on the one hand, between α_0^T and α^T, and, on the other hand, between the "internal" and the spectroscopic quadrupole moments. Hence we have

$$\alpha^T = \alpha_0^T J(2J-1)/(J+1)(2J+3). \quad (13)$$

Thus, the amplitude R^T of photon scattering by the nucleus in the laboratory system of coordinates is expressed through the "internal" polarizability by the following formula:

$$R^T = -\frac{\omega^3}{2\pi}\frac{J(2J-1)}{(J+1)(2J+3)}\alpha_0^T = R_0^T\frac{J(2J-1)}{(J+1)(2J+3)}, \quad (14)$$

i.e., the tensor polarizability vanishes for nuclei with spins 0 or $\frac{1}{2}$.

A model of tensor polarizability is contained in an extension of the hydrodynamic model of dipole vibrations of nuclei to nonspherical forms (Okamato[3] and Danos[4]). These authors have called attention to the fact that in a deformed nucleus the fluctuations of the densities of the neutron and proton liquids must take place with two neighboring characteristic frequencies. The order of magnitude of the ratio of these frequencies follows from the dimensional consideration, namely $\omega_1/\omega_2 \sim R_2/R_1$, where R_2 and R_1 are the smallest and largest radii of the nuclear surface. A detailed hydrodynamic analysis yields $\omega_1/\omega_2 = 0.91\,R_2/R_1$ if the surface is an ellipsoid with small eccentricity. This idea enabled the two authors mentioned to reach the conclusion that the experimentally observed giant resonance corresponds not to one characteristic frequency, but to two. Hence followed the successful explanation of the "line broadening" in the giant resonance for deformed nuclei. From this model it also follows that the wave of the fluctuations of the density differences of the neutron and proton liquids is propagated in the direction of the electric field of the photon. The latter inference signifies that the frequency ω_1 is excited when the electric vector of the photon is directed along the radius R_1, and ω_2 when the electric vector is directed along R_2. We do not adhere to the hydrodynamic model, but use the results of molecular optics and consider the nucleus as an aggregate of three linear oscillators, arranged perpendicularly to each other, with the oscillation frequencies ω_1 and $\omega_2 = \omega_3$, and the damping ratios γ_1 and $\gamma_2 = \gamma_3$. In this case the scattering amplitude in the coordinate system affixed to the nucleus is expressed by

$$R_0^T = \frac{\omega^3}{2\pi}\,\frac{2}{3}\left\{f_1\,\frac{\omega_1^2-\omega^2+i\gamma_1\omega}{(\omega_1^2-\omega^2)^2+\gamma_1^2\omega^2} - f_2\,\frac{\omega_2^2-\omega^2+i\gamma_2\omega}{(\omega_2^2-\omega^2)^2+\gamma_2^2\omega^2}\right\},$$

$$R_0^S = \frac{\omega^3}{2\pi}\left\{\frac{1}{3}f_1\,\frac{\omega_1^2-\omega^2+i\gamma_1\omega}{(\omega_1^2-\omega^2)^2+\gamma_1^2\omega^2} + \frac{2}{3}f_2\,\frac{\omega_2^2-\omega^2+i\gamma_2\omega}{(\omega_2^2-\omega^2)^2+\gamma_2^2\omega^2}\right\}. \tag{15}$$

The expression for the amplitude in the laboratory coordinate system follows from Eqs. (15), (14), and (2). The imaginary part of the amplitude determines the cross section for the absorption of γ-ray quanta by the nucleus in accordance with (6) and (7). From formula (15) it is seen that with $\omega_1 - \omega_2 \gtrsim \gamma_{1,2}$ the tensor polarizability for frequencies within the range of the giant resonance can exceed the scalar polarizability twofold, whence it follows that the above-mentioned effects can be observed fully.

As seen from (10) and (14), in comparatively simple experiments with elastic scattering by unoriented nuclei (R_0^T) the tensor polarizability will be manifested only for nuclei with sufficiently high spins. Thus, for example, in the case of In^{115}, which has a spin of $9/2$, the ratio of the contributions by the first and second components can be expected to reach 0.5. In this connection, as evident from formula (10), the anticipated angular distribution in elastic scattering is of the type $a + \cos^2\theta$, where $a = 1.5$, not 1 (the latter value corresponds to $R^T = 0$).*

The quantities γ_1, ω_1, γ_2, ω_2 and the oscillator strengths f_1 and f_2 are of substantial importance in all the estimates.

