ACCELERATION OF PLASMOIDS BY HIGH-FREQUENCY ELECTRIC FIELDS

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Submitted to JETP editor, January 23, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1909-1917 (June, 1959)

The possibility of accelerating completely ionized quasi-neutral plasmoids in moving high-frequency potential wells is indicated. If such wells are formed by two fields of different frequency the plasmoids can be accelerated by changing the frequency of one of the fields or by using a waveguide of variable cross section. Certain features of linear and cyclical plasma accelerators are analyzed.

1. INTRODUCTION

IN accelerating quasi-neutral plasmoids or charged particles of one sign it is necessary to set up strong fields which provide acceleration as well as spatial stability. The latter requirement is predominant since any system which localizes a fixed plasmoid can, in principle, be used for acceleration if the appropriate displacement of the localizing fields in space can be achieved.

There has been a great deal of recent work in plasma confinement in connection with the well known problem of a controlled thermonuclear reaction. One method is to use a weakly inhomogeneous high-frequency electromagnetic field. The forces (averaged over an oscillation period) which act on a particle in such a field do not depend on the sign of the charge; this feature lies at the basis of all methods of using high-frequency fields for plasma localization.* The inhomogeneous field of the required configuration is produced by an appropriate distribution of sources as well as the perturbations introduced by the plasma itself. In certain cases it has been possible to obtain a self-consistent electrodynamic solution: a plasma sphere in a spherical resonator,[2] a plasma cylinder in a circular waveguide,[3] a two-dimensional plasma layer between ideal planes,† a plane plasma boundary maintained by a plane wave which is normally incident on it.[4] In general form the problem can be solved only for a highly rarefied plasma (density small enough so that distortions introduced by the plasma can be neglected); in this case we actually deal with the localization of single charged particles.[5,6]

Below we consider certain principles for the acceleration of quasi-neutral plasmoids in high-frequency electromagnetic fields. The properties of the plasma are assumed to be as follows: a) the plasma is completely ionized; b) during the time in which the bunch is accelerated a Maxwellian distribution over average particle velocity is maintained; c) all the particles have non-relativistic velocities in the reference system fixed to the bunch; d) the dielectric permittivity of the plasma is approximately unity, i.e., the density of particles N satisfies the requirement $N \ll m_e\omega^2/4\pi e^2 = 3.1\times10^{-8}\omega^2$, where m_e is the mass of the electron, e is the charge of the electron and ω is the angular frequency of the external field. Although the last condition is not a fundamental limitation it does allow us to make a complete analysis of the kinematic part of the problem, i.e., the displacement of the localizing fields in space.

2. LOCALIZATION OF INDIVIDUAL CHARGED PARTICLES

It has been shown by Gaponov and the author[5,6] that charged particles can be localized close to absolute minima* of a high-frequency potential Φ defined by the relation

$$\Phi = (\eta^2/4\omega^2)\,|\mathbf{E}|^2, \quad (1)$$

where η is the charge-to-mass ratio and $\mathbf{E}$ is the amplitude of the electric field. If the profile of Φ is a potential well, it will trap particles that have velocities $|v_0| \le \sqrt{2\Delta\Phi}$ at the center (minimum

*This feature was first pointed out by V. I. Veksler, who proposed the electromagnetic radiation pressure for acceleration of plasma objects.[1]

†Diplomate paper by E. I. Yakubovich, Gor'kiĭ State University, 1958.

*In the presence of a fixed magnetic field localization is possible in regions of maximum E under the condition that $\omega < |\omega_H|$, where ω_H is the cyclotron frequency.[6]

value of Φ) where $\Delta\Phi$ is the minimum potential difference between the edge and center of the well.

We assume that this potential well is formed in a reference system K′ which moves in the +z direction with respect to the laboratory system K with a velocity v ($\beta = v/c$) and an acceleration w where

$$w(1-\beta^2)^{-3/2} = \tilde{w} = \text{const.} \quad (2)$$

Introducing in place of (1) some effective potential* Φ'_{eff}, which includes the potential of the inertia forces $\Phi'_{eff} = \Phi' + \tilde{w}z'$, we find that if particles are to be trapped the following inequality must be observed

$$|\partial\Phi'/\partial z'| > \tilde{w}. \quad (3)$$

The condition in (3) imposes an additional limitation on the potential Φ'; one cannot use a Φ with very steep slopes†: in the first place the time-of-flight of the particle through the region of inhomogeneous field ($L \sim \Phi/|\nabla\Phi|$) must be long enough to correspond to a large number of high-frequency periods ($L \gg v_0/\omega$); secondly, the amplitude of the high-frequency oscillations of the particle must be much smaller than L, $L \gg |\eta E/\omega^2|$.

