

| K-Baryon coupling                             | $R = 1.4 A^{1/3} \cdot 10^{-12}$ |            |            |          |     | $R = 1.2 A^{1/3} \cdot 10^{-12}$ |            |            |          |     |
|---|----------------------------------|------------|------------|----------|-----|----------------------------------|------------|------------|----------|-----|
|   | $U^{1K}$                         | $U^{2\pi}$ | $U^{K\pi}$ | $U^{2K}$ | $U$ | $U^{1K}$                         | $U^{2\pi}$ | $U^{K\pi}$ | $U^{2K}$ | $U$ |
| Pseudoscalar,<br>$f^2=0,08$                   | +1                               | -50        | +22        | -20      | -47 | +3                               | -78        | +34        | -32      | -73 |
| Scalar, $g_{\Lambda}^2 = g_{\Sigma}^2 = 1.1$  | +17                              | -50        | +25        | -21      | -29 | +25                              | -78        | +39        | -33      | -47 |
| Scalar, $g_{\Lambda}^2 = 3g_{\Sigma}^2 = 1.1$ | +17                              | -50        | +17        | -10      | -26 | +25                              | -78        | +26        | -16      | -43 |

is negative, guaranteeing a bound state for the  $\Lambda$  particle. The data in the table give information on the size of the contributions from the various processes to the potential energy of the  $\Lambda$  particle. In view of the exchange character of the  $1K$  and  $K\pi$  mesonic potentials, these make the interaction of the  $\Lambda$  particles with the nuclear matter nonlocal. However, in the approximation of "zero interaction range," we can also in this case introduce an effective potential for the  $\Lambda$  particles in the nucleus. The data of the table were obtained in this approximation. Owing to the effects of the Pauli principle in the system of nucleons, the contributions to the potential energy from the exchange-type  $1K$  and  $K\pi$  forces are strongly suppressed. They are positive and have about the same absolute value as the contributions from the  $2K$  forces.

The uncertainties in the experimental binding energies for  $\Lambda$  particles in heavy hypernuclei are quite large, and it is therefore impossible to obtain from them any accurate information about the well depth. Apparently, the depth should be 20 to 30 Mev for  $10 < A < 20$ . The depth of the square potential well in hyperfragments with  $A < 10$  is about 20 Mev. The estimates of Dalitz and Downs<sup>6</sup> indicate that the well depth in heavy hypernuclei can be 29 to 38 Mev. The comparison of these data with those of the table shows that the scalar variant is not in disagreement with these data, while the pseudoscalar variant gives too high values for the depth, using our assumed values for the coupling constants.

Since there are at present no accurate experimental data on the binding energies of heavy hypernuclei, it is not possible to make a detailed comparison.

The author thanks Prof. D. D. Ivanenko for his support in the completion of this work.

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Translated by R. Lipperheide  
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### ON THE AXIAL ASYMMETRY OF ATOMIC NUCLEI

D. A. ZAIKIN

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor November 26, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1570-1571 (May, 1959)

To investigate the equilibrium shape of atomic nuclei, we considered previously<sup>1</sup> the behavior of nucleons in an infinite ellipsoidal square well with semi-axes  $a_x r_0$ ,  $a_y r_0$ , and  $a_z r_0$ . This was done by means of a coordinate transformation which transforms the ellipsoid into a sphere of radius  $r_0$ . In the new coordinates the kinetic-energy operator of the nucleons has two parts,  $-\hbar^2/2M\Delta$  and  $\hat{V}$ . The second part,  $\hat{V}$ , can be expanded into a series in powers of the deformation with a linear leading term. It is convenient to choose as the deformation parameters  $\rho$  and  $\gamma$  which are connected with the semi-axes by the relations

$$a_x^{-1} + a_y^{-1} - 2 = \rho \cos \gamma, \quad a_y^{-1} - a_x^{-1} = \sqrt{3} \rho \sin \gamma. \quad (1)$$

It is easy to see that  $\gamma$  coincides with the parameter  $\gamma$  introduced by Bohr,<sup>2</sup> and  $\rho$  is in first order proportional<sup>2</sup> to  $\beta$ :  $\rho \approx (5/4\pi)^{1/2} \beta$ . The parameters  $\alpha$  and  $\delta$  of reference 1 are connected with  $\rho$  and  $\gamma$  by the relations

$$\alpha = \rho \cos \gamma, \quad \delta = -\sqrt{3} \rho \sin \gamma. \quad (2)$$

Considering  $\rho$ , and consequently  $\hat{V}$ , to be small

the problem can be treated by perturbation methods. The degeneracy in the quantum number  $m$  is already fully lifted in the first order (see reference 1). The nucleon states now can be classified by the quantum numbers  $n$ ,  $l$ ,  $|m|$ , and  $w$ , where  $w$  is the parity with respect to reflections in the  $(x, z)$  plane. The meaning of the other quantum numbers can be easily explained by going to the limits of axial symmetry and spherical symmetry.

