COMPARISON OF THE DIFFERENTIAL CROSS SECTIONS FOR THE (dp) AND (dt) REACTIONS

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The (dt) reaction is considered as a special case of a stripping reaction involving two complex systems. The reduced widths derived from the (dt) and (dp) reactions by choosing various triton wave functions are compared. The neutron wave function relative to the deuteron in the triton, which yields the best agreement with experiment is found. A value of ≈ 0.40 is derived for the probability of finding the triton in the (deuteron + neutron) state.

THE analysis of the angular distribution of the triton in the reaction (dt) shows that the mechanism of the reaction at deuteron energies of the order of several Mev is similar to the mechanism of the (dp) reaction, i.e., a stripping reaction is encountered. In connection with this, two interrelated problems arise: a) the comparison of the "reduced widths" γ^2 obtained for the reactions (dp) and (dt), and b) the determination of that part of the wave function of the triton which corresponds to the state (deuteron + neutron). Below, we consider both problems, and a part of the results is compared with results of other authors.[1,2]

As the analysis of the experimental data obtained in recent years shows (Belyaev, Zakhar'ev, and Neudachin, to be published), the "reduced width" γ^2 in the formulae of Butler does not represent the "square of the amplitude of the wave function of a nucleon on the surface of the nucleus" since it is not constant for varying energy of the deuteron but varies very rapidly. It is possible that this is due to the influence of the exchange effect in the stripping reaction.[3] The comparison of γ^2 from the (dp) and (dt) reactions is interesting as a check on the analogy between the two mechanisms, complementing the comparison of the angular distributions.

In contrast to the deuteron, the triton and He^3 are compound particles, on the wave functions of which one has to impose, apart from the vector coupling of the moments, the requirements of antisymmetry. The calculation was carried out by means of the Born approximation using plane waves which, for the ratio of the reduced widths, gives about the same results as the calculation with distorted waves.

The general expression for a stripping reaction between two coupled systems applied to the reaction (dt) is of the form (see also reference 1)

$$\frac{d\sigma}{d\Omega}=\frac{\mu_d\mu_t}{4\pi\mu_n^2}\frac{k_t}{k_d}\frac{(2S_t+1)}{(2S_d+1)(2S_n+1)}$$
$$\times n_t\,|\langle s^3\alpha\,|\,s^2\alpha_0\rangle|^2\,|G(p)|^2 Z^{-2}$$
$$\times[j_l'(Za)-g_l(\chi a)\,j_l(Za)]^2(2\mu_n/\hbar^2)\gamma_c^2, \qquad (1)$$

where $|<s^3\alpha\,|\,s^2\alpha_0>|^2=\frac{1}{2}$ is the parentage coefficient of the transition from the state s^2 of the deuteron with quantum numbers α_0 to the state s^3 of the triton with quantum numbers α; n_t is the number of nucleons in the triton; S_t, S_d, S_n — are the spin of the triton, deuteron, and neutron respectively; γ_c^2 is the reduced width;

$$\mu_d=M_dM_i/(M_i+M_d),\qquad \mu_t=M_tM_f/(M_f+M_t),$$
$$\mu_n=M_nM_f/(M_f+M_n),$$

M_i and M_f are the masses of the initial and final nuclei; k_t, k_d are the wave numbers,

$$k_d=\left(\frac{2M_d}{\hbar^2}\frac{M_i^2}{(M_i+M_d)^2}E_{d\,\mathrm{lab}}\right)^{1/2};$$
$$\mathbf{p}=\mathbf{k}_t/3-\mathbf{k}_d/2,\qquad \mathbf{Z}=\mathbf{k}_t-\mathbf{k}_dM_f/(M_j+M_n),$$
$$\chi=(2\mu_n\,|E_b\,|)^{1/2}/\hbar,$$

and E_b is the binding energy of the neutron in the initial nucleus. The interaction radius α is chosen for the best fit of the theoretical angular distribution with the experimental one. The function $j_l(x)$ can be expressed by a Bessel function of half-integral order:

$$j_l(x)=\sqrt{\pi x/2}\,J_{l+1/2}(x)=xj_{lC}(x),$$

where $j_{lC}(x)$ is the spherical Bessel function; $j'_l(x)$ is the derivate with respect to the radius: $j'_l(Za) = Zdj_l(x)/dx$ for $x = Za$; and $g_l(\chi a)$ are logarithmic derivatives on the boundary of the nucleus. These are given for $l = 0$, 1, and 2 by Yoshida.[4] The reduced widths can be expressed in units of the sum $\sum_c \gamma_c^2$ over all reaction channels, and one can obtain the non-dimensional quantity θ:

