RANGE-ENERGY RELATION FOR SOME SUBSTANCES

M. Z. MAKSIMOV

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From an analysis of the experimental and theoretical data on the stopping of charged particles in matter it is concluded that the specific loss and the range depend only on the ratio of E_a/a, the incident-particle energy per nucleon, to the mean excitation potential of atoms of the medium. Proceeding upon this assumption, a universal experimental curve is plotted from the experimental data, which yields the range for energies in the interval $0.5 \leqslant E_a/a \leqslant 200$ Mev for any substance whose excitation potential is known.

THE energy loss of a nonrelativistic particle with charge Z_a and mass number a upon passage through matter with atomic number Z and mass number A is given by the familiar formula

$$\varepsilon = \frac{dE}{dR} = \frac{4\pi Z_a^2 e^4}{mv^2} \frac{N_0 Z}{A} \ln \frac{2mv^2}{I_A}, \qquad (1)$$

where R is the range in mg/cm^2, e and m are the electron charge and mass, N_0 is the Avogadro number, v is the velocity of the particle and I_A is the mean excitation potential of atoms of the stopping material.

In the Bloch approximation ($I_A \sim Z$) specific losses are a function of only the single parameter v^2/Z.[1,2] Subsequent investigations have shown[3] that this approximation is not entirely justified. If the energy properties of atoms of the stopping material are represented approximately by the mean excitation potential, then, in accordance with dimensional considerations and approximation (1), the specific loss must depend only on the ratio of the mean kinetic energy acquired by atomic electrons of the medium to the mean excitation potential. With the incident particle energy E_a given in Mev and I_A in kev, a convenient dimensionless parameter is

$$y = E_a / aI_A. \qquad (2)$$

The formula obtained for the specific loss is then $\epsilon = cf(y)$, comparison of which with approximation (1) determines the constant c:

$$\varepsilon(E_a) = (Z / AI_A) Z_a^2 f(y). \qquad (3)$$

The obvious expression for the range is then

$$R(E_a) = \frac{AI_A^2 a}{ZZ_a^2} F(y), \qquad F(y) = \int_0^y \frac{dt}{f(t)}, \qquad (4)$$

where both f(y) and F(y) depend only on y and in approximation (1) f(y) is given by

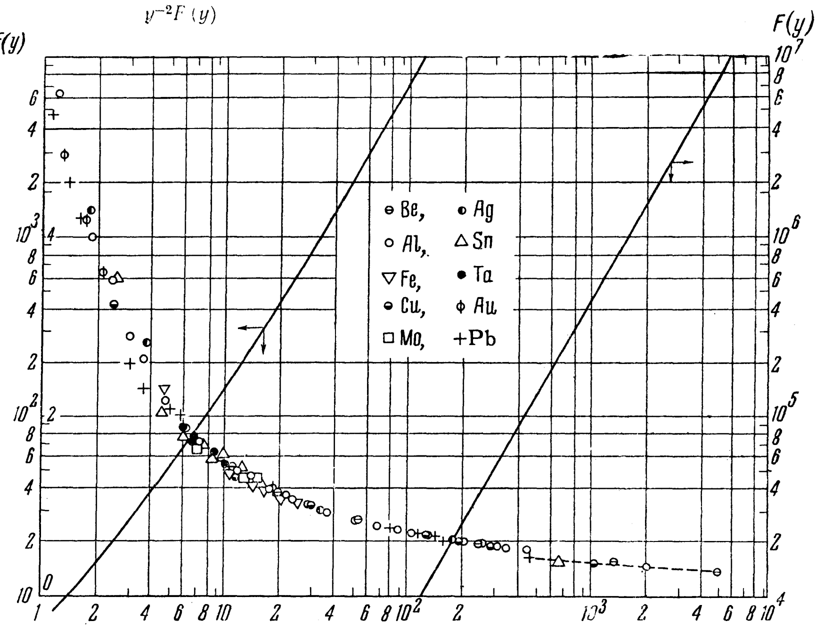
$$f(y) \approx (145/y) \ln(2.17y) \text{ kev}^2 - \text{cm}^2/\text{mg}. \qquad (5)$$

We conclude from (3) and (4) that when experimental data are plotted on a graph with y along one axis and $F(y) = (ZZ_a^2/AI_A^2 a) R(E)$ or $f(y) = (I_A A/ZZ_a^2)\epsilon(E)$ along the other axis we obtain universal curves for determining ranges and specific losses in different media when the mean excitation potentials are known.