The energies of the levels as a function of  $\rho$  and  $\gamma$  was calculated up to and including quadratic terms for  $s$ ,  $p$ ,  $d$ , and  $f$  states. Thus, for states with  $l = 3$  and  $m = 1$  the energy is given by

$$E_{n31\pm} = E_{n3}^0 \left\{ 1 + \frac{4}{15} \rho \left[ \sin\left(\frac{\pi}{6} \pm \gamma\right) - \sqrt{1 + 4 \cos^2\left(\frac{\pi}{6} \pm \gamma\right)} \right] + \rho^2 \left[ 1 - \frac{2}{15} \cos^2\left(\frac{\pi}{6} \pm \gamma\right) + \frac{12 \cos^3 \gamma \mp \sqrt{3} \sin \gamma}{15 \sqrt{1 + 4 \cos^2(\pi/6 \pm \gamma)}} + \frac{24}{7} \left( 5 + 4 \sin^2\left(\frac{\pi}{6} \pm \gamma\right) - \frac{44 \sin^3(\pi/6 \pm \gamma) - 35 \sin(\pi/6 \pm \gamma)}{\sqrt{1 + 4 \cos^2(\pi/6 \pm \gamma)}} \right) D_{n3}^- + \frac{72}{77} \left( 103 + 60 \sin^2\left(\frac{\pi}{6} \pm \gamma\right) - \frac{380 \sin^3(\pi/6 \pm \gamma) - 343 \sin(\pi/6 \pm \gamma)}{\sqrt{1 + 4 \cos^2(\pi/6 \pm \gamma)}} \right) D_{n3}^+ \right] \right\}, \quad (3)$$

where

$$E_{nl}^0 = \frac{\hbar^2 \mu_{nl}^2}{2M r_0^2},$$

$$D_{nl}^{\pm} = \mp \left\{ \frac{2(l \pm 3) + 1}{16[2(l \pm 1) + 1]^2} - \frac{\mu_{nl}^2}{8[2(l \pm 1) + 1]^3} \right\},$$

$\mu_{nl}$  is the  $n$ -th root of the spherical Bessel function  $j_l(x)$ . For the axially symmetric ( $\gamma = 0$ ) case these expressions go over into Moszkowski's expressions.<sup>3</sup>

Numerical evaluations of the energy were performed for different configurations. They show that for the case of a few particles above the closed shell, beginning with three, the minimum of the energy can correspond to an axially non-symmetric shape of the nucleus. So, for example, the configuration  $(1s)^6 (1p)^6 (1d)^4$  of one nucleon kind, corresponding to the nucleus  $Mg^{24}$  the minimum of the energy occurs at  $\beta \approx 0.3$ ,  $\gamma \approx 7^\circ$ ; for the configuration  $(1s)^2 (1p)^6 (1d)^{10} (2s)^2 (1f)^2$  corresponding to  $Ti^{44}$ , it occurs at  $\beta \approx 0.2$ ,  $\gamma \approx 5^\circ$ .

In this one has to keep in mind the rough character of the utilized model and the slow convergence of the perturbation series (this is particularly so for values of  $\gamma$  close to zero and for small  $m$ ). Therefore the obtained results can by

no means aspire to be in agreement with experiment. However, it follows from the above that the independent particle model in its simplest form contains the possibility of deviations of the nuclear equilibrium shape from axial symmetry. This result is in agreement with the results of similar calculations of Gursky<sup>4</sup> and Geřlikman<sup>5</sup> for the oscillator potential. Analogous results have been obtained for the unified model by Davydov and Filippov.<sup>6</sup>

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Translated by M. Danos  
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### A POSSIBLE METHOD FOR THE DETERMINATION OF THE DIRECTION OF POLARIZATION OF $\mu^-$ MESONS

V. A. DZHRBASHYAN

Physics Institute, Academy of Sciences,  
Armenian S.S.R.

Submitted to JETP editor December 18, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1572  
(May, 1959)

It is well known<sup>1,2</sup> that the law of lepton conservation will be completely verified only when it is shown experimentally that  $\mu^-$  mesons produced in the decay of  $\pi^-$  mesons are longitudinally polarized in the direction of their motion. Dolginov<sup>3</sup> suggested the use, for this purpose, of the angular distribution of circularly polarized  $\gamma$  rays emitted in the  $2p \rightarrow 1s$  transition in mesic atoms.

In the first order effect ( $2p \rightarrow 1s$  transition) the expression for the angular distribution (see, e.g., reference 4) naturally depends on the degree