THE ULTRAVIOLET ASYMPTOTIC VALUE FOR THE INTERACTION OF K MESONS WITH BARYONS

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The ultraviolet asymptotic value for the interaction of K mesons with baryons is studied for different kinds of relative baryon parity under the assumption of weak coupling.

1. INTRODUCTION

THE investigation of ultraviolet asymptotic values in quantum field theory can be carried out either by solving the Dyson integral equations for boson and fermion Green's functions together with a specific equation for the source particle,[1,2] or by the method of the renormalization group.[3,4] The weak coupling hypothesis is used in both cases. Moreover, these investigations are limited to the examination of one fermion field with a specific boson field (quantum electrodynamics or meson theory). It seems to us more logical to consider all possible (renormalizable) interactions in the study of the ultraviolet asymptotic value.

As an example we look at the ultraviolet asymptotic value for the interaction of K mesons with baryons (customarily called the theory of medium strong interactions). For this we take into consideration the interaction constants for K mesons with all baryons. The use of the weak-coupling hypothesis in meson theory is not well founded. But there is sufficiently strong proof that the K-baryon interaction constants are significantly less than the π-meson constant $(g_K^2/4\pi \ll g_\pi^2/4\pi)$.[5,6] It can be shown that there exists a reasonable, physical momentum interval in which the weak coupling assumption is legitimate. Special attention is given to the ultraviolet asymptotic value for different kinds of relative baryon parity.

2. THE RENORMALIZATION GROUP IN THE THEORY OF MEDIUM STRONG INTERACTIONS

Assuming charge independence for the interaction of K mesons with baryons, we write the Lagrangian of the system in the form

$$L = L_{\text{coup}} + L_{\text{int}},$$

$$L_{\text{int}} = g_1 \overline{N}_1 \Gamma_1^0 \Lambda K + g_2 N_1 \overline{\Gamma}_2^0 \boldsymbol{\tau} \textstyle\sum K + g_3 \overline{N}_2 \Gamma_3^0 \Lambda \overline{K} + g_4 N_2 \overline{\Gamma_4^0 \boldsymbol{\tau}} \sum \overline{K} + h(\overline{K}K)(\overline{K}K) + \text{Herm. conj.} \qquad (1)$$

where

$$N_1 = \begin{pmatrix} p \\ n \end{pmatrix}, \quad N_2 = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}, \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix},$$

$$\Lambda = \Lambda^0, \quad \textstyle\sum = \begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix},$$

$$\Gamma_i^0 = 1, \ \gamma_5 \quad (i = 1, \ldots, 4), \quad \overset{+}{\gamma}_5 = -\gamma_5.$$

The choice of 1 or γ_5 depends on the relative parity of the baryons. We represent the fermion Green's function in the form

$$G_B(p) = (S_B(p^2)\,\hat{p} + S_B'(p^2)\,m_B)(p^2 - m_B^2)^{-1}. \qquad (2a)$$

The index B on S and m means the baryons (N_1, N_2, Λ, Σ). We write the K-meson Green's function and the source particles as

$$D_K(p) = S_K(p^2)/(p^2 - m_K^2); \qquad (2b)$$

$$\Gamma_i = \Gamma_i^0 G_i, \quad G_i = G_i(p^2, q^2, (p-q)^2), \quad i = 1, \ldots, 4;$$
$$\square = \square(p'^2, q'^2, p^2, q^2, (p'-q)^2, (p+q)^2). \qquad (2c)$$

The source $\square$ corresponds to the diagrams with four incoming K-meson lines.

For brevity, we do not write out the renormalization group. There is the following system of invariant charges:

$$\sigma_1 = S_{N_1} S_\Lambda S_K G_1^2 g_1^2, \quad \sigma_2 = S_{N_1} S_\Sigma S_K G_2^2 g_2^2,$$
$$\sigma_3 = S_{N_2} S_\Lambda S_K G_3^2 g_3^2,$$
$$\sigma_4 = S_{N_2} S_\Sigma S_K G_4^2 g_4^2, \quad \rho = S_K^2 \square h. \qquad (3)$$

The Lie equations for the invariant charges (3) are given in Appendix 1. We have written down these equations for the ultraviolet asymptotic case.

