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ANGULAR DISTRIBUTION OF SHOWER PARTICLES IN STARS PRODUCED BY HIGH-ENERGY PARTICLES

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An analysis is carried out of the angular distribution of a unique star consisting of 20 + 15 + 59p, and probably produced by a proton of energy > 1000 Bev. The angular distribution of shower particles is characterized by two maxima and is explained by the anisotropic angular distribution and a power-law energy spectrum of the produced particles in the c.m.s. The presence of a large number of strongly ionized particles indicates that the Heitler-Terreaux theory does not give a correct description of π meson production. The angular distribution of 11 other showers with the same characteristic anisotropy is also investigated.

INTRODUCTION

In the scanning of an emulsion stack exposed to cosmic radiation at the latitude of Moscow at an altitude of ~ 30 km, an event was recorded, a shower 35 + 59p, in which a proton interacted with an emulsion nucleus. A general picture of this is given in Fig. 1. Half of all shower particles are found within a cone with the opening angle $\theta_{1/2} = 1°39'$, and the mean geometrical angle θ_g, calculated according to the formula

$$\ln \tan \theta_g = \frac{1}{n_s} \sum_{i=1}^{n_s} \ln \tan \theta_i$$

(where n_s is the number of charged particles in the shower), equals $2°42'$.

Assuming a symmetrical emission of particles in c.m.s. and their mono-energetic distribution, we obtain for the energy γ_c of the nucleon, in the c.m.s., $\gamma_c(\theta_{1/2}) = 34.7$ and $\gamma_c(\theta_g) = 21.2$. The length of the tunnel l, punched by the incident nucleon in the target nucleus,[1] in units of the nucleonic diameter d, is $l/d = 8$. Consequently, the interaction involved a heavy emulsion nucleus, as confirmed also by the presence of a large number of black and grey tracks $N_h = 35$.

Taking into account the collision with a tunnel containing approximately 8 nucleons, the energy of the incident nucleon is equal[2] to $E = 19 \times 10^3$ Bev. An estimate of the energy of the primary particle, taking into account the energy spectrum of the shower (secondary) particles and the energy spectrum of the shower-producing (primary) particles, was carried out similarly to that in reference 3. If the spectrum of shower-producing particles is $\sim E^{-2.7}$, and the spectrum of shower particles is $\sim E_0^{-2}$, then

$$\ln (2\gamma_c^2) = \bar{x} + \ln B^2 - 2.7\,\sigma^2 / n_s,$$

where

$$\bar{x} = \frac{1}{n_s} \sum_{i=1}^{n_s} \ln \frac{2}{\tan^2 \theta_i}, \qquad B = 0.6, \qquad \sigma^2 = \frac{1}{n_s} \sum_{i=1}^{n_s} (x_i - \bar{x})^2.$$

Hence, $\gamma_c = 8.6$, and the energy is $E = 1280$ Bev.

In the estimate of the energy it is necessary to take into account the fluctuations in the backward-