3. LOCALIZATION OF A PLASMOID

If a potential well is filled by a rarified plasma characterized by a kinetic temperature $T(V_T = kT/e)$ the particle density is given by the expression[3]

$$N(\mathbf{r}) = N(0)\exp(-m_e\Delta\Phi_e/2kT),$$

where k is the Boltzmann constant, $\Delta\Phi_e$ is the difference in the high-frequency potentials between the points‡ $\mathbf{r}$ and $\mathbf{r} = 0$. If this difference is considerably greater than $2kT/m_e$ in all directions the bunch is essentially localized at the center of the well.

Suppose that the localizing field and the plasmoid move with accelerated motion and that the condition in (2) is satisfied; then, in place of the potential Φ'_e it is necessary to introduce the effective potential Φ'_{eff} (analogous to the potential for the electron) which takes account of the increase in the force of inertia due to the factor m_i/m_e (m_i is the mass of the ion, $m_i \gg m_e$)

$$\Phi'_{eff} = \Phi'_e + (m_i/m_e)\tilde{w}z'. \quad (4)$$

As an example we consider a sinusoidal (in the z direction) potential profile

$$\Phi'(z') = \Phi'_0 + \Phi'_1 \cos 2h'z'. \quad (5)$$

Substituting (5) and (4) and seeking the position of the extrema in Φ'_{eff}

$$\sin 2h'z'_{extr} = \frac{m_i}{m_e}\frac{\tilde{w}}{2h'\Phi'_1} = \alpha, \quad (6)$$

for a potential difference $\Delta\Phi'_{eff}$ corresponding to a minimum potential differential between the edge and the center of the well, we have*

$$\Delta\Phi'_{eff} = 2\Phi_1\Psi(\alpha), \qquad \Psi(\alpha) = \sqrt{1-\alpha^2} - \alpha \arccos\alpha. \quad (7)$$

Using these formulas it is an easy matter to find the maximum possible acceleration of the bunch since the potential difference $\Delta\Phi'_{eff}$ which appears in (7) is determined by the condition of partial or total localization of a plasma at temperature T.

4. MOVING POTENTIAL PROFILES

As has been noted earlier,[7] a simple method of displacing a potential profile consists of using two traveling waves of different frequencies. Suppose that in a reference system K′ moving with uniform motion in the z direction the field is a superposition of two plane non-uniform waves of the same frequency ω':

$$\mathbf{E} = \mathbf{E}'_+(x, y)\exp(i\omega't' - ih'z') + \mathbf{E}'_-(x, y)\exp(i\omega't' + ih'z'), \quad (8)$$

then in the fixed system K this field corresponds to two traveling waves of different frequencies ω_+ and ω_-:

$$\mathbf{E} = \mathbf{E}_+(x, y)\exp(i\omega_+t - ih_+z) + \mathbf{E}_-(x, y)\exp(i\omega_-t + ih_-z). \quad (9)$$

The frequencies $\omega_\pm$ and the wave numbers $h_\pm$ are related to the corresponding quantities in the K′ system by the relations

$$k_\pm = (k' \pm h'\beta)/\sqrt{1-\beta^2}, \qquad h_\pm = (h' \pm k'\beta)/\sqrt{1-\beta^2}, \quad (10)$$

where $k_\pm = \omega_\pm/c$, $\beta = v/c$ while v is the velocity of the K′ system with respect to the K system.

Using the fields in Eq. (8) it is possible to produce various profiles for the high-frequency potential Φ'.[6] In particular, in a regular cylindrical

*The primed quantities refer to the moving coordinate system.

†It is precisely as a result of this situation that it is impossible to trap a plasmoid with a sharply defined boundary.

‡For simplicity, the charges of the electrons and ions are assumed to be the same in all cases; in the general case $e_i = Ze$, in place of $\Delta\Phi_e/2$ we write $Z\Delta\Phi_e/(Z+1)$. We may note, moreover, that for dense plasmoids the potential difference $\Delta\Phi_e$ must be determined from a self-consistent solution of the problem.