Let us look at a simpler problem. We notice that the system (A1) is invariant under the change $\sigma_1 \to \sigma_3$, $\sigma_3 \to \sigma_1$, $\sigma_2 \to \sigma_4$, $\sigma_4 \to \sigma_2$. This circumstance allows us to transform this system into a simpler one. Let $g_1 = g_3$, $g_3 = g_4$ in the Lagrangian (1). Polkinghorne employed such a Lagrangian.[7] Neglecting the mass differences between the baryons, which is fully justified in the ultraviolet asymptotic case, we get

$$\sigma_1 = \sigma_3, \qquad \sigma_2 = \sigma_4, \tag{4}$$

and the system (A1) leads to a system of three equations

$$d\sigma_i / dx = \sigma_i \sum_{j=1}^{2} \beta_i^j(\varepsilon)\sigma_j + \varkappa_1 \sigma_i \rho^2 \qquad (i = 1, 2);$$

$$d\rho / dx = 3F_0[\sigma_1^2 + \sigma_1\sigma_2(2+\varepsilon) + 9\sigma_2^2]$$

$$+ 2F_1[\sigma_1 + 3\sigma_2]\rho + F_2\rho^2, \tag{5}$$

where

$$\beta_i^j(\varepsilon) = \frac{1}{32\pi^2}\begin{pmatrix} 5 & 3 + 12(1-\varepsilon) \\ 1 + 4(1-\varepsilon) & 7 \end{pmatrix},$$

$x = \ln(p^2/\lambda^2)$, λ the normalization momentum.

The scalar functions S, G_i, and □ are determined by the equation

$$\frac{\partial}{\partial x}\ln S(x, g_1^2, g_2^2, h) = \frac{\partial}{\partial \xi} S^0(\xi, \sigma_1, \sigma_2, \rho)|_{\xi=1}; \tag{6}$$

S^0 is obtained with the aid of perturbation theory. The parameter ϵ takes on the two values ± 1. The choice of + or − depends on the type of relative parity of the baryons. We denote by I_{AB} the parity of baryon A relative to baryon B, then

$$\begin{aligned} \varepsilon = +1, \quad & \text{if } I_{\Lambda N_1} = I_{\Sigma N_1} = I_{\Lambda N_2} = I_{\Sigma N_2} = \pm 1, \\ & \text{or } I_{\Lambda N_1} = I_{\Sigma N_1} = -I_{\Lambda N_2} = -I_{\Sigma N_2} = \pm 1, \\ & \text{or } -I_{\Lambda N_1} = I_{\Sigma N_1} = -I_{\Lambda N_2} = I_{\Sigma N_2} = \pm 1; \\ \varepsilon = -1, \quad & \text{if } -I_{\Lambda N_1} = I_{\Sigma N_1} = I_{\Lambda N_2} = -I_{\Sigma N_2} = \pm 1. \end{aligned} \tag{7}$$

3. INVESTIGATION OF THE ULTRAVIOLET ASYMPTOTIC VALUE

We shall not take into account K-K meson scattering processes. Then in the version given by (4), the Lie equations have the form

$$d\sigma_1 / dx = \beta_1^1\sigma_1^1 + \beta_1^2(\varepsilon)\sigma_1\sigma_2,$$

$$d\sigma_2 / dx = \beta_2^1(\varepsilon)\sigma_1\sigma_2 + \beta_2^2\sigma_2^2. \tag{8}$$

We examine two cases.

