ON THE RELATIVISTIC RELATION BETWEEN POLARIZATION AND ASYMMETRY

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THE usual method for determining the polarization of particles consists of measuring the azimuthal asymmetry of the scattering of the polarized particles, and is based on the fact that the azimuthal asymmetry is equal to the product of the polarization of the incident particles and the polarization produced in the scattering of unpolarized particles. This relation was derived by Wolfenstein and Ashkin for the nonrelativistic case, from general principles of symmetry.[1,2] It is interesting to examine the question of the relation between asymmetry and polarization in the case of the scattering of relativistic particles.

The state of polarization of a relativistic particle with four-vector momentum p_μ can be described[3] by the density matrix

$$\rho = {}^1/_2 (1 + i\gamma_5\gamma_\mu\xi_\mu)\Lambda_p. \tag{1}$$

Here $\Lambda_p = (\gamma_\mu p_\mu + im)/2im$ is a projection operator, m is the rest mass of the particle, and $\xi_\mu = i\mathrm{Sp}\,\gamma_5\gamma_\mu\rho$ is a spacelike axial vector orthogonal to the vector p_μ ($\xi_\mu p_\mu = 0$). The degree of polarization is defined as $P = (\xi_\mu\xi_\mu)^{1/2}$

Let us consider first elastic scattering of particles with spin $\frac{1}{2}$ by spinless particles. By considerations of invariance the scattering matrix can be written in the form

$$M = A + B\gamma_\mu(q_\mu + q'_\mu), \tag{2}$$

where q_μ and q'_μ are the momenta of the particle with spin zero before and after the scattering, and A and B are arbitrary functions of the two independent invariants. The density matrix of the final state can be put in the following form:

$$\rho_{\mathrm{scat}} = \Lambda_{p'} M\rho_{\mathrm{inc}}\beta M^+\beta\Lambda_{p'}. \tag{3}$$

p_μ and p'_μ are the momenta of the particles with spin $\frac{1}{2}$ before and after the scattering.

From Eqs. (1) and (3) we get the following expression for the scattering cross section of the polarized beam (polarization vector ξ_μ^{inc}):

$$\sigma = \frac{1}{2}\mathrm{Sp}\,\Lambda_{p'}M\Lambda_p\beta M^+\beta\left(1 + \xi_\mu^{\mathrm{inc}}\frac{\mathrm{Sp}\Lambda_{p'}\,Mi\gamma_5\gamma_\mu\Lambda_p\beta M^+\beta}{\mathrm{Sp}\,\Lambda_{p'}\,M\Lambda_p\beta M^+\beta}\right). \tag{4}$$

By means of the expression (2) we can verify that

$$\mathrm{Sp}\,\Lambda_{p'}\,Mi\gamma_5\gamma_\mu\Lambda_p\beta M^+\beta = \mathrm{Sp}\,i\gamma_5\gamma_\mu\,\Lambda_{p'}\,M\Lambda_p\beta M^+\beta\Lambda_{p'}$$

and, consequently,

$$\sigma = \sigma_0(1 + \xi_\mu^{\mathrm{inc}}\,\xi_\mu^0), \tag{5}$$

where σ_0 is the scattering cross section of unpolarized particles and ξ_μ^0 is the polarization vector that appears from the scattering of unpolarized particles ($\xi_\mu^0 \sim \epsilon_{\mu\nu\rho\sigma}p_\nu p'_\rho q_\sigma$).

The formula (5) is also valid in the case of a reaction of the type $\frac{1}{2} + 0 \rightarrow \frac{1}{2} + 0$, if the product of the intrinsic parities of all the particles involved in the reaction is +1. On the other hand, if the product of the intrinsic parities is −1, then it is not hard to derive the following expression for the cross section of the reaction:

$$\sigma = \sigma_0(1 - \xi_\mu^{\mathrm{inc}}\,\xi_\mu^0). \tag{6}$$

It can be verified that Eq. (5) is also valid in the case of scattering of polarized particles of spin $\frac{1}{2}$ by unpolarized particles with spin $\frac{1}{2}$ (for example, for nucleon-nucleon scattering).

In the laboratory coordinate system ($\mathbf{q} = 0$), and also in any other system moving relative to the laboratory system with a velocity lying in the plane of the scattering (for example in the center-of-mass system), the fourth component ξ_4^0 of the polarization vector is zero and the vector $\boldsymbol{\xi}^0$ is directed along $[\mathbf{p}\times\mathbf{p}']$. Equation (5) then takes the following form:

$$\sigma = \sigma_0(1 + \boldsymbol{\xi}^{\mathrm{inc}}\boldsymbol{\xi}^0).$$

Thus in the usual double-scattering experiment the asymmetry ϵ is equal to $\xi_\mu^0\xi_\mu^0$, i.e., to the square of the relativistically invariant polarization. A measurement of the asymmetry of the scattering of an arbitrarily polarized beam makes it possible to determine the component of the spatial part of the polarization vector normal to the plane of the scattering.

[1] L. Wolfenstein, Phys. Rev. **75**, 1664 (1949).

[2] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952).

[3] L. Michel and A. Wightman, Phys. Rev. **98**, 1190 (A) (1955).

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335