*Here we are considering only the potential difference in the z direction.

lossless line two propagating waves of the same type form a profile of the form in (5) in which

$$\Phi_0' = (\eta/2\omega')^2 \, [|E_+'|^2 + |E_-'|^2],$$
$$2\Phi_1' = (\eta/\omega')^2 \, |E_+' E_-'|. \qquad (11)$$

Particles can be trapped in a finite region with cross section z = const either by virtue of the existence of an absolute minimum in $\Phi'(x, y, z')$ or, if there is no such minimum, by means of a supplementary fixed magnetic field $\mathbf{H}_0 = \mathbf{H}_{0z}z_0$. The trapping in the z direction occurs as a result of the sinusoidal shape of the profile (5), i.e., trapping is possible when $h' \neq 0$.

The waves in (9) in the K system can propagate in the same direction ($h_+h_- < 0$) or in opposite directions ($h_+h_- > 0$). Quantities referring to waves moving in opposite directions (anti-parallel) will be denoted by the subscript "a" while waves moving in the same direction (parallel) will be denoted by the subscript "p". In the general case it is also necessary to distinguish between the so-called fast waves ($h \leq k$) which exist in waveguides and multi-conductor lines with ideal smooth conducting surfaces and the slow waves ($h \geq k$) which exist in systems with corrugated boundaries or with inhomogeneous (cross section z = const) dielectrics. Although in both cases the dispersion equation is of the form $h_\pm^2 = k_\pm^2 - \kappa_\pm^2$, in the fast-wave case, if one considers waves in a lossless isotropic system the transverse wave number κ is pure real and cannot depend on frequency $\kappa_+ = \kappa_- = \kappa$ (regular waveguides); in the slow wave case κ is an imaginary quantity and, in principle, does depend on frequency ($\kappa_+ \neq \kappa_-$ if $k_+ \neq k_-$). In what follows, for simplicity we shall limit ourselves to regular waveguides and fast waves; the analysis may be carried out in general form without any specification of the waveguide cross section or the field configuration.*

Two important parameters determine the usefulness of the profile (5) from the point of view of acceleration of charged particles: the constant, h′, which characterizes the slope of the walls of the potential well, and the quantity $\beta = v/c$, which determines the displacement velocity of the potential wells. Using the dispersion equation $h_\pm^2 = k_\pm^2 - \kappa^2$ and introducing the dimensionless quantities $\gamma = k_+/k_-$, $p^2 = \kappa^2/k_-^2$, $s = h'/k_-$, from (10) we have

$$\beta = (k_+ - k_-)/(|h_+| \pm |h_-|)$$
$$= (\gamma - 1)/(\sqrt{\gamma^2 - p^2} \pm \sqrt{1 - p^2}), \qquad (12)$$

*It should be kept in mind, however, that slow-wave systems (helices, waveguides with ribbed walls, etc.) are preferable in certain respects. Aside from the lower values of the velocity of displacement of the profiles, in the slow-wave case larger values of h′ are achieved.

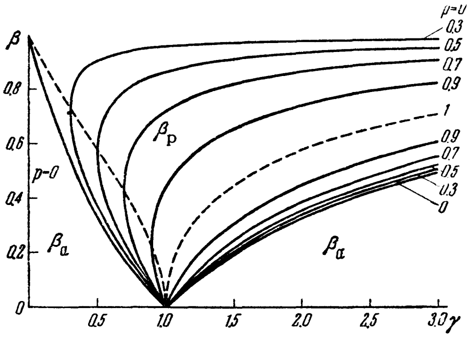

FIG. 1

$$s = [\gamma - p^2 \pm \sqrt{(\gamma^2 - p^2)(1 - p^2)}]^{1/2} / \sqrt{2}. \qquad (13)$$

The upper sign in Eqs. (12) and (13) refers to the anti-parallel waves; the lower sign refers to the parallel waves. In Figs. 1 and 2 are shown the relations between β_a, β_p, s_a, and s_p and γ for various values of p (the number on the curves). Inasmuch as exponentially decaying waves are not being considered,* γ can vary within the limits $p \leq \gamma < \infty$ where $0 \leq p \leq 1$. When $\gamma = p$ or $p = 1$ we have a situation in which the waveguide is excited at one of its critical frequencies, i.e., one of the fields in (9) becomes independent of the z coordinate. The regions of β_a and β_p, and correspondingly, s_a and s_p, are separated in Figs. 1 and 2 by dashed lines.