We are interested in the plot of F(y) which was based on the following experimental data: in the energy interval 0.2—2 Mev proton ranges were calculated from the experimental stopping powers of Be, Al, Cu, Au and Pb;[2,4] for 1—7 Mev, proton ranges were measured in Fe, Cu, Mo, Cd, Sn, Ta and Pb relative to Al;[5] for 6—18 Mev, absolute proton ranges were measured in Be, Al, Cu, Ag and Au;[6] for 40—120 Mev, the theoretical ranges in Al, Cu and Au were corrected experimentally;[7] finally, the ranges of 190-Mev deuterons (E_p = 95 Mev) were measured in Be, C, Cu, Ag, Pb, and U relative to Al.[8]

It is evident from (2) and (4) that the plotting of F(y) requires knowledge of the mean excitation potential, which has not been determined for all materials. The latest[3] of frequent[2,3,6,9] measurements for aluminum gives $I_{Al} \approx 0.166$ kev; therefore the experimental points for aluminum according to (2) and (4) were plotted first. Then from the data for other materials, making use of these same equations, suitable excitation potentials I_A were selected so that the experimental points would give the best fit to the curve obtained for aluminum. Closest agreement was obtained in the region 6—18 Mev, where absolute proton ranges were measured for several materials.

The results of the analysis are shown in the figure, where for convenience the mentioned experimental data are collected in the plot of $F(y)/y^2$.



Solid curves – Plots of $F(y) = (ZZ_a^2 / AI_A^2 a) R(E)$ mg/cm^2-kev^2 (outside scales on the left and right). Experimental points – values of $F(y)/y^2$ (inside non-logarithmic scale on the left)

As was expected, the experimental points lie in a satisfactory manner on a smooth curve over a wide range of y. The spread of the points results partly from experimental errors in determining ranges and energies $[F(y)/y^2 \approx R(E)/E^2]$. Thus, for example, the results obtained by Rybakov[5] show that the error in the function may be 5% or higher. Points (connected by the dashed line) were also plotted for protons with ≈ 340 Mev in Be, Al, Cu, Sn and Pb;[10] these also lie on a smooth curve but in a less satisfactory manner.

We also note, without giving the results, that experimental data on stopping powers[2,3,8] were used to plot $yf(y) = (A/ZaZ_a^2)\, E\epsilon(E)$ in the region $0.1 \leqslant y \leqslant 10^3$. The fit to a smooth curve is somewhat better than for $y^{-2}F(y)$; this may result from a smaller scattering effect. A calculation of yf(y) by means of the approximate formula (5) agrees with experiment even when $y \gtrsim 1$. However, the calculation of F(y), which is of interest here, by means of the same formula encounters difficulties since the region $y < 1$ is then included, where the formula is unsuitable [F(y) is obtained from (4) by an integration]. For sufficiently large y ($y \gtrsim 5$—10), where the effect of this region can be neglected, (5) gives satisfactory results for the calculation of F(y).

The figure contains the plot of F(y) while the table gives values of I_A for different materials; the plot and table combined enable us to calculate the range for any material at energies 0.5—200 Mev. The excitation potential of an univestigated material can be determined from the plot of $y^{-2}F(y)$, using the range at a single energy. In the absence of such information an estimate can be obtained from the table by means of linear interpolation in accordance with the Bloch approximation.

In conclusion the author wishes to thank M. M. Agrest for a discussion of the results.

Note added in proof, May 29, 1959. A satisfactory fit to the curve of $y^{-2}F(y)$ is also obtained with recently published experimental data on the range of 660-Mev protons in copper and of multiply charged ions with 40—100 Mev in copper and gold [V. P. Zrelov and G. D. Stoletov, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 658 (1959), Soviet Phys. JETP **9**, 461

Element	Z/A	I_A, kev	Element	Z/A	I_A, kev	Element	Z/A	I_A, kev
H	1.000	0.017*	Al	0.483	0.166	Sn	0.421	0.450
Li	0.430	0.038*	Cl	0.481	0.169*	Ta	0.405	0.730
Be	0.445	0.067	Fe	0.465	0.286	W	0.402	0.770*
C	0.500	0.084*	Cu	0.457	0.320	Au	0.400	0.935
N	0.500	0.085*	Mo	0.433	0.450	Pb	0.395	0.730
Air	0.500	0.089*	Ag	0.436	0.530	U	0.386	0.995*
O	0.500	0.097*	Cd	0.427	0.457			

*Data from reference 9, normalized to $I_{Al} = 0.166$ kev.

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