$$\theta^2 = (a2\mu_n/3\hbar^2)\,\gamma_c^2.$$

Furthermore, $(2\pi)^{-3/2}G(p)$ is the Fourier transform of a function whose square expresses the probability of finding a triton in the state (deuteron + neutron). We shall denote the neutron contained in the deuteron by n_0, the captured neutron by n_1, and the distances in the triton as $(n_0p) = x$, $(pn_1) = y$, and $(n_1n_0) = z$. We then have

$$G(p) = \int \exp\{ip(2y - x)\}\,\psi_t(p, n_0, n_1)\,\psi_d(p, n_0)\,d\tau$$

(the integration is carried out over all internal coordinates of the triton). For the deuteron, we assume the Hulthen wave function

$$\psi_d(p, n_0) = N_d(e^{-\alpha x} - e^{-\beta x})/x,$$

$$N_d^2 = [4\pi\ (1/2\alpha - 2/(\alpha + \beta) + 1/2\beta)]^{-1},$$

$$\alpha = 0.23\cdot 10^{13}\ \text{cm}^{-1},\quad \beta = 1.63\cdot 10^{13}\ \text{cm}^{-1}.$$

The problem of the triton has been studied to a much lesser extent than that of the deuteron. Therefore, there is still no acceptable wave function for the triton. We assume a function which is in good agreement with experimental values of the binding energies of H^3 and He^3, and with the difference of the binding energies of H^3 and He^3 for a certain choice of potential parameters. These are the Gaussian function

$$\psi_t(p, n_0, n_1) = N_{t_1}\exp\{-\gamma^2(x^2 + y^2 + z^2)\},$$
$$N_{t_1}^2 = 24\sqrt{3}\,\pi^{-3}\gamma^6,\qquad \gamma = 0.25\cdot 10^{13}\,\text{cm}^{-1}\ [5]$$

and the wave function of Irving[6]

$$\psi_t(p, n_0, n_1) = N_{t_2}\exp\{-\lambda(x^2 + y^2 + z^2)^{1/2}\}/(x^2 + y^2 + z^2)^n,$$
$$n = \tfrac{1}{4},\quad \lambda = 0.759\cdot 10^{13}\ \text{cm}^{-1},\quad N_{t_2}^2 = 4\sqrt{3}\,\pi^{-3}\lambda^5.$$

It is shown in several papers that the angular distribution of the reaction (dt) is well described by this function for a suitable choice of the interaction radius. In the present work, we have studied the applicability of the Gaussian and Irving wave functions to a calculation of the reduced width and of the angular distribution. For this purpose, it is necessary to know, apart from the dependence on p, the absolute value of G(p).

We shall give the results of the calculation of G(p): 1) for the triton, we assumed the Gaussian wave function

$$G(p) = 42.5\cdot 10^{-20}\exp\{-p^2/2\gamma^2\};$$

2) for the triton we used the Irving function

$$G(p) = N_dN_{t_2}(8\pi^2/3)\,\Gamma(7/2)\lambda^{-7/2}[I(\alpha) - I(\beta)]/p,$$

where

$$I(\alpha) = \int_0^1 (A^2 + B^2)^{-7/4}\sin\left(\frac{7}{2}\tan^{-1}\frac{B}{A}\right)u\,du,$$

$$B = \frac{\sqrt{2}\,p}{\lambda}(1 - u^2)^{1/2},\qquad A = 1 + \sqrt{\frac{2}{3}}\,\frac{\alpha}{\lambda}\,u$$

(see also reference 7).

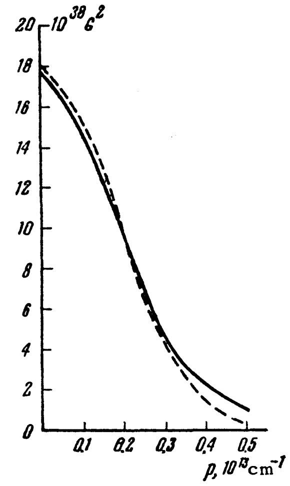


FIG. 1. $|G(p)|^2$ for two cases: Solid curve represents the triton function according to Irving; dotted curve – the Gaussian wave function.

The final results are given in Fig. 1. It can be seen from the figure that the value of the differential cross section depends very slightly on the choice (between variants 1 and 2) of the triton wave function. Therefore, in Table I, where the values of θ^2 calculated from the reaction (dt) according to formula (1) are compared with the reduced widths calculated from the reactions (dp), (pd), and (nd), the Irving function was used to calculate θ^2_{dt} for the corresponding levels. (Formulae for the cross sections are given, e.g., in reference 8.)