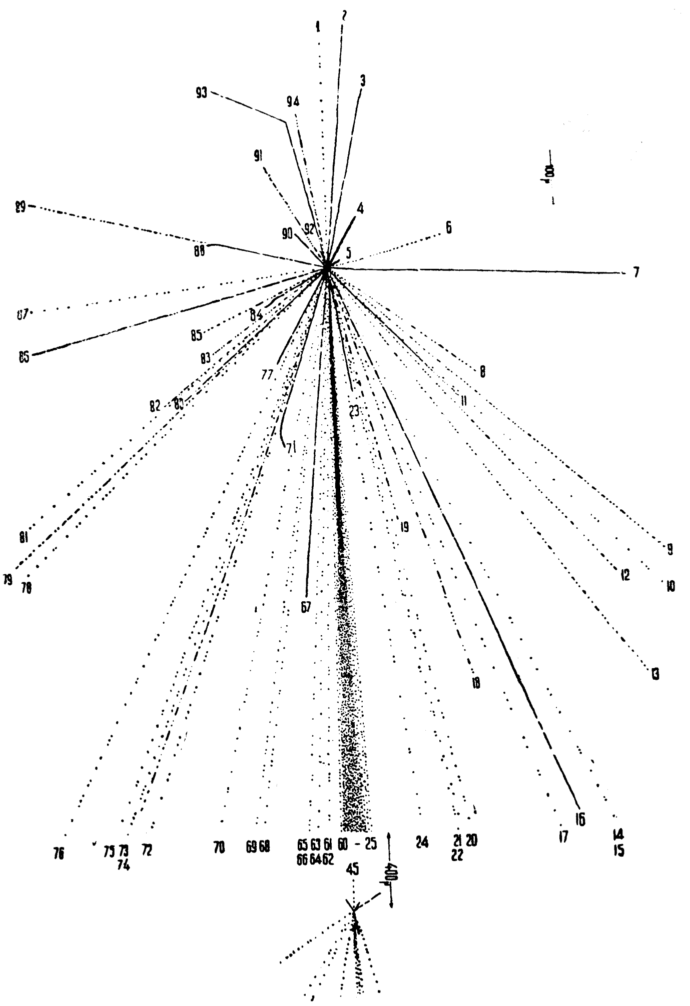

FIG. 1. Microgram of the shower 20 + 15 + 59 p and of the secondary interaction 4 + 8 (p or π) produced by a charged particle with energy E = 23.5 Bev.

forward distribution of the particles in c.m.s., and also the fluctuations in the angular distribution. The errors in the energy, due to fluctuations in the distribution of particles in c.m.s., are as a rule larger than the errors due to fluctuations of the angular distribution. Accordingly, we take into account only the errors ΔE due to the first factor. If the shower particles are anisotropically distributed in the c.m.s., $N(\theta^*) \sim \cos^2\theta^*$, then the fraction of particles f emitted in the angle θ_f^* is equal to

$$f = \int_0^{\theta_f^*} \sin\theta^* N(\theta^*)\, d\theta^* \Big/ \int_0^{\pi} \sin\theta^* N(\theta^*)\, d\theta^* = (1 - \cos^3\theta_f^*)/2.$$

If we take into account the fluctuations in the forward-backward distribution, then $n_s/2 + \sqrt{n_s/2}$ of the particles are contained in the angle θ_1^*, and $n_s/2 - \sqrt{n_s/2}$ are contained in the angle θ_2^*. The angles θ_1^* and θ_2^* can be found from the relation

$$f = \tfrac{1}{2}(1 - \cos^3\theta_f^*) = \tfrac{1}{2} \pm \sqrt{1/2n_s}.$$

The corresponding angles in the laboratory system, θ_{min} and θ_{max}, give the possible deviations from $\theta_{1/2}$ and are given by the formula:

$$\tan\theta_{\substack{min\\max}} = \sin\theta_f^* / \gamma_c(1 + \cos\theta_f^*).$$

The angles θ_{min} and θ_{max}, calculated for various forms of the angular distribution in the c.m.s. (isotropy, $\sim\cos^2\theta^*$, $\sim\cos^4\theta^*$), are given in Table I.

TABLE I

	$\theta_{1/2}$	θ_{max}	θ_{min}	γ_c^{max}	γ_c^{min}
Experiment	1°39′	6°01′	0°54′	63.7	9.7
Isotropic distribution	1°39′	1°59′	1°29′	38.6	28.8
$\sim\cos^2\theta^*$	1°39′	3°09′	0°52′	66.2	18.2
$\sim\cos^4\theta^*$	1°39′	4°01′	0°46′	74.7	14.2

Experimentally, allowance for the redistribution of particles in the c.m.s. system reduces to a determination of the angles θ_{min} and θ_{max}, which contain $n_s/2 \pm \sqrt{n_s/2}$ particles in the laboratory system. The values of the angles θ_{min} and θ_{max} for the shower 35 + 59p are also given in Table I.

