INELASTIC SCATTERING OF NUCLEONS ON Mg^{24} *AND* Si^{28} *NUCLEI*

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Inelastic scattering of nucleons on Mg^{24} and Si^{28} nuclei is investigated with account of Coulomb interaction for 1) excitation of collective nuclear levels, and 2) single-particle excitation in the field of a deformed nucleus. An analysis is made of the relation between the character of the angular distribution and magnitude of the effective cross section on the one hand, and the magnitude and the sign of the deformation on the other.

1. Many recently published papers deal with experimental investigations of inelastic scattering of nucleons and deuterons by nuclei.[1,2] This problem becomes even more interesting because a comparison of the experimental data with the results of the theoretical investigations can lead to many conclusions concerning the character of excitation of nuclei.

The present paper is devoted to a theoretical investigation of inelastic scattering of nucleons by Mg^{24} and Si^{28}. Two cases of excitation, one-particle and collective, are considered.

Unlike Sawicki,[3] who recently considered this problem, we also take Coulomb interaction into account and investigate the dependence of the character of angular distribution and of the magnitude of the effective scattering cross section on the magnitude and sign of the deformation.

2. We first consider single-particle excitation. Here, as is known, it is assumed that only one of the nucleons above the closed shell is excited, and that this nucleon moves in the field of the deformed nucleus.

The wave functions for the nuclear system at the beginning and the end of the process are respectively[4]

$$\Phi_0 = \sqrt{\frac{2I+1}{16\pi^2}}\varphi_{n_\beta n_\gamma}(\beta,\gamma)\,[\chi_\Omega D^I_{MK}(\theta_i) + (-1)^p \chi_{-\Omega} D^I_{M-K}(\theta_i)], \quad (1)$$

$$\Phi = \sqrt{\frac{2I'+1}{16\pi^2}}\varphi_{n'_\beta n'_\gamma}(\beta,\gamma)\,[\chi_{\Omega'} D^{I'}_{M'K'}(\theta_i) + (-1)^{p'} \chi_{-\Omega'} D^{I'}_{M'-K'}(\theta_i)], \quad (2)$$

where I, K, and M is the angular momentum of the nucleus and its projections along the symmetry axis of the nucleus and a stationary axis respectively; $\Omega = \Sigma\Omega_i$, where Ω_i is the projection of the momentum of the i-th nucleon (from among the nucleons above the closed shell) on the axis of the nucleus; χ_Ω is the anti-symmetrized wave function of the nucleons above the closed shell; the wave functions $\varphi(\beta, \gamma)$ and $D(\theta_i)$ describe the collective (vibrational and rotational) states of the nucleus, β and γ being parameters that characterize the deformation of the nucleus, while $\theta_i\,(\theta_1\theta_2\theta_3)$ are the Euler angles; $p = I + K - \Omega - (1/2)A$, where A is the mass number of the nucleus. (The primed symbols in Φ have the same meaning as in Φ_0, but pertain to the final states of the nuclear system.)

Since it is assumed that the incident nucleon interacts only with one nucleon of the nucleus, we can write

$$\chi_\Omega = \chi_{\Omega_i}\chi_{\Omega_0}, \quad (3)$$

where

$$\chi_{\Omega_i} = \sum_{l_i m_i \sigma_i} A_{l_i\Omega_i-\sigma_i} R_{Nl_i}(r_i) \times Y_{l_i m_i}(\vartheta_i,\varphi_i)\,S(\sigma_i)\,D^{l_i}_{m_i\,\Omega_i-\sigma_i} \quad (4)$$

is the wave function of the excited nucleon, referred to a stationary system of coordinates, χ_{Ω_0} is the wave function of the remaining nucleons outside the closed shell, with $R_{Nl_i}(r_i)$ being the radial wave function, $S(\sigma_i)$ the spin function, and $A_{l_i\Omega_i-\sigma_i}$ the coefficients of diagonalization, tabulated by Nilsson.[4] We choose the energy of interaction between the incident nucleon and the i-th nucleon of the nucleus in the form

$$V = -V_0\delta(\mathbf{r}-\mathbf{r}_i) + 4\pi e^2\varepsilon\sum_{l'm'}\frac{r_i^{l'}}{(2l'+1)\,r^{l'+1}}Y^*_{l'm'}(\vartheta,\varphi)\,Y_{l'm'}(\vartheta_i,\varphi_i), \quad (5)$$

where $\epsilon = 1$ when the incident and i-th nucleon of the nucleus are protons, and $\epsilon = 0$ in all other cases. The Coulomb interaction in expression (5) is chosen in the form used in reference 5, where it is assumed, first, that the electric interaction occurs only when $r > r_0$ (where r_0 is the electric radius of the nucleus), and second, that the wave functions of the nuclear nucleons are real in the region $r_i < r_0$.