It is apparent that $\beta_a(\gamma, p) \leq \beta_p(\gamma, p)$ and $s_a(\gamma, p) \geq s_p(\gamma, p)$, i.e., by using waves in the same direction it is always possible to achieve higher displacement velocities for the localizing fields; on the other hand, the anti-parallel fields always form potential wells with steeper sides. The profile with the steepest sides is formed by anti-parallel TEM waves which propagate with the velocity of light ($p = \kappa/k_- = 0$, $h = k$) for which, in accordance with Eq. (13), we have† $\beta_a = \sqrt{\gamma}$. For waves traveling in the same direction the most suitable mode of operation is the one with the maximum values of $p: p_{max} = 1$, $s_p = \sqrt{(\gamma - 1)/2}$.

If the frequencies ω_+ and ω_- are the same ($\gamma = 1$) the profile Φ_a' remains fixed ($\beta_a = 0$). The profile Φ_p' formally must be displaced with the

*We assume that the quantity h′ in Eq. (8) is pure real; an exponentially decaying or increasing field in the z-direction (9) ($\mathrm{Im}\, h \neq 0$) must correspond, in the K′ system, to a field with complex h′, as given by Eq. (10).

†The application of slow waves makes it possible to increase the value of s_a, which, as has already been noted, occurs at the expense of a reduction in the velocity β; if the dispersion equation is of the form $h^2 = k^2 + \tilde{\kappa}^2$, for s, in place of Eq. (13), we have

$$2s = [2\gamma + \tilde{p}_-^2 + \tilde{p}_+^2 \pm \sqrt{(\gamma^2 + \tilde{p}_+^2)(1 + \tilde{p}_-^2)}]^{1/2},$$

where $\tilde{p}_\pm^2 = \tilde{\kappa}_\pm^2/k_-^2$, whence it follows that $s_a > \sqrt{\gamma}$.

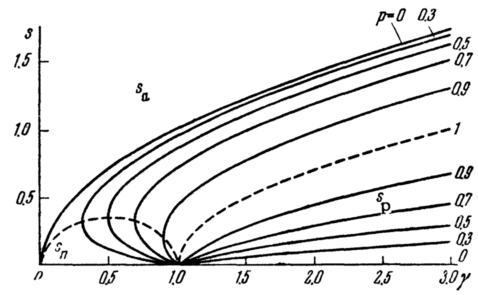

FIG. 2

group velocity of the wave $\beta_p = \sqrt{1-p^2}$; however the potential wells vanish since $s_p(1, p) = 0$.

As the value of γ increases the velocity of displacement of both profiles, Φ'_a and Φ'_p, increases, approaching the velocity of light; as far as purely technical considerations are concerned the velocity which can be achieved is determined by the conditions of localization of the plasma within the potential well. When $\gamma \gg p$, s is given approximately by

$$s \approx [\gamma(1 \pm \sqrt{1-p^2})/2]^{1/2}.$$

Thus s increases asymptotically in proportion to $\sqrt{\gamma}$; starting at some value of the velocity the conditions for applicability of the average description of the motion of the particles in the moving system $L \sim 1/h' \gg v_0/\omega'$ (v_0 is the mean velocity of the electrons in a bunch) is violated: a peculiar "tunneling" of the particles through the potential barriers takes place and it becomes impossible to trap particles, even at very large values of Φ'_1.

5. ACCELERATED POTENTIAL PROFILES

There are two possible methods for accelerating the potential profiles formed by the fields (8). The first method is to vary the frequency of the waves (9) as a function of time; the second method is to vary the propagation constants h_+ and h_- as functions of the z coordinate. In order to maintain the stationarity of the profile Φ' (unless this is done the potential description loses its significance) these changes must be slow as compared with the period $(2\pi/\omega)$ or the wavelength $(2\pi/h)$: $|dk/dt| \ll k^2c$, $|dh/dz| \ll h^2$.

