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*This possibility, although it is not without difficulties, is of interest because it may reduce the number of Hamiltonians of type (1).

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ALPHA DECAY CONSTANTS OF NON-SPHERICAL NUCLEI

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IN references 1 — 4, the author developed an analytical theory of alpha decay of non-spherical nuclei,* and simple formulas were given for the relative intensities of the fine structure lines. References 1 and 2 give expressions for wave functions which also make it possible to calculate the absolute decay probability. For even-even nuclei we obtain:

$$W = \frac{3\hbar\varkappa}{mR_0} w_\alpha \Gamma \left| \int_0^1 e^{\beta P_2(\mu)} d\mu \right|^2 \sum_J w_J, \tag{1}$$

Where W is the decay probability per unit time; $\sum_J w_J$ is the sum of the relative decay probabilities to all rotation levels of the daughter nucleus, including decay to the ground state $w_0 = 1$;

$$\Gamma = \exp\left\{-2 \int_{R_0}^{a_0} \sqrt{\varkappa_b^2 R_0/r - k^2}\, dr\right\}$$

$$= \exp\left\{-2\left(\frac{\varkappa_b^2 R_0}{k} \tan^{-1}\frac{\varkappa}{k} - \varkappa R_0\right)\right\};$$

$k = \sqrt{2mE}/\hbar$ is the wave number of an α particle at infinity; $\varkappa_b = \sqrt{4mZe^2/\hbar^2 R_0}$ is the wave number corresponding to the height of the Coulomb barrier for an α particle $2Ze^2/R_0$; m is the reduced mass; $\varkappa = \sqrt{\varkappa_b^2 - k^2}$; a_0 is the return point corresponding to the decay energy E to the ground state; the quantity β is given by the relation

$$\beta = [{}^4/_5\, \varkappa R_0 (1 - k^2/2\varkappa_b^2) - i2k^3 R_0/5\varkappa_b^2]\, \alpha_2$$

and is connected with the quadrupole deformation α_2 contained in the equation $R(\mu) = R_0\{1 + \Sigma \alpha_n P_n(\mu)\}$ for the form of the daughter-nucleus surface.

Let us explain the meaning of the "internal probability of formation of an α-particle" w_α. The wave function of the mother nucleus can be expressed in the form:

$$\Psi = \sum_{ik} \psi_{ik}(\mathbf{r}) \varphi_i^\alpha \varphi_k, \tag{2}$$

where φ_i^α and φ_k are the wave functions of the stationary state of internal motion of an α particle and a daughter nucleus respectively; $\mathbf{r}$ is the radius vector of an alpha particle relative to the center of mass of the daughter nucleus. Only the term $\varphi_{00}(\mathbf{r})\varphi_0^\alpha\varphi_0$ appears in alpha decay to the ground state. The function $\varphi_{00}(\mathbf{r})$ goes over continuously to the external wave function found in reference 1. Considering the nuclear substance to be homogenous and the mean free path of an alpha particle in it to be small compared to the size of the nucleus, it is natural to assume $\varphi_{00}(\mathbf{r}) = \text{const}$. Then

$$w_\alpha = \int |\psi_{00}|^2 dV = \frac{4}{3}\pi R_0^3 |\psi_{00}|^2 < 1.$$

Since there must be many excited states with short alpha-particle mean free paths in sum (2), we have

$$w_\alpha \ll 1. \tag{3}$$

With the help of formula (1), we can, from experimental data, determine w_α, for which the calculated results are shown in Table I.† Condition (3) is fulfilled only with $R_0 = 1.4 A^{1/3} \times 10^{-13}$ cm but when $R_0 = 1.0 A^{1/3} \times 10^{-13}$ the value of w_α increases by four or five orders of magnitude. With this, the good constancy of w_α in the entire region of alpha active nuclei confirms the reasonableness of the basic premises of the theory. Therefore, for alpha decay $R_0 \geq 1.4 A^{1/3} \times 10^{13}$ cm.