Taking plane wave functions for the incident nucleon at the beginning and the end of the process, we obtain for the differential cross section of inelastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\mu^2 V_0}{(2\pi\hbar^2)^2} \sqrt{1 + \frac{Q}{E_0'}} \frac{1}{2I+1} \sum |H_{if}|^2, \tag{6}$$

where

$$H_{if} = \frac{1}{2}\sqrt{(2I+1)(2I'+1)} \times \sum_{Ll}\sum_{l_i l_i'}\sum_{m_i \sigma_i} [C(Lll_il_i' m_i\sigma_i\Omega_i\Omega_i'KK') + (-1)^{p+p'} C(Lll_il_i' m_i\sigma_i, -\Omega_i, -\Omega_i', -K, -K')]\, \delta_{MM'}\, \delta_{\Omega_i'+K', \Omega_i+K}, \tag{7}$$

with

$$C(Lll_il_i'm_i\sigma_i\Omega_i\Omega_i'KK') = A_{l_i\Omega_i-\sigma_i} A'_{l_i'\Omega_i'-\sigma_i} \frac{i^l(2l+1)}{2L+1} J_{ll_il_i'}(q) \times \sqrt{\frac{2l_i+1}{2l_i'+1}} (l_il00 \mid l'0)(l_ilm_i0 \mid l_i'm_i)(l_iIm_iM \mid Lm_i+M) \times (l_i'I'm_iM \mid Lm_i+M)(l_iI\Omega_i-\sigma_iK \mid L\Omega_i+K-\sigma_i) \times (l_i'I'\Omega_i'-\sigma_iK \mid L\Omega'+K'-\sigma_i). \tag{8}$$

In Eq. (6) μ is the reduced mass, E_0' the energy of the incident nucleon in the center of mass system, Q the energy absorbed by the nucleus in the reaction, and the function J(q) which enters into (8) is equal to

$$J_{ll_il_i'}(q) = \int_0^\infty R_{Nl_i}(r_i) R_{Nl_i'}(r_i) f_l(qr_i) r_i^2 dr_i - \frac{e^2}{V_0}\frac{4\pi}{2l+1}\int_0^{r_0} r_i^{l+2} R_{Nl_i}(r_i) R_{Nl_i}(r_i)\, dr_i \times \int_{r_0}^\infty \frac{\exp(i\mathbf{qr})}{r^{l+1}} Y_{l0}(\vartheta, \varphi)\, d\mathbf{r}, \tag{9}$$

where r_0 is the electric radius of the nucleus, $q = |\mathbf{k} - \mathbf{k}'|$, and $f_l(qr)$ is the spherical Bessel function ($\mathbf{k}$ and $\mathbf{k}'$ are the wave vectors of the incident and scattered nucleon).

3. Let us apply the derived formulas to the scattering of protons by Mg^{24} and Si^{28} with excitation of the first level. In both cases we have $I = 0$, $K = 0$, $M = 0$, $I' = 2$, $K' = 0$. Therefore, in accordance with the selection rules $\Omega_i' + K' = \Omega_i + K$, we can write $\Omega_i = \Omega_i'$. It is known that in Mg^{24} the nucleons above the O^{16} shell $(N = 2)$ are in the state with $\Omega_i = \pm 1/2$ and $\pm 3/2$ at $\delta > 0$ and $\Omega_i = \pm 5/2$ and $\pm 1/2$ at $\delta < 0$ (δ is the deformation parameter). Consequently, the possible transitions in Mg^{24} are $1/2 \to 1/2$ and $3/2 \to 3/2$ (and also the transitions $-1/2 \to -1/2$ and $-3/2 \to -3/2$). We use the well known relation between the excitation energy and the deformation parameter