We consider the features of each of these methods. Suppose that the frequency ω_- is fixed while $\omega_+ = \omega_+(t)$; this is equivalent to a variation of γ in Eq. (12) with p = const (cf. Fig. 1). In β_a systems it is possible to start the acceleration from zero velocity ($\gamma = 1$, $\beta_a = 0$); in β_p systems it is necessary to inject a bunch which has first been accelerated. The magnitude of the initial velocity is determined by the conditions under which the potential well is formed at the input of the accelerator ($s_p > 0$). The appropriate values of the acceleration are found by differentiating Eq. (12). The infinite increase in acceleration for $\gamma \to p$ (cf. Fig. 1) excludes the possibility of choosing the variable frequency close to one of the critical frequencies of the waveguide; however, generally speaking the fixed frequency can be set equal to one of the critical frequencies (p = 1). If the frequencies coincide at the starting time t = 0 ($\gamma = 1$) and the condition $t d\gamma/dt \ll 1$ holds over the extent of the entire cycle, the acceleration, according to (12), is given by the relation

$$d\beta_a/dt = \tfrac{1}{2}(1-p^2)^{-1/2}\, d\gamma/dt,$$
$$d\beta_p/dt = \tfrac{1}{2}p^2(1-p^2)^{-1/2}\, d\gamma/dt.$$

Whence it follows, in particular, that a linear variation of frequency corresponds in the first period of acceleration to uniform acceleration of the profile.

An important feature of the second method of acceleration is the fact that fixed frequencies are used; thus it is possible to use resonance excitation of the electrodynamic system and no special nonlinear frequency tuning elements are required. A smooth variation of the propagation constant h(z) can be obtained in quasi-cylindrical waveguides or resonators in which the cross section and, consequently, the transverse wave number κ, vary slowly in the z direction. The appropriate values of the velocities in acceleration are easily determined by Eq. (15) where $d\beta/dt = \beta c(d\beta/dp) \times dp/dz$. Since the parameter p must lie within the limits $0 \le p \le 1$ for fast propagation, the maximum velocity differential, i.e., the difference between the velocities at the input and the output of the accelerator, must be [from Eq. (12)]:

$$\Delta(\beta_a)_{\max} = \sqrt{(\gamma-1)/(\gamma+1)} - (\gamma-1)/(\gamma+1),$$
$$(\Delta\beta_p)_{\max} = 1 - \sqrt{(\gamma-1)/(\gamma+1)}$$

In this case the initial velocities are different from zero, i.e., preliminary acceleration is required.

It should be kept in mind that in this method the displaced profile necessarily becomes distorted since the acceleration is based on a variation of $h_\pm$ and consequently h'; hence the maximum possible velocity found from Eq. (12) may be unusable because the conditions required for trapping the bunch may no longer hold.

6. CHOICE OF ACCELERATOR PARAMETERS

We now analyze some of the factors that enter into a choice of the parameters of a high-frequency plasma accelerator.

The point of departure is the fact that the conditions required for localizing the bunch must be observed. Taking account of Eqs. (7) and (8) and the requirement[6] for smooth variation of the field amplitude E, we have

$$L\omega^2/\eta_e \gg |E_\pm| \gg \sqrt{2}\omega\,(V_T/\eta_e\Psi(\alpha))^{1/2}, \quad (14)$$

where $V_T = kT/e$. Taking $L = \lambda/4 = \pi c/2\omega$ and substituting $\eta_e = 5.3 \times 10^{17}$, we write Eq. (14) in the form

$$5.1\cdot 10^8(\Psi/V_T)^{1/2}|E_\pm| \gg \omega \gg 1.1\cdot 10^7|E_\pm|. \quad (15)$$

Whence it follows that the trapping conditions impose a rather stringent limitation on the possible frequency range which can be used; it is convenient to choose ω close to the left-hand limit in Eq. (15) since the limitation on frequency from above is weaker. Secondly, the dimensions of the localization region ($L \sim \lambda/4$) are actually the factors which determine the allowable values of the electric field intensity E: for example with L = 5 cm, ($\omega = 10^{10}$), V_T = 3.3 cgs (10^3v), $\Psi = 0.5$, the field strength must be $\sim 10^2$ cgs (3×10^4v/cm); with E = 0.1 cgs (30v/cm), $\omega \approx 10^7$, i.e. $L \approx 50$ m. Thus, to contain a bunch in a relatively small region of space it is necessary to use high-frequency fields of rather high amplitude.

