SOVIET PHYSICS JETP VOLUME 36 (9), NUMBER 6 DECEMBER, 1959

TRANSITION PROBABILITIES BETWEEN THE LEVELS OF THE ROTATIONAL BAND OF NON-AXIAL NUCLEI

A. S. DAVYDOV and V. S. ROSTOVSKIĬ

Moscow State University

Submitted to JETP editor December 16, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1788-1796 (June, 1959)

The energies and the wave functions of the rotational states $(J \geq 4)$ of non-axial nuclei are calculated, and the reduced probabilities for E2 transitions between these states are derived. Conditions under which the rotational states can be characterized by a quantum number K are ascertained. It is shown that, when the shape of the nucleus deviates from axial symmetry, the interval rule 1 — 3.3 — 7 — 12 observed in the rotational band of axial nuclei is violated. The theory is compared with experiment.

THE rotational states of even-even nuclei have been studied by Davydov and Filippov[1-3] under the assumption that the equilibrium shape of the nucleus can be represented by a tri-axial ellipsoid. Analytical expressions for the energy of the rotational states with spin 2, 3, and 5 were found, and the transition probabilities between these states were calculated. In particular, it was shown that the theory made it possible to find a single-valued relation between the ratio of the energy of the two levels with spin 2 and the ratio of the reduced probabilities of E2 transitions from the second level (J = 2) to the first level (cascade transition) and directly to the ground state (direct transition).

In the present work, results are presented of numerical calculations of the energy of rotational states with spin 4, 6, and 8 for different values of the parameter γ which determines the deviation of the shape of the nucleus from axial symmetry.[4] The wave functions of these excited states and the transmission probabilities between them are calculated. In Sec. 3 the conditions are given under which the rotational states of nuclei may be described by approximate wave functions corresponding to states with a given value of the projection of the total momentum on axis 3 of the nucleus. Approximate formulae which determine the reduced probabilities of E2 transitions between rotational states of nuclei whose shape does not differ greatly from axially symmetric are derived. The theory is compared with experimental data in Sec. 4.

1. ENERGY OF EXCITED STATES WITH SPIN 4, 6, AND 8

The energy of rotation of a non-spherical even-even nucleus is given, in the adiabatic approximation, by the Schrödinger equation

$$(H - \varepsilon)\psi = 0, \qquad (1.1)$$

where ϵ is measured in units of $\hbar^2/4B\beta^2$, and the operator H is given by the formula

$$H = \frac{1}{2}\sum_{\lambda=1}^{3} \hat{J}_\lambda^2 \sin^{-2}(\gamma - \lambda 2\pi/3), \qquad (1.2)$$

where J_λ are the projections of the operator of the total angular momentum on the axes of the coordinate system fixed in the nucleus. The wave function corresponding to the state with total moment J, and fulfilling the conditions of symmetry found by Bohr,[4] can be represented in the form

$$\psi_{JM} = \sum_{K\geqslant 0} |JK\rangle A_K, \qquad (1.3)$$

where

$$|JK\rangle = [(2J+1)/16\pi^2(1+\delta_{K0})]^{1/2} \times\{D_{MK}^J + (-1)^J D_{M,-K}^J\}. \qquad (1.4)$$

The functions D_{MK}^J in Eq. (1.4) are functions of the Euler angles that determine the orientation of the principal axes of the nucleus in space. It can be shown that the wave functions (1.3) form the basis of a totally symmetric representation of the group D_2 (see reference 1), the elements of which are the rotation through 180° around each of the three principal axes of the nucleus.