For non-even nuclei the wave functions derived in reference 2 give

$$W = \frac{3\hbar\varkappa}{mR_0} w_\alpha \Gamma \sum_{l=0}^{2K}{}' (2l+1) \left| C_{KKl0}^{KK} \right|^2 \times e^{-\gamma_l l(l+1)} \left| \int_0^1 e^{\beta P_2(\mu)} P_l(\mu) d\mu \right|^2 \sum_J w_J, \qquad (4)$$

where K is the projection of the nuclear moment on the axis of symmetry; the prime for the sigma means that the summation is done only for even l; $C_{j_1 m_2 j_2 m_2}^{JM}$ are the Clebsch-Gordan coefficients; $\gamma_l = 2\kappa/\kappa_b^2 R_0$. Results of a comparison with an experiment are listed in Table II. The average value of w_α is twice as small as for even nuclei.

TABLE I

Nucleus	w_α
Ra^{222}	0.155
Ra^{224}	0.051
Ra^{226}	0.068
Ra^{228}	0.104
Th^{226}	0.057
Th^{228}	0.063
Th^{230}	0.094
Th^{232}	0.079
Th^{234}	0.201
U^{230}	0.113
U^{232}	0.094
U^{234}	0.088
U^{236}	0.134
U^{238}	0.110
Pu^{238}	0.090
Pu^{240}	0.096
Cm^{242}	0.094
Cm^{246}	0.107
Cm^{248}	0.189
Cf^{250}	0.095
	$\overline{w}_\alpha = 0.10$

TABLE II

Nucleus	K	w_α
Ac^{227}	$3/2$	0.0308
Th^{229}	$5/2$	0.0459
U^{235}	$1/2$	0.0413
Np^{237}	$5/2$	0.0517
Np^{239}	$5/2$	0.0724
Pu^{239}	$5/2$	0.0387
Cm^{245}	$9/2$	0.0361
Bk^{249}	$7/2$	0.0560
		$\overline{w}_\alpha = 0.047$

*In a recent article[5] it is stated that the 2^4-pole interaction of an alpha particle with a daughter nucleus was not taken into consideration in any of the earlier published articles; it is also asserted that the theoretical calculations for non-even nuclei were not brought to formulas comparable with experiment. Actually, reference 2 gives the derivations of simple formulas for the case of non-even nuclei and a comparison is made with experimental results. The 2^4-pole interaction of an alpha-particle with a daughter nucleus was considered in reference 3, published at an earlier date.

†The daughter nucleus is always shown in the tables.

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BREAK-UP OF CHARGED PARTICLES BY A NUCLEAR COULOMB FIELD

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A charged particle with charge Z_a and mass number a, in flying past a target nucleus (Z, A), interacts with its Coulomb field. In some cases the energy of this interaction is sufficient to dissociate the incident particle (nuclei of deuterium, beryllium, singly ionized molecules of hydrogen, and so forth). The theory of such processes was first developed by Dancoff[1] for the break-up of fast deuterons and later was carried over without special changes to the break-up of beryllium nuclei.[2] It was assumed that the disturbance (the energy of the Coulomb interaction) is time dependent, and so they used the conclusions of perturbation theory for this case.

Although quantum-mechanical methods are used in the references cited, the intermediate calculations contain a number of approximations which can hardly be justified (see, for instance, the simplification made in computing the integral I_r and the further impulse integration in reference 1). Besides, the final results are complicated and cannot always be applied to concrete evaluation. In this connection it is interesting to note an easier way of computing cross sections of the aforementioned processes — by applying a classical analysis analogous to the derivation of the Thompson formulas for the ionization of atoms by impact.[3]

Let the complex particle (Z_a, a) be capable of decaying into the parts (Z_1, a_1) and (Z_2, a_2), and let ϵ_0 be the binding energy for this decay. Then, in interacting with the Coulomb field of the target nucleus, each of these parts will receive an additional momentum equal in order of magnitude to (see reference 3)

$$p_i \approx (2ZZ_i e^2 / vb) [1 + (ZZ_a e^2 / \mu_a v^2)^2]^{-1/2}; \qquad (1)$$