The allowable values of $\tilde{w}$ can be determined from Eq. (6) if it is assumed that the quantity Φ_1', in accordance with Eq. (11), is approximately $\Phi_1' \approx \frac{1}{2}(\eta_e/\omega)^2 E_{av}^2$ where E_{av} is the average value of the electric field in the traveling wave:

$$\tilde{w}_{av} = \alpha_{av}(m_e/m_i)\,(\eta_e/\omega_{av})^2\,ksE_{av}^2.$$

For example, with $\omega = 10^{10}$, $\alpha = 0.4$, $\Psi = 0.5$, $m_i/m_e = 1.8 \times 10^3$, s = 0.6, and $E_{av} \approx 10^2$ it is possible to obtain an acceleration $\tilde{w}_{av} \approx 10^{15}$ cm/sec^2.

The acceleration and maximum velocity which can be realized in practice depend on the method of displacing the localizing field and on the electrodynamic nature of the system; in particular, if we require that there be a propagating mode of only one kind in the waveguide* it is necessary to vary the generator frequency in such a way that it remains between the first and second critical frequencies of the waveguide:† $c\kappa_2 \geq \omega \geq c\kappa_1$; this requirement and that given in (18) set additional limitations on the range of ω and determine the maximum possible velocity at the output of the accelerator β_{max}.

From the technical point of view perhaps the most important factor which limits the applicability of high-frequency plasma accelerators is the power required to form the localizing field. If non-resonance excitation is used this power is approximately equal to the mean Poynting vector flux in the traveling wave and can be estimated from the relation

$$P_{TM} = \frac{c}{8\pi}\frac{k}{h}E_{av}^2 S, \qquad P_{TE} = \frac{c}{8\pi}\frac{h}{k}E_{av}^2 S, \quad (16)$$

where S is the effective cross section of the waveguide.

In resonance excitation of an appropriate cylindrical configuration of length l the power P is estimated taking account of the Q of the system:

$$P_Q = \frac{\omega}{8\pi Q}\int |\mathbf{E}|^2\,dV \approx \frac{c}{16\pi Q}(kl)\,E_{av}^2 S. \quad (17)$$

For example with λ = 20 cm, $S \approx \lambda^2/4 = 10^2$cm^2, h/k = 0.6 and $E_{av} \approx 10^2$cgs, $P_{TE} \approx 7 \times 10^{14}$ joules (7×10^7 watts); similarly, for excitation of an appropriate resonator $kl = 10^3$, $Q = 10^4$, $P_Q \approx 6 \times 10^{13}$ joules (6×10^6 watts). A further increase in the amplitude of the field would require the use of sources of enormous power, such as are generally used in pulsed rather than continuous operation.

In conclusion we present typical data for a linear accelerator for hydrogen plasma at a temperature $V_T = 10^3$ ev, which uses anti-parallel TE_{01} waves in a waveguide of rectangular cross section:

a) $\omega_- = 6.3 \times 10^{10}$sec^{-1} ($\lambda_- = 3$ cm), p = 0.9, $\Delta\omega/\omega = 0.27$, $\beta_{max} = 0.2$, $s_{av} = 0.6$, $E_{av} = 10^3$ cgs, $w_{av} = 2 \times 10^{16}$cm/sec^2, $P_{TE} \approx 3 \times 10^8$ watts; for operation with pulses of duration $\Delta t = 3 \times 10^{-7}$ sec the length of the accelerator is l = 9 m.

b) $\omega = 10^{10}$sec^{-1} (λ = 20 cm), p = 0.9, $\Delta\omega/\omega_- = 0.27$, $\beta_{max} = 0.2$, $s_{av} = 0.6$, $E_{av} = 10^2$cgs, $w_{av} = 10^{15}$ cm/sec^2, $P_{TE} = 7 \times 10^7$ watts, $\Delta t = 6 \times 10^{-6}$ sec, $l = 2 \times 10^2$ m.