Substituting Eq. (1.3) in Eq. (1.1), and making use of the value of the matrix element of the operator of the rotational energy (1.2) acting on the wave functions (1.4),

$$\langle JK | H | JK\rangle = \frac{\alpha+\beta}{4}[J(J+1)-K^2]+\frac{\delta K^2}{2},$$

$$\langle JK+2 | H | JK\rangle = \frac{\alpha-\beta}{8}\{(1+\delta_{K0})(J-K)$$

$$\times (J-K-1)(J+K+1)(J+K+2)\}^{1/2},$$

$$\alpha = \sin^{-2}(\gamma - 2\pi/3),\quad \beta = \sin^{-2}(\gamma + 2\pi/3),$$

$$\delta = \sin^{-2}\gamma,\quad \delta_{K0} = \begin{cases}0, & \text{for } K\neq 0\\ 1, & \text{for } K=0\end{cases},$$

we obtain for each value of J a system of algebraic equations for the value of the coefficients A_K in the wave functions (1.3). For instance, for J = 4, the Schrödinger equation (1.1) is reduced to a system of equations

$$[5(\alpha+\beta)-\varepsilon]A_0 + (3\sqrt{5}/2)(\alpha-\beta)A_2 = 0,$$

$$(3\sqrt{5}/2)(\alpha-\beta)A_0 + [4(\alpha+\beta)+2\delta-\varepsilon]A_0$$

$$+ (\sqrt{7}/2)(\alpha-\beta)A_4 = 0,$$

$$(\sqrt{7}/2)(\alpha-\beta)A_2 + [(\alpha+\beta)+8\delta-\varepsilon]A_4 = 0. \quad (1.5)$$

The energy of the corresponding rotational states is determined from the condition that the system (1.5) has a solution. Having solved each of the systems of equations (for corresponding values of ϵ) we can determine the wave functions of these states.

The wave function (1.3) and the values of the energy of the states with spin 2, 3, and 5 can be expressed in terms of the parameter γ by analytical functions. The formulae are given in references 1 and 2. For the energy of the states with spin 4, 6, and 8, equations containing the third, fourth, and fifth power of ϵ are obtained respectively.

The results of a numerical solution of these equations for several values of γ are given in Table I. The number in parentheses next to ϵ indicates the value of the spin in the excited state, whereas the index below denotes the number of the level with a given spin.

2. PROBABILITIES OF ELECTRIC QUADRUPOLE TRANSITIONS IN THE ROTATIONAL BAND

The reduced probability of electric quadrupole transitions between the two states described by the functions ψ_{Jmi} and $\psi_{J'm'f}$ is given by the expression

$$B(E2;\ i\to f) = \frac{5}{16\pi(2J+1)}\sum_{m'\mu m} |(J'm'f|\hat{Q}_{2\mu}|Jmi)|^2, \quad (2.1)$$

where

$$\hat{Q}_{2\mu} = eQ_0\left\{D^2_{\mu 0}\cos\gamma + \frac{1}{\sqrt{2}}(D^2_{\mu 2}+D^2_{\mu,-2})\sin\gamma\right\},$$

$$Q_0 = 3Z\beta R^2/\sqrt{5\pi}.$$

The wave functions of the states with spin 2 and 3 are given in reference 1. Coefficients A_K of the wave functions of the state with spin 4

$$\psi_{4mi} = \sqrt{9/8\pi^2}\left\{A_{0i}D^4_{m0} + \frac{1}{\sqrt{2}}A_{2i}(D^4_{m2}+D^4_{m,-2})\right.$$

$$\left. + \frac{1}{\sqrt{2}}A_{4i}(D^4_{m4}+D^4_{m,-4})\right\}, \quad (2.2)$$

can be calculated by solving the system of equation (1.5) for each of the three roots of the equation determining the value of the energy of these levels. The values of the coefficients A_{Ki} for three levels with spin 4 are given in Table II. Also presented in the table are the values of the coefficients B_{Ki} of the wave functions

$$\psi_{6mi} = \sqrt{13/8\pi^2}\left\{B_{0i}D^6_{m0} + \frac{1}{\sqrt{2}}B_{2i}(D^6_{m2}+D^6_{m,-2})\right.$$

$$+ \frac{1}{\sqrt{2}}B_{4i}(D^6_{m4}+D^6_{m,-4})$$

$$\left. + \frac{1}{\sqrt{2}}B_{6i}(D^6_{m6}+D^6_{m,-6})\right\} \quad (2.3)$$

of the states with spin 6.