$$\Delta E = \varkappa\hbar\omega_0 [r_{\alpha_1}^{N\Omega_i}(\delta) - r_{\alpha_2}^{N\Omega_i}(\delta)], \tag{10}$$

where $\hbar\omega_0 = \hbar^2/2Mr_0'^2$, where r_0' is a parameter that enters into the expression for the oscillator potential, M the mass of the nucleon, and $r_\alpha^{N\Omega_i}$ the eigenvalue of the additional term in the interaction taking into account the deformation of the nucleus and the spin-orbit forces; the index α indicates the number of the level with given Ω_i according to Nilsson.[4] Nilsson also tabulated the values of $r_\alpha^{N\Omega}$ for different values of the deformation parameter δ (at $\kappa = 0.05$). By finding the density of the nucleons in the nucleus on the basis of the oscillator wave functions, and by stipulating that the point of maximum slope of the density curve correspond to the boundary of the nucleus, we can determine the value of the parameter r_0'. Assuming for the nuclear radius $R_0 = 1.45\,A^{1/3} \times 10^{-13}$ cm, we find approximately the same value for Mg^{24} and Si^{28}, $r_0' = 1.9 \times 10^{-13}$ cm. Using the experimental value of the excitation energy ΔE, we can determine the deformation parameter δ from Eq. 10. In the case of Mg^{24} the excitation energy of the first level is 1.37 Mev. Assuming the transition $(1/2)_{\alpha_1} \to (1/2)_{\alpha_2}$ to take place, we obtain from Eq. (10) for the deformation parameter two values of δ, namely $\delta = 0.17$ and $\delta = -0.22$. If, on the other hand, we assume a $(3/2)_{\alpha_1} \to (3/2)_{\alpha_2}$ transition, then the deformation parameter is found to be practically zero. Thus, only a $1/2 \to 1/2$ transition is possible in the Mg^{24} deformed nucleus in the case of single-particle excitation.

A different result is obtained in the case of the Si^{28} nucleus, in which the levels $\pm 5/2$, $\pm 1/2$, $\pm 3/2$ are filled at $\delta < 0.17$ and the levels $\pm 1/2$, $\pm 3/2$, $\pm 1/2$ are filled at $\delta > 0.17$. On the basis of the selection rule $\Omega_i = \Omega_i'$ we have the same transitions as in the case of Mg^{24}, namely $1/2 \to 1/2$ and $3/2 \to 3/2$. (In the outer shell of Si^{28} there is only one level with $\Omega_i = \pm 5/2$.)

Considering that the excitation energy of the nearest level in Si^{28} is 1.77 Mev, we obtain for the deformation parameter $\delta = 0.1$ and $\delta = 0.3$

for the transition $1/2 \to 1/2$ and $\delta = 0.1$ for the transition $3/2 \to 3/2$

4. From (7) and (8) we have for the $1/2 \to 1/2$ transition

$$H_{if} = \tfrac{1}{2}\{J_{220}(q)[A_{00}(A'_{20}+A'_{21}) + A'_{00}(A_{20}+A_{21})] + (\sqrt{5}/7)J_{222}(q)(A_{20}+A_{21})(2A'_{20}+A'_{21})\}, \qquad (11)$$

and for the $3/2 \to 3/2$ transition

$$H_{if} = (\sqrt{5}/7)J_{222}(q)(A_{21}+A_{22})(A'_{22}+A'_{21}), \qquad (12)$$

where

$$J_{220} = \frac{1}{3\sqrt{10}}\Big\{(qr'_0)^2[(qr'_0)^2-4]\exp[-(qr'_0)^2/2] - \frac{12}{5}\frac{\sqrt{2\pi}e^2}{V'_0R_0}\Big(\frac{r'_0}{R_0}\Big)^2 a\Big(\frac{r_0}{r'_0}\Big)\varepsilon\frac{f_1(qr_0)}{qr_0}\Big\}$$