A. $\epsilon = -1$. From Eq. (8) we get a formula determining the dependence of σ_1 on σ_2. In the (σ_1, σ_2) plane we have the family of integral curves shown in Fig. 1. The straight line $\sigma_1 = 5\sigma_2$ is stationary, so that it is sufficient to solve Eq. (8) for this value. We get

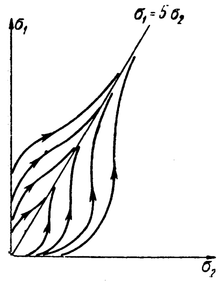

FIG. 1

$$\sigma_1 = 5\sigma_2, \qquad \sigma_2 = \left[1 - \frac{13}{8\pi^2}g_2^2 x\right]^{-1}. \tag{9}$$

With the help of (9) we find the form of the scalar functions S and G_i:

$$S_i = \left[1 - \frac{13}{8\pi^2}g_2^2 x\right]^{-\alpha_i/13}. \tag{10}$$

The constants α_i are

$S_i =$	$S_{N_1} = S_{N_2}$	S_Λ	S_Σ	S_K	$G_1 = G_3$	$G_2 = G_4$
$\alpha_i =$	2	5	1	16	−5	−3

B. $\epsilon = +1$. The integral curves in the (σ_1, σ_2) plane for this case are shown in Fig. 2. The particular solution for the straight line $\sigma_1 = \sigma_2$ is not stationary. The integral curves go either to the asymptotic $\sigma_1 = \sigma_2^5$ or to $\sigma_1 = \sigma_2^{3/7}$. We solve (8) for these asymptotes.

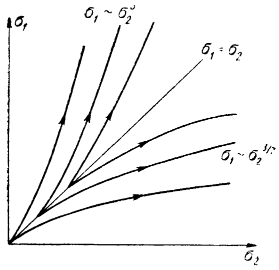

FIG. 2

a) The asymptote $\sigma_1 = \sigma_2^5$

$$\sigma_1 = g_1^2\left[1 - \frac{5}{32\pi^2}g_1^2 x\right]^{-1},$$

$$\sigma_2 = g_2^2\left[1 - \frac{5}{32\pi^2}g_1^2 x\right]^{-1/5}; \tag{11}$$

b) The asymptote $\sigma_1 = \sigma_2^{3/7}$

$$\sigma_1 = g_1^2\left[1 - \frac{7}{32\pi^2}g_2^2 x\right]^{-3/7},$$

$$\sigma_2 = g_2^2\left[1 - \frac{7}{32\pi^2}g_2^2 x\right]^{-1}. \tag{12}$$

The scalar functions S and G_i, found with the aid of (11) and (12), are respectively

S_i	α_i	β_i	α'_i	β'_i	S_i	α_i	β_i	α'_i	β'_i
$S_{N_1} = S_{N_2}$	1/5	3/4	3/7	1/4	S_K	8/5	6	24/7	2
S_Λ	4/5	0	0	1	$G_1 = G_3$	—4/5	—3	—12/7	—1
S_Σ	0	1	4/7	0	$G_2 = G_4$	—4/5	—3	—12/7	—1

$$S_i = \left[1 - \frac{5}{32\pi^2} g_1^2 x\right]^{-\alpha_i} \exp\left\{-\beta_i \frac{g_2^2}{g_1^2}\left[1 - \frac{5}{32\pi^2} g_1^2 x\right]^{4/5}\right\},$$

$$S_i = \left[1 - \frac{7}{32\pi^2} g_2^2 x\right]^{-\alpha'_i} \times \exp\left\{-\beta'_i \frac{g_1^2}{g_2^2}\left[1 - \frac{7}{32\pi^2} g_2^2 x\right]^{4/7}\right\}. \quad (13)$$

The values α_i, β_i, α'_i, and β'_i are given in the table.