Using Eq. (2.1) and the wave function with J = 2 and 4, one can calculate the reduced transition probabilities between these states. Thus, for the electric quadrupole transition, the reduced probabilities (in units of $e^2Q_0^2/16\pi$) are given by

TABLE I. Dependence of the rotational energy (in units of $\hbar^2/4B\beta^2$) of even-even nuclei on the parameter γ

$\gamma°$	0	5	10	15	20	22.5	25	27.5	30
$\varepsilon_1(4)$	13.33	13.54	14.12	15.00	15.81	16.01	16.06	16.02	16
$\varepsilon_2(4)$	∞	274.1	77.73	42.48	32.26	30.92	31.21	32.69	34
$\varepsilon_3(4)$	∞	1056	268.1	122.5	71.92	58.51	49.20	42.85	40
$\varepsilon_1(6)$	28	28.42	29.51	30.65	30.78	30.51	30.33	30.06	30
$\varepsilon_2(6)$	∞	270.5	93.51	60.46	54.52	55.74	57.33	59.71	60
$\varepsilon_1(8)$	48	48.60	50.26	50.87	49.60	48.94	48.38	48.10	48

TABLE II. Coefficients determining the wave functions (2.2) and (2.3) of states with spin 4 and 6 respectively

$\gamma°$	0	5	10	15	20	22.5	25	27.5	30
A_{01}	1	1	0.999	0.993	0.955	0.909	0.852	0.792	0.739
A_{21}	0	0.003	0.030	0.114	0.296	0.414	0.522	0.605	0.661
A_{41}	0	10^{-6}	10^{-4}	0.001	0.010	0.022	0.042	0.076	0.125
A_{02}	0	−0.003	−0.03	−0.114	−0.296	−0.415	−0.523	−0.602	−0.559
A_{22}	1	1	0.999	0.993	0.954	0.907	0.842	0.754	0.500
A_{42}	0	$5\cdot10^{-4}$	0.004	0.015	0.043	0.074	0.128	0.264	0.661
B_{01}	1	1	0.998	0.973	0.878	0.817	0.766	0.714	0.672
B_{21}	0	0.0075	0.065	0.232	0.476	0.570	0.633	0.674	0.695
B_{41}	0	10^{-4}	$6\cdot10^{-4}$	$8\cdot10^{-3}$	0.043	0.081	0.113	0.189	0.254
B_{61}	0	10^{-7}	10^{-6}	$6\cdot10^{-5}$	$9\cdot10^{-4}$	$2.8\cdot10^{-3}$	$6.1\cdot10^{-3}$	0.015	0.031

the formula

$$b(E2;\ 4i\rightarrow 2f)=\tfrac{5}{126}\{\cos\gamma\cdot[6A_{0i}a_f+\sqrt{15}A_{2i}b_f]+\sin\gamma\cdot[\sqrt{15}A_{2i}a_f+A_{0i}b_f+\sqrt{35}A_{4i}b_f]\}^2, \quad (2.4)$$

where a_f and b_f are coefficients that determine the wave functions of spin 2 (see reference 1). The reduced transition probabilities between states with spin 4 are given by the expression

$$b(E2;\ 4i\rightarrow 4f)=(77)^{-1}\{2\cos\gamma\cdot[7A_{4i}A_{4f}-5A_{0i}A_{0f}-2A_{2i}A_{2f}]+\sqrt{3}\sin\gamma\cdot[3\sqrt{5}(A_{2i}A_{0f}+A_{0i}A_{2f})+\sqrt{7}(A_{2i}A_{4f}+A_{4i}A_{2f})]\}^2. \quad (2.5)$$

The reduced probabilities of a transition between states with spin 4 and 3 are given by the expression