It can be seen from the comparison that the experimental data agree satisfactorily with theoretical data for an anisotropic distribution; the higher the degree of anisotropy, the better the agreement. Such a conclusion confirms our assumption about the character of the anisotropy ($\sim\cos^{2n}\theta^*$) of the analyzed 35 + 59p shower.

The energy determined, taking the fluctuations into account, is equal to

$$E = \left(19.2 \begin{smallmatrix} +50.7 \\ -14.0 \end{smallmatrix}\right) \cdot 10^3 \text{ Bev}.$$

For such a large energy of the primary particle the star should contain, according to Heitler and Terreaux,[4] a small number of heavily ionizing particles (3 or 4), since the energy of the excitation of the nucleus would be small in this case. However, the analyzed shower contains $N_h = 35$. Hence it follows that the theory of Heitler and Terreaux cannot explain the experimental results.

ANGULAR DISTRIBUTION OF SHOWER PARTICLES OF THE STAR 35 + 59p

The differential angular distribution of shower particles of the star 35 + 59p is given in Fig. 2. It has two different maxima for the narrow and for the diffused cones. If we assume[5] that in the case of a collision between a nucleon and a nucleus

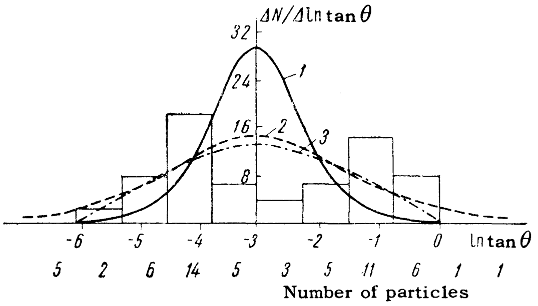

FIG. 2. Histogram of the angular distribution of shower particles of the star in the laboratory system. Curve 1 – isotropic distribution in the c.m.s.; curve 2 – angular distribution according to Heisenberg; curve 3 – according to Landau.

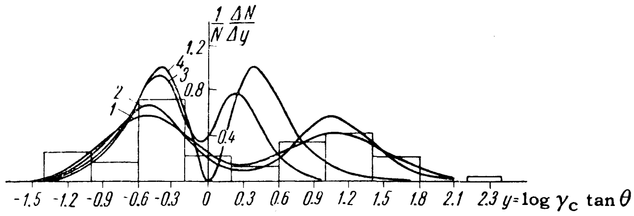

FIG. 3. Differential angular distribution of shower particles in the laboratory system. Curves 1 and 2 – Gaussian and isotropic distributions in the c.m.s., calculated for each maximum separately; curves 3 and 4 – for anisotropy $\cos^2\theta^*$ in the c.m.s. taking into account the energy spectrum of the produced particles, and assuming them to be monoenergetic, respectively.

the angular distribution does not differ essentially from the angular distribution of mesons produced in a nucleon-nucleon collision, then, as can be seen from Fig. 2, the theories of Heisenberg[6] and Landau[7] do not explain the observed angular distribution [the probability of a fit is $P(\chi^2) < 0.01$].

The possibility of explaining each maximum of the histogram by an isotropic angular distribution of the produced particles (Fig. 3, curve 2) in a certain coordinate system [$P(\chi^2) \sim 0.75$] makes it possible to use the Takagi model for the production of secondary particles.[8] According to that model, the mesons are produced not at the moment of collision but considerably later. After a collision in the c.m.s. the nucleons are strongly excited and move away from the center of collisions, each conserving its initial energy but having a smaller momentum and, consequently, a larger mass. If we neglect the angular momentum, the angular distribution of produced mesons will be isotropic in a system of coordinates where the excited nucleon is at rest. The emission of mesons by each nucleon follows the statistical theory of Fermi.[9] Thus, in the c.m.s., two sources of mesons are present, and the angular distribution of the produced meson is anisotropic.