We shall find which curve is followed by a given point in the three-dimensional space $(\rho, \sigma_1, \sigma_2)$ when x goes to infinity. To do this we determine the integral curves of the system of equations got by dividing the third equation in (5) by the first and the second:

$$\frac{d\rho}{d\sigma_1} = \frac{3F_0[\sigma_1^2 + \sigma_1\sigma_2(2+\varepsilon) + 9\sigma_2^2] + 2F_1[\sigma_1 + 3\sigma_2]\rho + F_2\rho^2}{\beta_1^1\sigma_1^2 + \beta_1^2(\varepsilon)\sigma_1\sigma_2 + \varkappa_1\sigma_1\rho^2},$$

$$\frac{d\rho}{d\sigma_2} = \frac{3F_0[\sigma_1^2 + \sigma_1\sigma_2(2+\varepsilon) + 9\sigma_2^2] + 2F_1[\sigma_1 + 3\sigma_2]\rho + F_2\rho^2}{\beta_2^1(\varepsilon)\sigma_1\sigma_2 + \beta_2^2\sigma_2^2 + \varkappa_1\sigma_2\rho^2}. \quad (14)$$

The three dimensional space ρ, σ_1, σ_2 $(\sigma_1, \sigma_2 \geq 0)$ is divided into three regions by the surfaces ρ_1, ρ_2, the roots of the trinomials quadratic in ρ in the numerators in the right hand terms in (14); $\rho_1 > \rho_2$:

$$\text{I) } \rho \geqslant \rho_1(\sigma_1, \sigma_2, \varepsilon),$$
$$\text{II) } \rho_2(\sigma_1, \sigma_2, \varepsilon) \leqslant \rho \leqslant \rho_1(\sigma_1, \sigma_2, \varepsilon),$$
$$\text{III) } \rho \leqslant \rho_2(\sigma_1, \sigma_2, \varepsilon). \quad (15)$$

The derivatives $d\rho/d\sigma_1$, $d\rho/d\sigma_2$ are positive in region II and negative in regions I and III. Near the origin of the coordinates we neglect the $\sigma_1\rho^2$ and $\sigma_2\rho^2$ terms and write the solution of (14) in the form [see (A2)]

$$\sigma_1 = \lambda_1\rho, \qquad \sigma_2 = \lambda_2\rho. \quad (16)$$

For the ratio $\lambda = \lambda_1/\lambda_2$ we get three values $\lambda = 0$, $\lambda = \infty$, and $0 < \lambda(\epsilon) < \infty$. The first two give a family of integral curves corresponding to the planes (σ_2, ρ): $\sigma_1 = 0$ and (σ_1, ρ): $\sigma_2 = 0$. A detailed analysis shows that in these planes the family is qualitatively the same as in reference 8 (see Fig. 3). In the case $0 < \lambda(\epsilon) < \infty$ [see (A4)] we have a family of integral curves in the plane which includes the ρ axis and which forms with the (σ_2, ρ) plane an angle θ such that

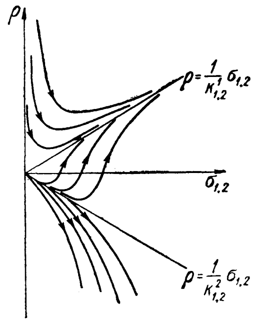

FIG. 3

$$\tan\theta = \lambda(\varepsilon). \quad (17)$$

It appears that the integral curves in this plane are qualitatively the same as those in the (σ_1, ρ) for (σ_2, ρ) planes but the straight lines (A2) or (A3) should be respectively replaced by those given in (A4).

Points lying in the neighborhood of the origin on these three planes can only move along the integral curves as x goes to infinity and cannot leave these planes. Points somewhat away from the (σ_1, ρ) and (σ_2, ρ) planes will move along curves removed from these planes as x goes to infinity. This means that the (σ_1, ρ) and (σ_2, ρ) planes are not stationary. To clear up the stationary quality of the plane corresponding to $0 < \lambda(\epsilon) < \infty$, we look at the two cases $\epsilon = +1$ and $\epsilon = -1$. In the neighborhood of the origin the quantity $d\sigma_1/d\sigma_2$ does not depend on ρ and is equal to

$$d\sigma_1/d\sigma_2 = (\beta_1^1\sigma_1^2 + \beta_1^2(\varepsilon)\sigma_1\sigma_2)/(\beta_2^1(\varepsilon)\sigma_1\sigma_2 + \beta_2^2\sigma_2^2). \quad (18)$$

Equation (18) determines the lines of intersection of the integral curves of the system (14) with the plane ρ = const. In the $\epsilon = -1$ case they are shown in Fig. 1. We see that the plane forming the angle given by $\tan\theta = \lambda(-1) = 5$ with the (σ_2, ρ) plane is stationary. For the $\epsilon = +1$ case the analogous curves are shown in Fig. 2, from which it follows that the plane given by $\tan\theta = \lambda(+1) = 1$ is not stationary.