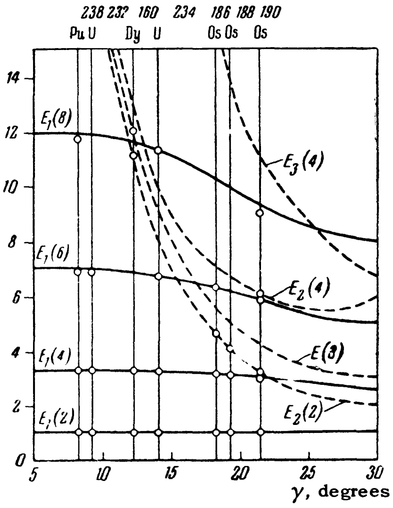
$$b(E2;\ 4i\rightarrow 3)=\tfrac{1}{9}\{2\sqrt{3}\cos\gamma\cdot A_{2i}+\sin\gamma\,[\sqrt{5}A_{0i}-\sqrt{7}A_{4i}]\}^2. \quad (2.6)$$

Finally, we shall give the equation determining the reduced probabilities for the transition $6i \rightarrow 4f$:

$$b(E2;\ 6i\rightarrow 4f)=\tfrac{5}{143}\{\cos\gamma\,[3\sqrt{5}B_{0i}A_{0f}+4\sqrt{2.1}B_{2i}A_{2f}+3B_{4i}A_{4f}]+\sin\gamma\,[\sqrt{3}B_{0i}A_{2f}+B_{2i}A_{4f}/\sqrt{10}+\sqrt{14}B_{2i}A_{0f}+\sqrt{21}B_{4i}A_{2f}+\sqrt{49.5}B_{6i}A_{4f}]\}^2. \quad (2.7)$$

For the sake of brevity, we shall call the energy levels 0, 21, 41, 61, and 81 (where the first figure denotes the spin of the level and the second the number of the level) the levels of the "principal rotational band." These energy levels are represented in the figure by solid lines. For $\gamma \rightarrow 0$, the states of the "principal rotational band" pass over to the levels of the axially symmetrical nucleus. All other energy levels represented in the figure (by dotted lines) tend to infinity for $\gamma \rightarrow 0$. We shall call these energy levels "anomalous."

Using the values of coefficients determining the wave functions, one can calculate from Table II and reference 1 the probabilities of electric quadrupole transitions between various rotational states of the nucleus. The values of some of these probabilities are given in Table III.



Taking the results of reference 1 and the data of Table III into account, we see that the values of reduced probabilities for electric quadrupole transitions between different rotational states of an even-even nucleus can be split into three types:

1. Transitions with reduced probabilities (in units of $e^2Q_0^2/16\pi$) on the order of unity. These transitions include the cascade transitions between the levels of the principal rotational band and the cascade transitions between "anomalous" rotational levels. For example, the transitions $3 \rightarrow 22$, $42 \rightarrow 3$, and $42 \rightarrow 22$ belong to this type.

2. Transitions beween the levels of the principal rotational band and "anomalous" rotational levels with another value of the spin. Examples of this are the transitions $3 \rightarrow 21$, $41 \rightarrow 22$, $42 \rightarrow 21$, and $61 \rightarrow 42$. The reduced probabilities of such transitions are equal to 0 for $\gamma = 0°$ or 30°, and are unlikely to occur for other values of γ.

TABLE III. Reduced probabilities for electric quadrupole transitions (in units of $e^2Q_0^2/16\pi$) between certain rotational states of even-even nuclei

γ°	0	5	10	15	20	22.5	25	27.5	30
$b(E2;\ 3 \to 41)$	0	0,0060	0.034	0.130	0.406	0.619	0,821	0.955	1.00
$b(E2;\ 41 \to 21)$	1.429	1.418	1.395	1.377	1.371	1.366	1.365	1.378	1.389
$b(E2;\ 41 \to 22)$	0	$3.5\cdot10^{-4}$	0.0023	0.010	0.033	0.044	0,039	0.016	0
$b(E2;\ 42 \to 21)$	0	$4.1\cdot10^{-3}$	0.011	0.008	$4\cdot10^{-4}$	0.009	0.021	0.018	0
$b(E2;\ 42 \to 22)$	0,595	0.591	0,575	0,543	0,481	0.447	0,435	0.484	0.595
$b(E2;\ 42 \to 3)$	1,333	1.323	1.282	1.172	0.978	0,680	0,448	0.210	0
$b(E2;\ 42 \to 41)$	0	0.0138	0.0624	0.167	0.313	0.339	0.311	0.271	0.273
$b(E2;\ 61 \to 41)$	1,573	1,563	1.547	1,562	1.623	1.671	1.703	1.725	1.731
$b(E2;\ 61 \to 42)$	0	10^{-3}	$7.7\cdot10^{-3}$	0.035	0.052	0.033	0,011	0,0023	0