7. POSSIBILITY OF CYCLIC ACCELERATION OF A PLASMA

The basic difficulty in cyclic acceleration of a plasma lies in the containment of a quasi-neutral plasmoid over a stable closed trajectory. Although the use of weakly inhomogeneous high-frequency fields for this purpose is possible in principle, as will be shown below, these can only be used for small velocities. For example, suppose that it is necessary to achieve stable motion of a plasmoid about a circle of radius $r_\perp = r_0$, which is considerably greater than the dimensions of the plasmoid

*The appearance of higher propagating modes may not only reduce the efficiency of the accelerator but can also cause delocalization of the bunch. Even if the system is excited by an external source which produces only the required mode the latter can be transformed into higher modes directly at the plasma bunches if the dielectric constant of these bunches varies greatly from unity.

†From the point of view of the most favorable conditions it turns out best to use the TEM mode in multiconductor lines for which $\varkappa_1 = 0$ and ω is limited from below only by the condition in (15).

itself. We set up a toroidal channel for the high-frequency potential $\Phi(r_\perp)$ in such a way that the circle $r_\perp = r_0$ lies within the toroidal potential well.* In order to compensate for the centrifugal forces it is necessary that the sides of the potential well be rather steep. In the non-relativistic approximation this limitation, similar in meaning to (3), is written in the form

$$|\partial\Phi / \partial r_\perp|_{r_\perp = r_0} > m_i v_\theta^2 / m_e r_0. \quad (18)$$

Here $v_\theta = \beta_\theta c$ is the mean linear velocity of the plasmoid. Assuming that $\partial\Phi/\partial r_\perp \sim \Phi/L$ and substituting the value of Φ in Eq. (18) from Eq. (1), we find the condition which limits the field intensity E from below for the field which forms the potential barrier with respect to $r_\perp = r_\theta$,

$$|E| / \beta_\theta > (L / r_0)^{1/2} (m_i / m_e)^{1/2} 2\omega c / \eta_e. \quad (19)$$

As an example we take the values $\omega = 2 \times 10^{10}$ sec^{-1}, $m_i/m_e = 1.8 \times 10^3$, $L/r_0 \sim 10^{-2}$. Substitution of these values in Eq. (19) yields $|E|/\beta_\theta > 10^4$. Consequently even with velocities $\beta_\theta \sim 10^{-1}$ one requires fields with intensities of $E \sim 10^3$ cgs $\times (3 \times 10^5$ v/cm) and, by virtue of Eqs. (16) and (17), sources with powers of the order of $10^7 - 10^9$ watts. Whence it follows that in cyclical plasma accelerators with radial high-frequency focusing the velocities which can be achieved are smaller than those that can be obtained in corresponding linear accelerators; most of the power required from the high-frequency sources goes into the fields which constrain the plasmoids in the radial direction.

Acceleration of plasmoids can be realized by displacement of supplementary potential wells produced by waves which rotate in azimuth. In this connection it is interesting to note that if inside a waveguide system which closes on itself we excite two standing waves of different frequencies it is possible to distinguish four potential profiles, which rotate in pairs in opposite directions with velocities

$$\beta_a = \pm (k_+ - k_-) / (h_+ + h_-), \qquad \beta_p = \pm (k_+ - k_-) / (h_+ - h_-),$$

which offers the possibility of obtaining simultaneous displacement and even acceleration of plasmoids in opposite directions.

*A toroidal channel of this kind can be produced, for example, by means of a TM_{01} mode in a corrugated metal torus of circular cross section, a TE_{01} (usual profile) or a TE_{11} mode (inverted profile) in a smooth-walled torus of circular cross section; symmetric waves in a helical conducting torus, and so on. One of the examples of the production of a two-dimensional circular potential well in a cylindrical resonator has been given for example by Veksler and Kovrizhnykh.[8]

8. CONCLUSION

Thus, high-frequency weakly inhomogeneous fields can be used for accelerating quasi-neutral plasmoids. This possibility has been demonstrated with plasmoids of low density; however, in principle, many of the points relating to the acceleration mechanism, in particular the method of displacing the localizing fields, remain valid for acceleration of dense plasmoids. The high-frequency field is distorted significantly only in the immediate vicinity of the plasmoid; the field is unchanged (in the absence of higher modes) at far distances. Obviously, however, for a complete description of the characteristics of such accelerators it will be necessary to find a self-consistent solution of the problem of the localization of dense plasma inside an accelerated high frequency potential well.

The author is indebted to A. V. Gaponov, V. B. Gil'denburg, and S. B. Mochenev for useful discussion of the results.

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Translated by H. Lashinsky
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