and

$$J_{222} = \frac{1}{15}\Big\{(qr'_0)^2[7-(qr'_0)^2]\exp[-(qr'_0)^2/2] - \frac{4}{5}\frac{\sqrt{2\pi}e^2}{V'_0R_0}\Big(\frac{r'_0}{R_0}\Big)^2 a'\Big(\frac{r_0}{r'_0}\Big)\varepsilon\frac{f_1(qr_0)}{qr_0}\Big\},$$

with

$$V_0 = V'_0R_0^3, \quad a\Big(\frac{r_0}{r'_0}\Big) = \int_{r_0/r'_0}^{\infty} e^{-x^2/2}x^6\Big(1-\frac{x^2}{3}\Big)dx,$$

$$a'\Big(\frac{r_0}{r'_0}\Big) = \int_{r_0/r'_0}^{\infty} e^{-x^2/2}x^8dx.$$

It is readily seen from (12) that in the $3/2 \to 3/2$ transition the relative angular distribution is independent of the deformation parameter, since the latter influences only the coefficients A_{ik} and A'_{ik}, which pertain to levels α_1 and α_2 respectively.

Let us compare the results obtained with the experimental data. In the case of Mg^{24}, as indicated above, only the $1/2 \to 1/2$ transition is possible in the presence of deformation. Curve 1 of Fig. 1 shows the theoretical angular distribution, obtained on the basis of Eq. (11) at $\delta = 0.2$; the vertical lines represent the experimental values of the cross section.[1] (We have replaced the exact value $\delta = 0.17$ by the approximate value $\delta = 0.2$, since the wave functions available in the literature[4] have been tabulated for δ in the interval from 0.3 to -0.3 in steps of 0.1). In plotting curve 1, we have assumed for the electric radius of Mg^{24} a value[5] $r_0 = 6 \times 10^{-13}$ cm. It is easy to see that the presence of a maximum in the angular distribution is determined essentially by the nuclear term of the interaction, and the location of the maximum depends substantially on R_0. We determine the parameter V'_0 from the condition that the maximum point on the theoretical curve coincide with the corresponding experimental value of the cross section at the same scattering angle. At $\delta = 0.2$ we obtain $V'_0 = 0.94$ Mev. The angular distribution at negative deformation, $\delta = -0.2$, differs little from the distribution at $\delta = 0.2$. Taking the same value $V'_0 = 0.94$ Mev, the only difference is that the magnitude of the maximum cross section in the case of $\delta = -0.2$ is approximately one order of magnitude less than for $\delta = 0.2$, and the position of the maximum is shifted approximately 10° towards the larger angles.

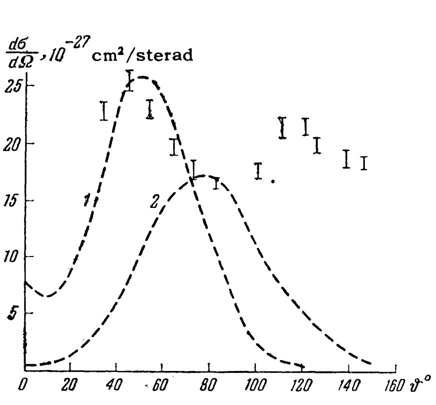

FIG. 1

As seen from Fig. 1, the theoretical curve 1 is in satisfactory agreement with the experimental data, both with respect to the position of the maximum and with respect to its shape, in the interval of angles from 0 to 70°. Thus, allowance for the Coulomb interaction has improved the agreement between theory and experiment in the region of small angles. It should be noted that we would obtain for the parameter δ a different value were we to stipulate that the experimental value of the maximum cross section coincide with the point of maximum not on the curve corresponding to the deformation parameter $\delta = 0.2$, but on the curve corresponding to the deformation parameter $\delta = -0.2$. It can be shown, however, that this would not influence the position of the maximum, since in this respect the angular distribution shows poorer agreement with experiment in the case of negative deformation.

