ROTATIONAL STATES OF ODD NUCLEI WITHOUT AXIAL SYMMETRY

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A theory is developed for the rotational states of odd nuclei whose ground state spin is due to the angular momentum $j = \frac{1}{2}$ of the outer electron. The energy of the rotational states is obtained as a function of the parameter γ $(0 \leq \gamma \leq \pi/3)$ which determines the deviation of the nuclear shape from axial symmetry.

INTRODUCTION

It was shown by Davydov and Filippov[1] that many properties of the first excited states of even-even nuclei (the order of succession of the spins of the excited states, their energies, and the probabilities for electromagnetic transitions between them) can be readily explained by assuming that the equilibrium shape of the nucleus can in first approximation be represented by a three-axial ellipsoid, which is characterized by two parameters, β and γ (see the work of Bohr[2]). The relations

$$a_0 = \beta \cos \gamma, \qquad a_1 = a_{-1} = 0, \qquad a_2 = a_{-2} = (\beta/\sqrt{2}) \sin \gamma$$

connect the parameters β and γ with the quantities a_μ which determine the shape of the nucleus:

$$R(\vartheta, \varphi) = R_0 + R_0 \sum_{\mu=-2}^{2} a_\mu Y_{2\mu}(\vartheta, \varphi)$$

in a coordinate system attached to the nucleus. Changing the "asymmetry" parameter γ from 0 to $\pi/3$, with a fixed value β, induces a change of the nuclear shape from a prolate to an oblate ellipsoid of revolution. The value $\gamma = 30°$ corresponds to an intermediate shape of the nucleus between the prolate and the oblate ellipsoids of revolution.

The possibility of departing from the axial symmetry of the equilibrium shape of the nucleus has been investigated in the papers of Geilikman,[3] Zaikin,[4] and Davydov and Fillippov.[5] In the present paper we consider the rotational states of odd nuclei under the assumption that the shape of the nucleus is determined by the fixed equilibrium values of the parameters β and γ, with the odd nucleon in a state with the definite total angular momentum $j = \frac{1}{2}$.

In keeping with the fact that we wish to consider the general case with arbitrary values of the parameter γ, we shall not assume that the projection of the total angular momentum $j_3 = \Omega$ is an integral of the motion. Instead, we seek exact solutions of the Schrödinger equation giving the rotational energy of the nucleus without neglecting (as was done in the papers of Bohr,[2] Bohr and Mottelson,[6] Davidson and Feenberg,[7] and others) the nondiagonal matrix elements connected with the different values of Ω.

In section 1 we obtain the energies of the rotational states of the nucleus for fixed parameters β and γ. We show, in particular, that for $\gamma = 0$ the exact solutions coincide with the solutions obtained in the aforementioned papers with neglect of the nondiagonal matrix elements of the operator of the rotational energy of the nucleus. In Sec. 2 we compare the theory with the experimental data for the nucleus W^{183}.

1. ROTATIONAL ENERGY LEVELS OF ODD NUCLEI WITH THE OUTER NUCLEON IN THE STATE $j = \frac{1}{2}$

We consider, as a model of the nucleus, a system consisting of a single outer nucleon in the state $j = \frac{1}{2}$ and a nuclear core with the shape of a three-axial ellipsoid. In other words, we assume that the core of the nucleus consists of nucleons belonging to filled shells as well as of "paired" nucleons of the outer shell, which are responsible for the deviation of the shape of the nuclear core from spherical symmetry.

If I_λ are the projections of the total angular momentum operator on the three principal axes of the ellipsoid describing the shape of the nucleus, and j_λ the corresponding projections of the total angular momentum of the outer nucleon, then the nuclear Hamiltonian which conserves the total angular momentum of the nucleon can, in the adiabatic approximation (neglecting the operator of the kinetic energy corresponding to changes of β and γ) be written in the form proposed by Bohr:[2]

$$H = \hbar^2 (8B\beta^2)^{-1} \sum_\lambda \frac{(\hat{I}_\lambda - \hat{j}_\lambda)^2}{\sin^2(\gamma - 2\pi\lambda/3)} + H_p + H_{int},$$

Here the first term is the rotational Hamiltonian; H_p is the Hamiltonian of the outer nucleon;

$$H_{int} = T\beta\{\cos\gamma\,(3\hat{j}_3^2 - \hat{j}^2) + \sqrt{3}\sin\gamma\,(\hat{j}_1^2 - \hat{j}_2^2)\}$$

is the Hamiltonian describing the interaction of the outer nucleon with the deformation of the nuclear surface.