We have assumed that, for the choice of a good triton wave function, the interaction radius chosen to describe the angular distribution of the (dt) reaction should coincide with the interaction radius for the corresponding (pd) reaction[14] and the reduced widths obtained from the (dt) reaction should be approximately equal to θ^2_{pd}. Since the accuracy of the ratio θ^2 for identical targets and for different states of the final nucleus is greater than the accuracy of the absolute values, then the ratio θ^2 for two levels of Li from reactions (dt) and (pd) should coincide even better. It can be seen from Table I that all these requirements are

TABLE I

Reaction	E_d, Mev c.m.s.	a, f	θ^2	Error, %	Reference
$Li^7(pd)Li^6$	7.7	5.5	0.052	20	[9]
$Li^7(pd)Li^{6*}$	6.07	5.5	0.036	20	[9]
$C^{12}(dp)C^{13}$	5.87	5.3	0.060	10	[10]
	2.02	5.8	0.030	3	[13]
$Li^6(nd)He^5$	6.48	4.5	0.090	5	[11]
$Be^9(pd)Be^8$	12.3	3	0.025	20	[9]
$Li^7(dt)Li^6$	8.7	6	0.067	20	[12]
$Li^7(dt)Li^{6*}$	8 7	6.5	0.038	20	[12]
$C^{13}(dt)C^{12}$	2.47	6.25	0.050	3	[13]
$Li^6(dt)Li^5$	11.3	5.5	0.140	20	[14]
$Be^9(dt)Be^8$	13.4	4.0	0.061	20	[14]

*Transition to the 2.187-Mev level; in all other cases, transition to the ground state.

fulfilled for wave functions of a triton in both Irving and Gaussian forms. Besides, the functions do not give the correct asymptotic behavior of the wave function f_0 of the neutron in the triton with respect to a deuteron described by the function $\exp(-\lambda R)/R$, where $\lambda = 0.45$ as calculated from the binding energy of the neutron in the triton.

One can try to determine the function f_0 which best satisfies the condition given above directly from experimental data.

Making use of the series expansion (see reference 15).

$$\psi_t(\mathbf{r}RS_tM_{S_t}) = \sum_{i,\, M_{S_d} m_{S_n}} A_{iM_{S_d}} m_{S_n} \psi_{d_i}(\mathbf{r}S_dM_{S_d})\, C_i f_i(RS_n m_{S_n})$$

(where $\mathbf{r}$ is the vector between the particles in the deuteron, $\mathbf{R}$ is the distance from the middle of $\mathbf{r}$ to the second neutron in the triton, $C_i f_i$ is the wave function of this neutron with a normalization factor, and i denotes the different states of the deuteron in the triton), we can substitute in the formula for the cross section of the (dt) reaction:

$$G^2(p)\,\theta_{dt}^2 = (2\pi)^3 A_0^2 C_0^2 \Phi^2(k)\,\theta_{dt}^{2'}. \quad (2)$$

where $k = 2p$ is the momentum of the neutron in the triton with respect to the deuteron, $\Phi(k)$ is the Fourier transform of the function $f_0(R)$, chosen to be equal to one for $k = 0$, and A_0^2 is the probability of finding a triton in the state (deuteron + neutron).

Vlasov and Oglobin[14] give the variation of $\Phi^2(k)$ for $k = (0.3 - 0.55) \times 10^{13}\ cm^{-1}$, and mention that $\Phi^2(k)$ describes the angular distribution of the reaction $Li^7(dt)Li^6$ and $Li^7(dt)Li^{6*}$ with the same interaction radius. For $k \approx 0$ to 0.34, according to data of Holmgren et al.,[13] $\Phi(k) = 1/(k^2 + \lambda^2)$ and $\lambda = 0.45$, which gives the correct asymptotic exponential behavior of $f_0(R)$ (see also reference 1). It was also found that $\Phi(k)$ for small and large k join smoothly (see Fig. 2,

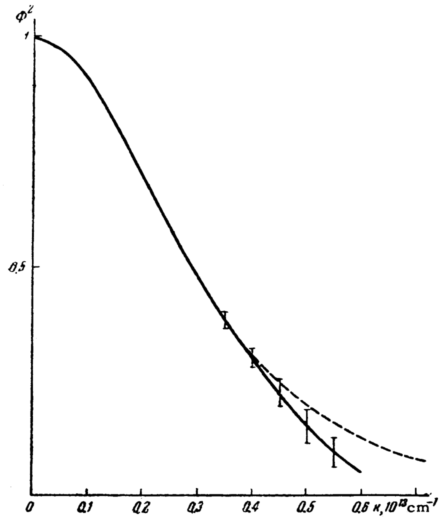
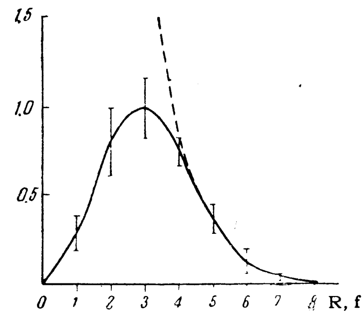


FIG. 2. Fourier transform $\Phi^2(k)$ determined from experimental data (solid curve). Dotted curve – transform $(k^2 + \lambda^2)^{-2}$ of the asymptote of f_0.

where, for large k, an average $\Phi(k)$ is taken for $Li^7(dt)Li^6$ at $E_d = 20$ Mev and $E_d = 14.4$ Mev).