Perhaps the most thorough hydrodynamic estimate of ω_1, ω_2 is that by Levinger,[6] who utilized a summation rule similar to the known Kramers-Heisenberg formula,

$$\alpha = \frac{1}{2\pi^2}\int \frac{\sigma}{\omega^2}\,d\omega$$

and evaluated the static polarizability of the nucleus in the coordinate system affixed to the nucleus. The results coincided with the calculations by Okamoto and Danos: $\omega_1 \sim 1/R_1$ and $\omega_2 \sim 1/R_2$. With regard to γ_1 and γ_2 it is impossible to say anything definite at the present time. Experiment appears to indicate that γ_1 and γ_2 are substantially different. This circumstance cannot be explained within the framework of the simple hydrodynamic model. In view of this, a consideration of other models is warranted.

Soga and Fujita[7] calculated the one nucleon electric dipole transitions in deformed nuclei, using the shell model, and showed that the transitions are grouped around the two frequencies $\omega_1 \sim 1/R_1$ and $\omega_2 \sim 1/R_2$. Of course, such an approach employs many detailed assumptions about the form of the wave functions for the ground and excited states of the nuclei. We shall show that in the independent-particle model the conclusion that $\omega_1 \sim 1/R_1$ and $\omega_2 \sim 1/R_2$ follows from very general notions about the nucleus, without recourse to the mentioned assumptions.

*Recently E. Fuller and E. Hayward (preprint and private communication from D. Levinger) made an attempt to interpret their data on the elastic scattering of γ-ray quanta from Ta from the viewpoint of the existence of tensor polarizability in this nucleus (on the basis of the Okamoto-Danos hydrodynamic model). However, they used classical averaging for the nuclear orientation. Our Eqs. (10) and (14) differ from theirs, by the factor $J(2J-1)(2J+3)(J+1)$. Inclusion of this factor will substantially change the result of their analysis. The author avails himself of this occasion to thank D. Levinger and E. Fuller for the interesting information.

We consider the summation rule

$$\sigma_{-1} = \int \frac{\sigma}{\omega} d\omega = 4\pi^2 \langle 0 | d_z^2 | 0 \rangle \tag{16}$$

in a coordinate system affixed to the nucleus; $\mathbf{d} = \sum_i e_i \mathbf{r}_i$ is the nuclear dipole moment operator and $|0\rangle$ represents the ground state of the nucleus. The right-hand part can be written

$$\langle 0 | d_z^2 | 0 \rangle = e^2 \langle 0 | \sum_i (z_i^2 - \tfrac{1}{3} r_i^2) | 0 \rangle + \tfrac{1}{3} \langle 0 | d^2 | 0 \rangle$$
$$- \langle 0 | \sum_{i \neq j} z_i z_j - \tfrac{1}{3} r_i r_j | 0 \rangle. \tag{17}$$

As can be seen without difficulty, in this expression the first term is proportional to the quadrupole moment of the nucleus, the second term to the first moment of the cross section σ_{-1} for unoriented nuclei, and the last term is connected with the positional correlations of the protons in the nucleus. As is well known, the basic sources of correlation of the type interesting to us are the relationship $\sum \mathbf{r}_k = 0$ (where $\mathbf{r}_k$ represents the coordinates of both protons and neutrons) and Pauli's exclusion principle. We shall take these effects into account and estimate the last term. To take the relationship $\sum \mathbf{r}_k = 0$ into account, we introduce, as usual, the effective charges $e_p = eN/A$ for the proton and $e_n = -eZ/A$ for the neutron; and since Pauli's principle imposes a restriction only upon similar particles, we discard terms associated with proton-neutron correlations:

$$e^{-2} \langle 0 | d_z^2 | 0 \rangle = (N/A)^2 \langle 0 \left| \sum\nolimits_p (z_i^2 - \tfrac{1}{3} r_i^2) \right| 0 \rangle$$
$$+ (Z/A)^2 \langle 0 \left| \sum\nolimits_n (z_i^2 - \tfrac{1}{3} r_i^2) \right| 0 \rangle + \langle 0 \left| \sum\nolimits_{p,\, i \neq j} [z_i z_j - \tfrac{1}{3} r_i r_j] \right| 0 \rangle$$
$$+ \langle 0 \left| \sum\nolimits_{n,\, i = j} [z_i z_j - \tfrac{1}{3} r_i r_j] \right| 0 \rangle + \tfrac{1}{3} \langle 0 | d^2 | 0 \rangle, \tag{18}$$

where $\sum_{p(n)}$ indicates summation over all the protons (or neutrons) in the nucleus.