It can be assumed that the observed two maxima of the differential angular distribution of Fig. 3 are the results of emission of mesons by two independent centers moving in different directions in c.m.s. These centers are fully equivalent: 30 particles are contained in a narrow angle, while 29 are contained in the diffused one. If the coordinate systems fixed on the emitting centers move in the laboratory system with velocities v_1 and v_2, and the velocity of c.m.s. is v_c, then the centers move in the c.m.s. with velocities $\bar{v}_1$ and $\bar{v}_2$, which can be found from the transformation formula

$$\bar{v}_1 = \frac{v_1 - v_c}{1 - v_1 v_c / c^2}, \qquad \bar{v}_2 = \frac{v_c - v_2}{1 - v_c v_2 / c^2},$$

where

$$v_1 = \sqrt{\gamma_1^2 - 1} / \gamma_1, \qquad v_2 = \sqrt{\gamma_2^2 - 1} / \gamma_2,$$
$$v_c = \sqrt{\gamma_c^2 - 1} / \gamma_c.$$

Here γ_1 and γ_2 are the energies of the emitting centers in the laboratory system. These are determined from the angular distribution of the narrow and wide cones separately. From these formulae we find, in the c.m.s. $\bar{\gamma}_1 = 2.85$ and $\bar{\gamma}_2 = 2.95$; we can assume that $\bar{\gamma}_1 = \bar{\gamma}_2 \equiv \bar{\gamma}$.

We plotted the angular distribution in the c.m.s. for different values of $\bar{\gamma}$ (1.5 and 2.5) and compared them with the experimental histogram constructed under the assumption $\beta_c = \beta_\pi^*$. Both curves are different from the histogram; consequently, the model shown above does not describe the observed meson distribution in the c.m.s. even for $\beta_c = \beta_\pi^*$. If we calculate $\bar{\gamma}$, γ_c', and the inelasticity factor k, as was done in reference 10, then the obtained values $\bar{\gamma} = 3.02$ and $\gamma_c' = \sqrt{\gamma_1\gamma_2} = 21.52$ differ very little from our values of $\bar{\gamma}_1$, $\bar{\gamma}_2$, and γ_c, and the inelasticity factor $k = 1.5 n_s E_\pi \bar{\gamma} / 2\gamma_c' \approx 3$ for $E_\pi = 0.5$ Bev is substantially larger than unity, which has no physical meaning.

For a full analysis of the star under consideration, from the point of view of the Takagi model, we calculated the excited masses of the colliding particles. Applying the energy and momentum conservation laws to a nucleon-nucleon collision, we obtained $M_1/M_2 = M_1^*/M_2^* \approx 1$ for the ratio of the excited masses in the laboratory system and in the c.m.s. For a collision of a nucleon and a nucleus, the ratio is greatly different from unity: $M_n/M_{tunnel} = 0.09$. After such a detailed analysis, we came to the conclusion that the Takagi model does not explain all features of the observed showers.

Next, to ascertain the shape of the true angular distribution of the mesons, each maximum of the histogram was approximated by a Gaussian curve with a distribution dispersion of shower particles

$\sigma_1 = 0.39$ for the narrow cone and $\sigma_2 = 0.44$ for the diffused cone (see Fig. 3, curve 1). Then, the function describing the angular distribution in the laboratory system is of the form

$$N(y) = 0.55\,(1.03\exp\{-(y+0.52)^2/0.30\} + 0.90\exp\{-(y-1.08)^2/0.39\}),$$

where $y = \log \gamma_c \tan\theta$.

To determine the character of the anisotropy it will be necessary to find a function of the angular distribution $N(\theta^*)$ in the c.m.s., using the formulae for transformation of the angles. In view of the complicated computations involved, the analytic form of this function was not found even for $\beta_c = \beta_\pi^*$; a graphical analysis shows that the form of the anisotropy of such a shower does not agree with the anisotropy predicted according to the Takagi model.