3. Transitions between levels with identical value of the spin. Examples of this are the transitions $22 \to 21$ and $42 \to 41$. The reduced probabilities of such transitions are equal to 0 for $\gamma = 0°$, and then markedly increase with increasing γ, attaining maximum values of the order of unity for $\gamma = 30°$. The transition $3 \to 41$ belongs to this group.

3. THE QUANTUM NUMBER K AND ITS SELECTION RULE

The wave functions (1.3) of rotational states of even-even nuclei are represented by linear combinations* of functions (1.4) corresponding to a state with a given value of the projection of the total momentum (quantum number K) of the nucleus on the axis 3 of the coordinate system fixed in the nucleus. The value of the coefficients that determine the contribution of different terms of such linear combination depends on the parameter γ.

As can be seen from Table II and reference 1, the wave functions of the rotational states of the nucleus can be approximated for $\gamma < 15°$ by expressions containing only one value of K. Thus, the wave function of states having spin 2 for $\gamma < 15°$ can be replaced by the approximate functions

$$\psi_{21}^0 = |20\rangle = (5/8\pi^2)^{1/2} D_{m0}^2,$$
$$\psi_{22}^0 = |22\rangle = (5/16\pi^2)^{1/2} (D_{m2}^2 + D_{m,-2}^2). \quad (3.1)$$

Under the same condition, the wave function of states having spin 4 are approximated by the expression

$$\psi_{41}^0 = |40\rangle = (9/8\pi^2)^{1/2} D_{m0}^4,$$
$$\psi_{42}^0 = |42\rangle = (9/16\pi^2)^{1/2} (D_{m2}^4 + D_{m,-2}^4),$$
$$\psi_{43}^0 = |44\rangle = (9/16\pi^2)^{1/2} (D_{m4}^4 + D_{m,-4}^4) \quad (3.2)$$

and so on.

In the cases ($\gamma < 15°$) where the rotational motion of the nucleus can be described by approximate functions of type (3.1) and (3.2), the rotational states can be characterized by two quantum numbers J and K. In this approximation, the levels of the principal rotational band are characterized by the value K = 0. "Anomalous" rotational states can then also be subdivided into system of levels with various K = 2, 4, 6 . . .

The reduced probabilities of electric quadrupole transition between states described by approximate functions $|JK\rangle$ different from 0 (in units of $e^2Q_0^2/16\pi$) have the form

$$b_0(E2;\ JK \to J'K) = 5(2J0K|J'K)^2 \cos^2\gamma,$$
$$b_0(E2;\ JK \to J',\ K+2) = \tfrac{5}{2}(1+\delta_{K0})(2J2K|J',\ K+2)^2 \sin^2\gamma,$$
$$b_0(E2;\ JK \to J',\ K-2) = \tfrac{5}{2}(1+\delta_{K2})(2J,\ -2K|J',\ K-2)^2 \sin^2\gamma.$$

The rules with respect to the probabilities of transitions given at the end of the last paragraph are, in this approximation, reduced to the selection rule

$$\Delta K = 0 \quad (3.3)$$

for the most probable transitions. Transitions violating the condition (3.3) are called K-forbidden transitions.

In view of the fact that the quantum number K is an approximate one, Eq. (3.3) is applicable only for nuclei with $\gamma < 15°$. For $\gamma > 15°$, one should use the accurate functions (1.3). The results of Sec. 2 make it possible to estimate the error which occurs when the approximate functions (3.1) and (3.2) are used instead of the accurate function (1.3).