To estimate the correlations associated with Pauli's principle, we employ a method first applied by Khokhlov[8]* to an analogous problem. We introduce a function expressing the distribution of the coordinates of two protons

$$n(\mathbf{r}', \mathbf{r}) = \frac{1}{Z(Z-1)} \sum_{p_1 \neq p_2} \langle 0 | \delta(\mathbf{r} - \mathbf{r}_{p_1}) \delta(\mathbf{r}' - \mathbf{r}_{p_2}) | 0 \rangle$$

(the summation extends over all the protons in the nucleus).

We shall define

$$\overline{r_\mu r'_\nu} = \int r_\mu r'_\nu n(\mathbf{r}', \mathbf{r})\, d\mathbf{r} d\mathbf{r}',$$
$$\overline{r_\mu^2} = \int r_\mu^2 n(\mathbf{r})\, d\mathbf{r}, \qquad n(\mathbf{r}) = \int n(\mathbf{r}', \mathbf{r})\, d\mathbf{r}', \tag{19}$$

where $\bar{r}_\mu$ is manifestly equal to zero. Inasmuch as the dimension a of the region in which the correlations due to Pauli's principle make themselves felt is much smaller than the dimensions of the nucleus, it is possible to adopt the following expression for $n(\mathbf{r}', \mathbf{r})$

$$n(\mathbf{r}', \mathbf{r}) = n(\mathbf{r})\, n(\mathbf{r}')$$
$$- \Omega [\delta(\mathbf{r}' - \mathbf{r}) n\left(\frac{\mathbf{r}' + \mathbf{r}}{2}\right) - n(\mathbf{r})\, n(\mathbf{r}')] \tag{20}$$

and to consider that this function is the same for neutrons as for protons. The quantity $\Omega \sim (a/R)^3$ can be found by using the actual form of the wave functions of the nucleus. However, as will be shown below, for our purposes the value of this quantity is generally insignificant. If the expression (20) and our definitions (19) are employed, then, as easily verified, Eq. (18) assumes the form

$$e^{-2} \langle 0 | d_z^2 | 0 \rangle = [(N/A)^2 Z + (Z/A)^2 N]\, \overline{(z^2 - {}^1\!/_3 r^2)}$$
$$- \Omega [(N/A)^2 Z(Z-1) + (Z/A)^2 N(N-1)]\, \overline{(z^2 - {}^1\!/_3 r^2)}$$
$$+ {}^1\!/_3 \overline{r^2} [(N/A)^2 Z + (Z/A)^2 N] - \Omega [(N/A)^2 Z(Z-1)$$
$$+ (Z/A)^2 N(N-1)]\, \overline{r^2}/3. \tag{21}$$

Hence we find the dependence of σ_{-1} on the angle between the electric vector of the photon and the symmetry axis of the nucleus:

$$\frac{1}{4\pi^2} \sigma_{-1} = F(Z, A, \Omega) \left[\frac{1}{2Z} Q^0 \left(\cos^2\beta - \frac{1}{3}\right) + \frac{1}{3} \overline{r^2}\right], \tag{22}$$

where $F(Z, A, \Omega)$ is a general factor the form of which follows from (21) and Q^0 is the "internal" quadrupole moment of the nucleus.

From (22) it is easy to get for oriented nuclei

$$\frac{1}{4\pi^2} \hat{\sigma}_{-1} = F(Z, A, \Omega)$$
$$\times \left[\frac{1}{2Z} Q \frac{2(J+1)}{(2J-1)} \left(\frac{\hat{J}_z^2}{J(J+1)} - \frac{1}{3}\right) + \frac{1}{3} \overline{r^2}\right], \tag{23}$$

where Q denotes the spectroscopic quadrupole moment of the nucleus. As already[2] pointed out, formula (23) is exact for deuterium, with $F = 1$.*

*The author is grateful to Yu. K. Khokhlov for pointing out that the effect of the correlations connected with Pauli's principle can be easily estimated by this method.

*The static tensor polarizability of the deuteron was calculated by Yu. I. Bregadze. The ratio of the tensor part of the polarizability to the scalar one came out equal to ~ 1.5%.

From (22) we obtain

$$(\sigma_{-1}^{(1)} - \sigma_{-1}^{(2)}) / \sigma_{-1}^{0} = 3Q^0 / 2Z\bar{r^2} \approx 5Q^0 / 2ZR^2. \quad (24)$$

Here $\sigma_{-1}^{(1)}$ is the greatest value of σ_{-1} (at $\beta = 0$) while $\sigma_{-1}^{(2)}$ is its least value (at $\beta = \pi/2$); σ_{-1}^{0} is the moment of the cross section for unoriented nuclei, and R is the radius of the nucleus.