To get a fuller family of integral curves we solve (14) and (5) for the case $|\rho| \gg \sigma_1, \sigma_2$. The solution is

$$\rho = h/(1 - hF_2x),$$
$$\sigma_1 = \sigma_2 = g_1^2 \exp\left[\frac{\varkappa_1}{F_2}(\rho - h)\right]$$
$$= g_1^2 \exp\left[h^2\varkappa_1 x/(1 - hF_2x)\right]. \quad (19)$$

A. Case $\epsilon = -1$. There exists a surface $\rho = \varphi(\sigma_1, \sigma_2, -1)$, in which the lines (A2) with $k_1^2 < 0$, (A3) with $k_2^2 < 0$, and (A4) with $\lambda_2^2(-1) < 0$ lie. This surface lies under the surface

$$\rho = \rho_2(\sigma_1, \sigma_2, -1), \qquad \varphi(\sigma_1, \sigma_2, -1) \leqslant \rho_2(\sigma_1, \sigma_2, -1).$$

There are two regions of the space $(\rho, \sigma_1, \sigma_2)$:

$$\text{I) } \rho \geqslant \varphi(\sigma_1, \sigma_2, -1); \quad \text{II) } \rho \leqslant \varphi(\sigma_1, \sigma_2, -1)$$
$$(\sigma_1, \sigma_2 \geqslant 0). \quad (20)$$

Each point $(\rho, \sigma_1, \sigma_2)$ in region I moves onto the line (A4) with $\lambda_2^1(-1) > 0$, as x goes to ∞. Every point in region II, as x goes to ∞, moves along curves whose asymptotic form is given by formula (19) with $\rho < 0$. Since in this case the solution (A4) with $\lambda_2^1(-1) > 0$ is stationary, it is sufficient to solve Eq. (5) for this line. We get the formulas determining the invariant coupling constants (A4) with $\lambda_2^1(-1) > 0$ and Eq. (9). The functions S and G found with the aid of these formulas have the form (10). We add here the formula for the source

$$\square = \left[1 - \frac{13}{8\pi^2} g_2^2 x\right]^{\alpha_\square}, \qquad \alpha_\square > 0. \quad (21)$$

We note that the sources G_i depend on three arguments and that it is therefore possible to have various kinds of ultraviolet asymptotic values; but here the situation is the same as in meson theory: they all coincide. The source $\square$ depends on six arguments [see (2c)]. It seems that in this case the ultraviolet asymptotes do not all coincide and that $\alpha_\square$ depends on the kind of ultraviolet asymptote (for details, see the work of Ginzburg and Shirkov[9]).

B. Case $\epsilon = +1$. There exists a surface $\rho = \varphi(\sigma_1, \sigma_2, +1)$ in which lie the lines (A2) with $k_1^2 < 0$, (A3) with $k_2^2 < 0$, and (A4) with $\lambda_2^2(+1) < 0$. This surface lies under the surface

$$\rho = \rho_2(\sigma_1, \sigma_2, +1) < 0, \qquad \varphi(\sigma_1, \sigma_2, +1) \leqslant \rho_2(\sigma_1, \sigma_2, +1).$$

There are three regions of the space:

$$\text{I) } \rho \geqslant \varphi(\sigma_1, \sigma_2, +1), \qquad \sigma_1 \geqslant \sigma_2 \geqslant 0;$$
$$\text{II) } \rho \geqslant \varphi(\sigma_1, \sigma_2, +1), \qquad 0 \leqslant \sigma_1 \leqslant \sigma_2;$$
$$\text{III) } \rho \leqslant \varphi(\sigma_1, \sigma_2, +1), \qquad \sigma_1, \sigma_2 > 0. \quad (22)$$

As x goes to infinity, points in region I, moving along the integral curves, go onto the curves (A5), points in region II go onto curves (A6), and points in region III onto curves (19). In this case there is no stationary solution in the form of a straight line and therefore the source terms and Green's functions behave according to a law different from the one in electrodynamics or in meson theory.