It should be mentioned that in several of papers,[5-8] in the analysis of the relative transition intensities, certain excited states with spin 2 and 3 were assigned the quantum number K = 2.

*Only the function of the rotational state with spin 3 corresponds to a given value of K = 2 (see reference 1).

The authors of these papers regarded these excited states as so-called γ-oscillations, and arbitrarily assumed the frequency of these oscillations. If we consider such states as rotational states of a non-axial nucleus, then the mutual position of states and the relative transition probabilities between them can be readily explained. In that case, the theory used only one parameter γ, which is determined in an unique way from the ratio of the energy of two levels with spin 2 (see following section).

4. COMPARISON WITH EXPERIMENT

In order to facilitate the comparison of the results obtained with experimental data, we show in the figure the ratio of the energy of rotational states with different values of spin to the energy of the first excited state plotted as a function of the parameter γ. The parameter γ is determined in an unique* way in the interval $0 < \gamma < 30°$ from the ratio of the energies of two levels with spin 2 by means of the formula

$$E_2(2)/E_1(2) = \left[1 + \sqrt{1 - \tfrac{8}{9}\sin^2 3\gamma}\,\right] \times \left[1 - \sqrt{1 - \tfrac{8}{9}\sin^2 3\gamma}\,\right]^{-1}. \qquad (4.1)$$

Having thus determined the value of γ, we can, with the help of the figure, find the position of the remaining rotational states with different values of the spin. It can be seen from the figure that the deviation of the shape of the nucleus from a rotational ellipsoid leads to the violation of the interval rule $1 - 3.3 - 7 - 12$ in the principal band, which can be observed in the rotational band of axially symmetrical nuclei. Thus, for instance for $\gamma = 30°$, the principal rotation level should satisfy the interval rule $1 - 2.67 - 5 - 8$.

The experimental values of the ratios of the excitation energy of nuclei Os^{190}, Dy^{160}, U^{232}, Pu^{238} (data of reference 9), U^{234} (data of reference 10) and Os^{188}, Os^{186} (data of reference 14) to the energy of their first excited state are denoted by points in the figure. It can be seen that the theory predicts correctly the sequence of spins and the experimental energy ratios. A slight deviation of the experimental points from theoretical values can be accounted for by introducing a correction term

$$-aJ^2(J+1)^2, \qquad (4.2)$$

which takes the connection between the rotation and internal excitation of the nucleus into account. The value of this correction term can be used as a criterion of the applicability of the adiabatic approximation. It should be noted that, in an analysis of rotational spectra from the point of view of the assumption about the actual form of the nucleus, the deviation of experimental energy ratios from the interval rule $1 - 3.3 - 7 - 12$ was assumed to be wholly due to the violation of the adiabatic condition. This has led to a great overestimate of the role of the correction term (4.2). In fact, the deviation from the interval rule for axial nuclei is mainly due to the violation of the axial symmetry of the nucleus. Of special interest, in that respect, are the experimental data on the levels 2, 4, 6, and 8 of the Os^{190} nucleus, the position of which has been determined by Scharf-Goldhaber et al.[11] and Aten et al.[12] in the study of cascade γ transtions in the decay of the isomere Os^{190} with a decay time of ten minutes and spin 10^-. As has been noticed by the authors,[11,12] the experimentally-observed sequence of spins corresponds well to the sequence of spins of the levels of the nucleus Hf^{180}, which has a well-defined rotational spectrum. However, the observed ratios of energies are substantially different from those obtained theoretically for axially symmetrical nuclei. It has been mentioned in this paper that the experimentally observed ratios $1 - 2.93 - 5.62 - 8.93$ cannot be obtained theoretically even by applying the correction (4.2).

Apart from the variation of the interval rule for the levels of the principal rotational band, the violation of axial symmetry of a nucleus leads to the appearance of new rotational levels ("anomalous"), such as, for instance, the second levels with spin 2 and 4 in Os^{190}. These levels do not appear in the decay of the isomer of Os^{190} with ten minutes half life, but they appear in the K-capture decay of Ir^{190}.