If, as usual, the "position of the maximum" of the giant resonance is defined as

$$\bar{\omega} = \int \sigma d\omega \Big/ \int \frac{\sigma}{\omega} d\omega,$$

then the width of the maximum for the giant resonance follows from (24):

$$(\overline{\omega^{(1)}} - \overline{\omega^{(2)}}) / \bar{\omega}_0 \approx 5Q^0 / 2ZR^2.$$

Comparing this formula with the results secured by Okamoto and Danos, we find that the results of a calculation based on the independent-particle model agree, accurate to a numerical factor, with those obtained on the basis of the collective hydrodynamic model.

Analogous calculations with the aid of formula (23) yield an expression for the "shift of the maximum"[2] for completely oriented nuclei:

$$(\omega_{\parallel} - \omega_{\perp}) / \omega_0 \approx 5Q / 4ZR^2.$$

Here Q is the spectroscopic quadrupole moment. It must be emphasized that essentially we have made use here of the assumption about the absence in the nucleus of any correlations other than those with a radius much smaller than the nuclear dimensions.*

Thus, the use of various models and very general assumptions yields one and the same result, $\omega_1 \sim 1/R_1$ and $\omega_2 \sim 1/R_2$. The calculation of the widths γ_1 and γ_2 on the basis of the independent-particle model requires special investigations.

4. DISCUSSION

An examination of the various models of the nucleus shows that it is hardly possible to doubt the existence of tensor polarizability in atomic nuclei. In any event, it can be affirmed that tensor polarizability exists in the nucleus of deuterium.

The theory of the polarizability of atomic nuclei ensues as an extension of the theory of molecular polarizability.[5] Its non-Hermitian character is an important distinguishing feature of the tensor polarizability of atomic nuclei in comparison with that of molecules. Another distinction of the theory of nuclear polarizability is that the effects considered correspond to the experimental methods of nuclear physics. For example, one of the basic methods for the investigation of molecular polarizability — the study of the depolarization of scattered radiation — is hardly likely to be applied to the investigation of nuclear polarizability in the near future.

It seems to us that experiments with oriented nuclei must be considered the principal source of information about the tensor polarizability of atomic nuclei, although experiments with the elastic scattering of γ-ray quanta by nuclei might compete with this method. At the same time it is necessary to consider the possibility of an experimental study of Raman scattering of γ-ray quanta by nuclei. This method might yield a great deal of valuable information about nuclear parameters. A recently initiated study[9] of (γ, γ') reactions (which are equivalent to "nuclear luminescence" in the terminology of molecular optics), may also furnish many very valuable facts about the tensor polarizability of atomic nuclei. However, the interpretation of these experiments is rather ambiguous (it is not clear how many and what kind of transitions have occurred before the nucleus reaches its metastable level).

Quantitative estimates of the order of magnitude of the effects follow from Eqs. (2), (7), (10), (14), and (15). As can be seen from (15), at $\gamma_{1,2} \lesssim \Delta\omega$, the value of R_0^T exceeds R^S twofold in the frequency range $\omega \sim \omega_1$, whereas in the frequency range $\omega \sim \omega_2$ the value of R_0^T is equal to R^S but is opposite in sign. Hence comes the very peculiar dependence of the difference of the cross sections $\sigma_\perp - \sigma_\parallel$ for the absorption of γ-ray quanta by completely oriented nuclei [see formulas (7) and (8)]. For nuclei with spins $> 5/2$ this difference is of the same order of magnitude as the cross sections themselves. This effect is certainly measurable even if the degree of nuclei orientation has been established meagerly and unreliably, since the energy dependence of the quantity R^T is of prime interest.

In the experiments on elastic scattering, the effects are also large (see text for estimates). These conclusions are almost independent of the models used. Identical values of the two Okamoto-Danos frequencies, $\omega_1 \sim 1/R_1$ and $\omega_2 \sim 1/R_2$, were secured from very different models. A discussion of the remaining parameters of formula (15) on the basis of the models is difficult at present because of the lack of experimental data.

*We note that our results will not be altered if the existence of strong neutron-proton correlations with a small radius is assumed.

At any rate, the investigation of tensor polarizability will provide more precise and more complete information about the shape of the nucleus than any other existing methods. In addition, experimental data on the quantities γ_1 and γ_2 can help substantially in understanding the mechanism of the absorption of γ-ray quanta by nuclei.

It must be emphasized that the effects under discussion can be observed only with nuclei possessing large spins. In this regard we point out once more the necessity of exploring the experimental possibilities in the study of Raman scattering, which is not subject to this restriction.

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