4. CONCLUSION

1. The investigated integral curves in the $(\rho, \sigma_1, \sigma_2)$ space are not closed. This means that there exists for them, just as in electrodynamics and meson theory, a way of going beyond the limits of the weak coupling for increasing momenta. In our view the existence of closed integral curves in the theory considered is scarcely probable: we should be left inside the limits of the weak coupling theory. However, it is still useful to investigate in detail Eqs. (A1) or (5).

2. The behavior of Green's function and of the source terms essentially depends on the relative parity of the baryons. For some kinds of relative baryon parity (7) with $\epsilon = -1$ there exists a stationary solution in the form of a straight line in the space $(\rho, \sigma_1, \sigma_2)$, and then the Green's function and the source terms behave according to the standard law (10), (21).[1-4] For other sources (7) with $\epsilon = +1$ there is no stationary solution in the form of a line in the $(\rho, \sigma_1, \sigma_2)$ space. The connection between the invariant charges is shown by formulas, either (A5) or (A6). Here the Green's functions and the source terms behave entirely differently [see (13)].

3. The extent to which the assumption which we made of weak coupling is valid $(\rho, \sigma_1, \sigma_2 \ll 1)$ will be clear if one succeeds in determining exactly the magnitude of the constant g_i in (1). From the estimates got in references 5 and 6, it evidently follows that $g_1^2/4\pi \sim 1$. For such a value of the constant the poles of the invariant charges (9) and (11) are distributed in the neighborhood, respectively, of $|p| \sim E \sim 2.5$ Bev and $|p| \sim E \sim 9$ Bev.

4. In the investigation of the ultraviolet asymptotes, we should have taken into account the interaction of π mesons with baryons, which could change our results in an essential way. For the present we lay this question aside.

The author expresses his deep thanks to D. V. Shirkov for numerous remarks and for constant interest in the work.

APPENDIX

1. The Lie equations for the ultraviolet asymptotic case.

$$\frac{d\sigma_i}{dx} = \sigma_i \sum_{s=1}^{4} \varkappa_i^s \sigma_s + \varepsilon \varkappa_i^0 [\sigma_1 \sigma_2 \sigma_3 \sigma_4]^{1/2} + \varkappa_1 \sigma_i \rho^2 \qquad (i = 1, \ldots, 4),$$

$$\frac{d\rho}{dx} = F_0 [\sigma_1^2 + 3\sigma_1\sigma_2 + 9\sigma_2^2 + \sigma_1\sigma_3 + 3\varepsilon [\sigma_1\sigma_2\sigma_3\sigma_4]^{1/2} + 9\sigma_2\sigma_4 + \sigma_3^2 + 3\sigma_3\sigma_4 + 9\sigma_4^2] + F_1 [\sigma_1 + 3\sigma_2 + \sigma_3 + 3\sigma_4]\rho + F_2\rho^2; \qquad \text{(A1)}$$

$$\varkappa_i^s = \frac{1}{32\pi^2} \begin{vmatrix} 3 & 3 & 2 & 12 \\ 1 & 5 & 4 & 2 \\ 2 & 12 & 3 & 3 \\ 4 & 2 & 1 & 5 \end{vmatrix};$$

$$\varkappa_1^0 = \varkappa_3^0 = -3/8\pi^2, \qquad \varkappa_2^0 = \varkappa_4^0 = -1/8\pi^2;$$

$$F_0 = 2F_1 = -2F_2/3 = 1/4\pi^2, \qquad \varkappa_1 > 0.$$

2. We look for the solution of (14) in the neighborhood of the origin in the form (16). For $\lambda = \lambda_1/\lambda_2$ we get