The features of the spectrum of the Os^{190} nucleus are due to the large value of $\gamma = 21.4°$ corresponding to an experimental ratio of energies $E_{22}/E_{21} = 3.15$. Using the value of $\gamma = 21.4$, one can calculate the values of the given probabilities and the relative probabilities of electric quadrupole transitions in the nucleus Os^{190}. These values are given in Table IV, together with the transitions connected with the theoretical level with spin 3, a level which has not yet been observed experimentally. With Os^{190} nucleus as an example we have shown that the theory of non-axial nuclei makes it possible to calculate the relative probabilities of electric quadrupole transitions

*We disregard the ambiguity due to the fact that, in even-even nuclei, the rotational energy and the transition probability between them are the same for various γ and $\pi/3 - \gamma$ (see reference 1 and 3).

TABLE IV. Relative transition probabilities between various rotational levels of the Os^{190} nucleus

E2-transition	Transition energy (kev)	Reduced probability $(e^2Q_0^2/16\pi)$	Relative probability	E2-transition	Transition energy (kev)	Reduced probability $(e^2Q_0^2/16\pi)$	Relative probability
$21 \to 0$	186	0.934	1	$3 \to 41$	244	0.515	2.14
$41 \to 21$	360	1.33	39.3	$61 \to 41$	500	1.6	251
$22 \to 21$	400	0.467	21.5	$42 \to 22$	540	0.461	100
$22 \to 0$	586	0.066	21.9	$42 \to 41$	580	0.319	100
$22 \to 41$	40	0.071	$3.5 \cdot 10^{-5}$	$42 \to 21$	940	0.007	24.6
$3 \to 21$	604	0.18	69	$42 \to 3$	336	0.813	16.5
$3 \to 22$	204	1.60	2.9	$42 \to 61$	80	0.06	$9 \cdot 10^{-4}$

between all rotational states. The values of the relative transition intensities found provide a qualitative explanation for the observed decay schemes of the excited states of Os^{190}. Thus, for instance, a decay of the isomeric state of Os^{190} takes place only through the series of cascade transitions through rotational levels of the main rotational band without a marked excitation of the "anomalous" rotational levels. The "anomalous" rotational levels 4^+ and 2^+ are excited in the K-capture decay of the Ir^{190} nucleus. It follows from Table IV that the nucleus, after emitting an E2 photon, can pass from the excited states corresponding to an "anomalous" rotational level 4^+ either into the rotational level with spin 4^+ in the principal rotational band with subsequent cascade emission of 2 photons, or (with the same probability) into the level 2^+ corresponding to an "anomalous" rotational level. From this state, the transition either occurs to the state with spin 2^+ of the principal rotational band, or (roughly with the same probability) directly to the ground state. According to the theory, the Os^{190} nucleus should have an excited state (~ 790 kev) with spin 3^+. However, as can be seen from Table IV, the excitation of this rotational level is not very probable since, from the higher levels 4^+ and 6^+, transitions to other rotational states are much more probable. It is possible that, because of that, this rotational state has so far not been discovered.

In reference 1, a comparison has already been made of the theoretical values for transition probabilities with experimental data for levels having spin 2 and 3. Since the writing of reference 1 new experimental data have appeared on the ratio of the reduced probabilities of cascade and direct transitions. Table V presents the experimental data (McGowan[13,6]) and theoretical values, obtained from the formula

$$\frac{b(E2;\ 22 \to 21)}{b(E2;\ 22 \to 0)} = \frac{20 \sin^2 3\gamma}{7\{9 - 8\sin^2 3\gamma - [3 - 2\sin^2 3\gamma]\sqrt{9 - 8\sin^2 3\gamma}\}}.$$

The parameter γ is determined from the ratio (4.1). The experimental data[13] and theoretical values

$$\frac{b(E2;\ 22 \to 21)}{b(E2;\ 21 \to 0)} = \frac{20 \sin^2 3\gamma}{7\{9 - 8\sin^2 3\gamma + [3 - 2\sin^2 3\gamma]\sqrt{9 - 8\sin^2 3\gamma}\}}$$

are also given for the ratio of the reduced transition probabilities $(22 \to 21)$ and $(21 \to 0)$. It can be concluded from Table V that this theory makes it possible to establish a single-valued relation between the given ratio of the reduced transition probabilities and the ratio of the energy of both levels with spin 2.