Figure 2 shows the angular distribution of protons inelastically scattered by Si^{28}. As noted above, we have in this case $1/2 \to 1/2$ transitions with deformations $\delta = 0.1$ and $\delta = -0.3$, and $3/2 \to 3/2$ transitions with deformations $\delta = 0.1$. The theoretical angular distributions differ little from each other in all cases. We therefore show only one curve, corresponding to the $1/2 \to 1/2$ transition with a deformation parameter $\delta = 0.1$. If V'_0 is determined here, too, from the condition that the maximum point on the theoretical curve

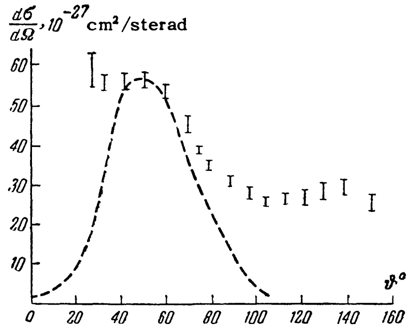

FIG. 2

correspond to the experimental value,[6] we find that, unlike Mg^{24}, allowance for the Coulomb interaction does not lead to any significant correction. (We obtain here $V_0' = 12.2$ Mev.) Therefore agreement is obtained with experiment only as regards the position of the principal maximum. As to the magnitude of the cross section, we indicate that the principal maximum value in the case of $1/2 \rightarrow 1/2$ and $3/2 \rightarrow 3/2$ transitions with deformation parameter $\delta = 0.1$ and the $1/2 \rightarrow 1/2$ transition with deformation parameter $\delta = -0.3$ are related approximately as 1:10:30 if we take for all the cases the same value of V_0', namely 12.2 Mev.

5. It is easy to obtain the angular distribution of inelastically scattered protons by Mg^{24} and Si^{28} under the assumption that the collective level is excited, provided we use the previously derived[5] formula, which pertains to the scattering of deuterons. Modifying this formula to account for the fact that we deal here with proton scattering, we obtain

$$\frac{d\sigma}{d\Omega} = \mu^2 \frac{(V_0 R_0^2)^2}{\pi \hbar^4} \beta^2 \sqrt{1 + \frac{Q}{E_0'}} \left[f_2(qR_0) + \frac{0,6Ze^2}{R_0 V_0} \frac{f_1(qr_0)}{qr_0} \right]. \tag{13}$$

In reference 5 we used a value of β determined from energy of the rotational level. Considering, however, that the value of β thus obtained is approximately twice the experimental value of the deformation, we use for Mg^{24} $\beta = 0.42$.[3] It must be noted, that the deformation parameter $\delta = 0.95\beta$, determined from the energy of the single-particle level, differs from the deformation determined from the data on collective excitation. This is natural, since by attributing the entire excitation of the nucleus to one particle, it is necessary apparently to make a corresponding change in the field of deformation.

Curve 2 of Fig. 1 shows the angular distribution obtained from Eq. (13) for Mg^{24}. The first term of this equation corresponds to the nuclear interaction. The angular distribution due to this interaction has a maximum, something not obtained with the Coulomb interaction. The position of the maximum in the angular distribution, with allowance only of the nuclear term in the interaction, is independent of the parameter V_0' and is close to 80°. If we require here that the value of the cross section at the maximum of angular distribution coincide with the corresponding experimental value of the cross section, then the parameter V_0' is found to be such that the Coulomb term influences the distribution only at small angles, and does not change the position of the maximum. An analogous result is obtained also in the case of Si^{28}. Here, too, the position of the maximum of the angular-distribution curve is independent of the parameter V_0' and is close to 80°.

After comparing the results obtained for single-particle and collective excitations, we can arrive at the following conclusions:

1. For the same value of the radius of the equilibrium sphere, on which the angular distribution depends substantially, the position of the maxima in the distribution are found to be different, depending on the character of excitation of the nucleus. For the value we have assumed, $R_0 = 4.2 \times 10^{-13}$ cm, the maximum in the angular distribution for Mg^{24}, in single-particle excitation, is close to the origin and is in better agreement with the experimental data. When R_0 is increased, the maximum in the angular distribution shifts in both excitations towards the smaller angles. For example, at $R_0 = 6 \times 10^{-13}$ cm, the position of the maximum in the angular distribution, in the case of collective excitation, is in better agreement with experiment, but such a value of R_0 must be considered as excessive and not in agreement with other data.

2. In our method of analysis, the presence of a second maximum on the experimental curve of angular distribution cannot be explained for either case of excitation.

3. The relative angular distribution, connected with the collective excitation, is independent of the magnitude and sign of the deformation, whereas in the case of single-particle excitation such a dependence exists, although it is weak.

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