a) $\lambda = \infty, \qquad \sigma_1 = k_{1,2}^1 \rho,$

$$k_{1,2}^1 = -\frac{2F_1 - \beta_1^1}{6F_0} \pm \left[\frac{(2F_1 - \beta_1^1)^2}{(6F_0)^2} - \frac{F_2}{3F_0}\right]^{1/2}; \qquad \text{(A2)}$$

b) $\lambda = 0, \ \sigma_2 = k_{1,2}^2 \rho,$

$$k_{1,2}^2 = -\frac{6F_1 - \beta_2^2}{54F_0} \pm \left[\frac{(6F_1 - \beta_2^2)^2}{(54F_0)^2} - \frac{F^2}{27F_0}\right]^{1/2}; \qquad \text{(A3)}$$

c) $\sigma_1 = \lambda(\varepsilon), \quad \lambda_2^{1,2}(\varepsilon)\rho, \quad \sigma_2 = \lambda_2^{1,2}(\varepsilon)\rho, \quad \lambda(+1) = 1,$ $\lambda(-1) = 5,$

$$\lambda_2^{1,2}(+1) = \frac{(4F_1 - 1/8\pi^2)}{3F_0 \cdot 13} \pm \left[\left(\frac{4F_1 - 1/8\pi^2}{3F_0 \cdot 13}\right)^2 - \frac{F_2}{3F_0 \cdot 13}\right]^{1/2},$$

$$\lambda_2^{1,2}(-1) = -\frac{[8F_1 - 13/16\pi^2]}{3F_0 \cdot 39} \pm \left[\left(\frac{8F_1 - 13/16\pi^2}{3F_0 \cdot 39}\right)^2 - \frac{F_2}{3F_0 \cdot 39}\right]^{1/2}. \qquad \text{(A4)}$$

3. The asymptotic solutions of systems (5) and (13). For the case $|\rho| \sim \sigma_1 \gg \sigma_2$

$$\sigma_1 = \sigma_2^5 = g_1^2 \left[1 - \frac{5}{32\pi^2} g_1^2 x\right]^{-1},$$

$$[\rho/\sigma_1 - a_1]^{A/\alpha_2} [\rho/\sigma_1 - a_2]^{-A/\alpha_2} = c\sigma_1, \qquad \text{(A5)}$$

where

$$c = g_1^{-2} [h/g_1^2 - a_1]^{A/\alpha_2} [h/g_2^2 - a_2]^{-A/\alpha_2},$$

$$a_{1,2} = -\alpha_1/2\alpha_2 \pm [(\alpha_1/2\alpha_2)^2 - \alpha_0/\alpha_2]^{1/2},$$

$$A = [(\alpha_1/\alpha_2)^2 - 4\alpha_0/\alpha_2]^{1/2}, \qquad \alpha_0 = 3F_0/\beta_1^1,$$

$$\alpha_1 = 2F_1/\beta_1^1 - 1, \qquad \alpha_2 = F_2/\beta_1^1.$$

For the case $|\rho| \sim \sigma_2 \gg \sigma_1$

$$\sigma_1 = \sigma_2^{3/7}, \qquad \sigma_2 = g_2^2 \left[1 - \frac{7}{32\pi^2} g_2^2 x\right]^{-1},$$

$$[\rho/\sigma_2 - a_1']^{A'/\alpha_2'} [\rho/\sigma_2 - a_2']^{-A'/\alpha_2'} = c'\sigma_2, \qquad \text{(A6)}$$

where A', c', $a_{1,2}'$ have the same structure as in the preceding case, but one must make the substitutions

$$\alpha_0 \to \alpha_0' = 27F_0/\beta_2^2, \qquad \alpha_1 \to \alpha_1' = 6F_1/\beta_2^2 - 1,$$

$$\alpha_2 \to \alpha_2' = F_2/\beta_2^2, \qquad g_1 \to g_2.$$

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