Experimental values of the ratio of intensity of E2 transitions $3 \to 41$ and $3 \to 21$ in the Sm^{152} nucleus are given by Nathan and Waggoner.[5]

TABLE V. Ratio of the reduced probabilities of electric quadrupole transitions for certain nuclei

Nucleus	E_{22}/E_{21}	γ	$b(E2;\ 22 \to 21)/b(E2;\ 22 \to 0)$		$b(E2;\ 22 \to 21)/b(E2;\ 21 \to 0)$	
			Theory	Experiment	Theory	Experiment
Th^{232}	15.4	10.3	1.7	1.8	0.052	0.014
Gd^{154}	8.11	14.0	2.6	1.94	—	—
W^{184}	8.02	14.1	3.0	2.36	0.18	0.12
Os^{188}	4.08	19.3	4.5	2.7	0.32	0.19
Os^{190}	2.98	21.4	7.1	9.7	0.50	0.52
Os^{192}	2.37	25.4	21	9.1	0.90	0.68
Te^{122}	2.23	26.3	40	30	1.29	2.8

This ratio corresponds to the ratio of reduced probabilities

$$\{b(E2;\ 3\rightarrow 41)/b(E2;\ 3\rightarrow 21)\}_{\mathbf{exp}} = 1.88.$$

From the ratio $E_{22}/E_{21} = 8.9$ for Sm^{152} nucleus, it follows that $\gamma = 13.5°$. The theory then leads to the value

$$\{b(E2;\ 3\rightarrow 41)/b(E2;\ 3\rightarrow 21)\}_{\mathbf{theor}} = 1.37,$$

which is in agreement with the experimental ratio.

[1] A. S. Davydov and G. F. Filippov, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 440 (1958), Soviet Phys. JETP **8**, 303 (1959).

[2] A. S. Davydov and G. F. Filippov, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 703 (1958), Soviet Phys. JETP **8**, 488 (1959).

[3] A. S. Davydov and G. F. Filippov, Nuclear Phys. **8**, 237 (1958).

[4] A. Bohr, Dan. Mat.-Fys. Medd. **27**, 14 (1952). A. Bohr and B. Mottelson, Dan. Mat.-Fys. Medd. **27**, 16 (1953).

[5] O. Nathan and M. Waggoner, Nuclear Phys. **2**, 548 (1956/57).

[6] O. Nathan, Nuclear Phys. **4**, 125 (1957).

[7] J. Juliano and F. Stephens, Phys. Rev. **108**, 341 (1958).

[8] G. Hickman and M. Wiedenbeck, Phys. Rev. **111**, 539 (1958).

[9] B. S. Dzhelepov and L. K. Peker, Схемы распада радиоактивных ядер, (Decay Schemes of Radioactive Nuclei) U.S.S.R. Acad. Sci. Press 1958.

[10] Strominger, Hollander, and Seaborg, Revs. Modern Phys. **30**, 585 (1958).

[11] Scharf-Goldhaber, Alburger, Harbottle, and McKeown, Bull. Amer. Phys. Soc. **2**, 25 (1957); preprint, 1958.

[12] Aten, de Feyfer, Sterk, and Wapstra, Physica **21**, 740, 990 (1955).

[13] F. McGowan, Paper presented at the International Nuclear Physics Conference in Paris, July 1958.

[14] F. McGowan and P. Stelson, Bull. Amer. Phys. Soc. **3**, 228 (1958).

[15] R. Diamond and J. Hollander, Nuclear Phys. **8**, 143 (1958).

Translated by H. Kasha
363