An investigation of the angular distribution of the shower 35 + 59p was next carried out in the laboratory system under the assumption of an anisotropy of the type $\sim\cos^{2n}\theta^*$ in the c.m.s. The curve corresponding to the anisotropy $\cos^2\theta^*$ in the c.m.s. has two symmetric maxima in the laboratory system and is strongly different from the observed angular distribution (see Fig. 3, curve 4). Account of the energy spectrum of the shower particles and of an anisotropy of the type $\cos^{2n}\theta^*$ yields a curve with two different maxima, which is closer to the true angular distribution (Fig. 3, curve 3). With increasing degree of anisotropy, and with an energy spectrum proportional to E_0^{-2}, the asymmetry in the position of the maxima increases (the maxima move away and decrease). Thus, taking into the account the power-law energy spectrum and the anisotropy of the angular distribution in the c.m.s. provides a possibility of explaining the angular distribution of shower particles in the laboratory system.

For a complete description of the shower, we give the integral angular distribution of the shower particles, taking into account the errors of the measurements (Fig. 4).

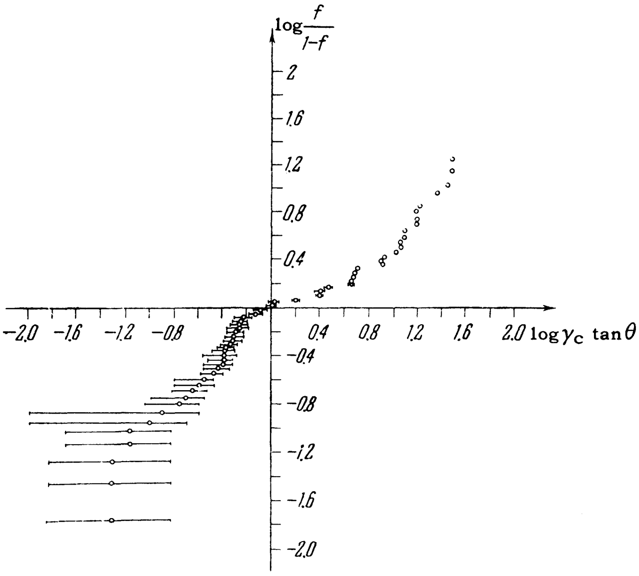

FIG. 4. Integral angular distribution of shower particles of the star 35 + 59 p.

DIFFERENTIAL ANGULAR DISTRIBUTION OF SHOWER PARTICLES IN SHOWERS WITH PRIMARY ENERGY E > 100 Bev

If a sharp division of particles into narrow and diffuse cones is observed in the shower, then the histogram of the differential angular distribution always has two maxima. The positions of the maxima and of the minimum in the shower differ and depend on the energy of the primary particle and, apparently, also on other features of each shower. Showers with such distributions are observed among the stars produced by nucleons with energies > 100 Bev (Table II). Table II lists data on 12 showers (N_g and N_b, are the numbers of grey and black tracks respectively): the summary histogram of their angular distribution is given in Fig. 5. This figure shows also the curve of the

TABLE II

No.	$N_b + N_g + n_s$	θ_g	$\gamma_c(\theta_g)$	$\theta_{1/2}$	$\gamma_c(\theta_{1/2})$	l/d	$E\cdot 10^3$, Bev
1	0+0+20p	0°50′	68.5	0°41′	83.8	1	13
2	7+2+27p	9°39′	5.9	7°00′	8.1	8	0.9
3	19+4+38p	10°52′	5.2	10°22′	5.5	13	0.4
4	16+3+39p	7°17′	7.8	8°21′	6.8	12	0.7
5	7+2+18p	4°40′	12.1	3°30′	16.3	4	1.9
6	11+5+39p	8°43′	6.5	13°41′	4.1	16	0.2
7	4+3+24p	10°14′	5.5	5°03′	11.3	6	1.4
8	3+1+23p	6°02′	9.5	5°13′	10.9	6	1.3
9	3+2+27p	4°00′	14.3	3°04′	18.7	5	3.2
10	20+15+59p	2°42′	21.2	1°39′	34.7	8.5	19
11	2+2+36p	3°41′	15.5	6°28′	8.8	9	1.2
12	10+3+23p	5°06′	11.2	5°07′	11.2	6	1.4