Choosing in a rather arbitrary way the asymptotic behavior of $\Phi^2(k)$ for large momenta (which, however, does not introduce a large error), we find the distribution $Rf_0(R)$, Fig. 3. It has a maximum for $R = 3f$, and, beginning with $R \approx 4.5f$, gives the asymptotic dependence $\exp(-\lambda R)$. Thus, $f_0(R)$ satisfies the physical requirements

FIG. 3. The function $R^2f_0^2(R)$ (solid curve) and the function proportional to $\exp(-2\lambda R)$ (dotted curve). Arbitrary units.



of the proper asymptotic behavior at large distances and has a maximum for R of the order of the deuteron radius. These properties of functions $f_0(R)$ are mentioned by Werner,[1] who calculated f_0 by solving the Schrödinger equation for the second neutron in the triton with a Gaussian potential. The normalizing coefficient of the function f_0 is given by the equation

$$C^2 = \left(4\pi \int_0^\infty f_0^2(R) R^2 dR\right)^{-1} = 4 \cdot 10^{-39} \text{ cm}^{-1}.$$

We can now find A_0^2 from different experiments. The interaction radius and the reduced width for the (dt) reaction were taken from the corresponding (dp), (pd), and (nd) reactions (see Table II). The low accuracy of the experimental data obtained from the (pd) reactions involving Be and from the reactions of low-energy deuterons with carbon should be noted. The average value $A_0^2 = 0.37$ (20% error) is greater than the upper limit 0.11 given by Werner.

TABLE II

Reaction	a, f	$\theta_{dt}^2 = \theta_{pd}^2$	A_0^2	Reference
$Li^7(dt)Li^6$	5.5	0.052	0.360	[9, 12]
$Li^7(dt)Li^{6*}$	5.5	0.036	0.377	[9, 12]
$Li^6(dt)Li^5$	4.5	0.090	0.372	[11, 14]
$Be^9(dt)Be^8$	3.0	0.025	0.390	[9, 14]
$C^{13}(dt)C^{12}$	5.8	0.030	0.330	[13]

*See footnote for Table I.

We note that Werner has compared the (dt) and (pd) reactions for different energies; if the comparison is made at the same energy using the data of Holmgren et al.[13] the value 0.11 should be increased by a factor of two.

The deviation from our value of 0.4 can be explained by the fact that the function f_0, and consequently the normalizing coefficient for it, have been obtained from experimental data and are different from f_0 and C^2 calculated by Werner for a definite choice of the interaction.

Recently El Nadi and Hadid[2] estimated A_0^2, using the calculations of Werner, from the reaction $Li^7(dt)Li^6$ (reference 12) and $Li^7(pd)Li^6$ (reference 9) from $F^{19}(dt)F^{18}$ (reference 16), and from $F^{19}(pd)F^{18}$ (reference 9) for the ground states of the final nuclei. The estimate $A_0^2 < 0.06$ obtained from the reactions with Li is doubtful, since the authors maintain that, for a choice of $\Phi(k)$ similar to that of Werner, the value $a = 6\phi$ describes the angular distribution of the (dt) reaction, while, in references 12 and 17, $a = 6$ is assumed for $\Phi(k)$ calculated from the wave function of the triton according to Irving. From the reactions involving fluorine a still smaller value $A_0^2 < 0.02$ has been obtained in reference 2. From the same experimental data, we obtain $A_0^2 \approx 0.06$. This value is in disagreement with five values of A_0^2, two of which are in agreement between themselves, obtained from data of three experiments[12-14] (see Table II).

To obtain a correct formula for the experimental angular distribution of the (dt) reaction with the same interaction radii as in the corresponding (pd) reaction, and to calculate the reduced width θ_{dt}^2, it is necessary to substitute Eq. (2) into Eq. (1) and to substitute the above-given values of A_0^2 and C_0^2.

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Note added in proof (April 29, 1959): The angular distribution of the reaction $C^{13}(dp)C^{14}$ for the ground state of C^{14} (reference 18), and the reaction $C^{14}(dt)C^{13}$ (reference 19) for the ground state of C^{13} and the 3.08-Mev and 3.68-Mev excited states are satisfactorily described by the choice $a = 5.4\phi$. The value of A_0^2 is 0.3 (100% error).

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