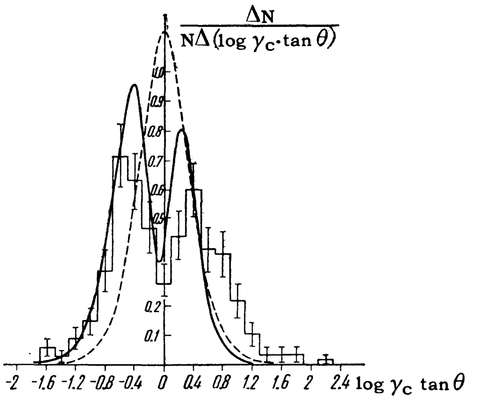

FIG. 5. Total histogram of the angular distribution of shower particles of 12 showers with E > 100 Bev. The dotted curve corresponds to an isotropic distribution in the c.m.s.; the solid curve – to a distribution $\cos^2\theta^*$ and to the energy spectrum of shower particles.

differential distribution, calculated under the condition that the energy spectrum of the produced mesons is proportional to E_0^{-2}, and that the distribution in the c.m.s. is anisotropic, proportional to $\cos^2\theta^*$. This curve describes satisfactorily the experimental histogram.

ANGULAR DISTRIBUTION OF BLACK AND GREY TRACKS OF THE STAR 35 + 59p

As has been stated above, the interaction involved a heavy nucleus of the emulsion (Ag, Br). If we assume that the incident proton traversed the diameter of the nucleus, then according to the theory of Heitler and Terreaux[4] the excitation energy U ~ 150 Mev, and the observed number of heavily ionizing particles should be equal to 3 or 4.

However, in the star analyzed, the number of black tracks is $N_b = 20$ and the experimentally estimated lower limit of the excitation energy is ~ 350 Mev, which is considerably greater than the value predicted by the Heitler-Terreaux theory. According to the evaporation theory (U = 800 Mev, see reference 11), the emission of particles from an excited nucleus should be isotropic. This has not been observed for the black tracks of the star 35 + 59p; the ratio of the number of particles going in the forward direction to the number of particles in the backward direction equals 1.5. We have tried to explain the deviation from isotropy by assuming that a velocity is imparted to the target nucleus, owing to the transfer of a large momentum to it from the primary particle. This velocity is found to be equal to v/c = 0.02 under the assumption that the evaporated particles are protons. For such a velocity of the nucleus, the excess of evaporated protons in the lower hemisphere equals $0.2/E_p^{1/2} = 0.07$. The experimentally observed excess is much larger than this value.

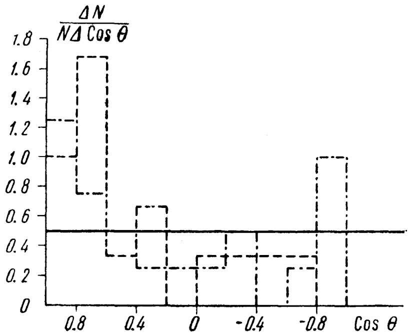

FIG. 6. Differential angular distribution of the grey (dotted) and black (dashed) tracks of the star 35 + 59 p. The straight line corresponds to an isotropic distribution.

Thus, the velocity imparted to the target nucleons (even if we assume that multiply-charged particles are among the evaporated ones) does not explain the observed anisotropy of the black tracks. This is confirmed by the fact that the correlation coefficient[12] of black and grey tracks equals K = 0.43 ± 0.29.

The differential angular distribution of the grey tracks is anisotropic. The ratio of the number of tracks going in the forward direction to those going in the backward direction equals 2.75. The following mechanism is proposed for the production of grey tracks: the incident nucleon interacts with a tunnel filled with nuclear matter, inside of which the mechanism of multiple-particle production is effective. The particles which get outside the tunnel emit additional nucleons in passing through the nucleus. These recoil nucleons we establish as the grey tracks.

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