In the states with $j = \frac{1}{2}$, $H_{int} = 0$, and the rotational energy of the nucleus (without changes in the internal state) can be obtained from the eigenvalues of the operator

$$H_r = \hbar^2 (8B\beta^2)^{-1} \sum_\lambda (I_\lambda - j_\lambda)^2 / \sin^2\left(\gamma - \frac{2\pi}{3}\lambda\right). \quad (1.1)$$

The wave function for the rotational state of the odd nucleus satisfying the symmetry conditions given by Bohr[2] (invariance of the nuclear shape under rotations of 180° about each of the axes 1, 2, and 3) and corresponding to the total angular momentum I of the nucleus can be written in the form

$$\psi_I = \sum_{K,\Omega} |IjK\Omega\rangle A_{K\Omega}, \quad (1.2)$$

where

$$|IjK\Omega\rangle = \left(\frac{2I+1}{16\pi^2}\right)^{1/2} \{\varphi_\Omega D^I_{MK} + (-1)^{I-j}\varphi_{-\Omega} D^I_{M,-K}\}, \quad (1.3)$$

with

$$K - \Omega = 2\nu, \quad \nu = 0, \pm 1, \pm 2, \pm \ldots \quad (1.4)$$

The wave function φ_Ω in (1.3) determines the state of the outer nucleon with total angular momentum j and projection Ω; D^I_{MK} are irreducible representations of the three-dimensional rotation group, which depend on the Eulerian angles specifying the orientation of the principal axes in space; K is the projection of the total angular momentum on the third axis of the nucleus, and M is the projection of the total angular momentum on the z axis of the fixed system of coordinates.

Using the known values of the matrix elements of the operator (1.1) with respect to the wave functions φ_Ω and of the quantities D^I_{MK}, we can transform the Schrödinger equation

$$\{H_r - E(I)\}\psi_I = 0$$

to a system of linear, homogeneous, algebraic equations for the unknown coefficients $A_{K\Omega}$. The rotational energy E(I) corresponding to the total angular momentum I of the nucleus is given by the secular equation for this system of equations.

For $I = \frac{1}{2}$ we conclude from (1.4) that there exists only one state with $K = \Omega = \frac{1}{2}$. The wave function for this state,

$$\psi_{1/2} = (8\pi^2)^{-1/2}\{\varphi_{1/2} D^{1/2}_{M1/2} + \varphi_{-1/2} D^{1/2}_{M,-1/2}\} \quad (1.5)$$

satisfies the equation

$$[H_r - E(1/2)]\psi_{1/2} = 0,$$

with $E(\frac{1}{2}) = 0$. The function (1.5) therefore corresponds to the ground state of the nucleus. The spin of the ground state is $\frac{1}{2}$.

For $I = 3/2$ formula (1.4) gives two possibilities, $K = \Omega = \frac{1}{2}$ and $\Omega = \frac{1}{2}$, $K = -3/2$. The wave function (1.2) will then be a linear combination of the two functions

$$|3/2, 1/2, 1/2, 1/2\rangle = (2\pi)^{-1}\{\varphi_{1/2} D^{3/2}_{M1/2} - \varphi_{-1/2} D^{3/2}_{M,-1/2}\},$$

$$|3/2, 1/2, -3/2, 1/2\rangle = (2\pi)^{-1}\{\varphi_{1/2} D^{3/2}_{M,-3/2} - \varphi_{-1/2} D^{3/2}_{M3/2}\}.$$

The energy of the two rotational states belonging to the spin 3/2, measured in the units $\hbar^2/B\beta^2$, is given by the formula

$$\varepsilon_{1,2}(3/2) = 9(1 \mp \sqrt{1 - 8/9 \sin^2 3\gamma})/4\sin^2 3\gamma. \quad (1.6)$$

For $I = 5/2$ the following values are possible

$$K = \Omega = 1/2; \quad \Omega = 1/2, \quad K = -3/2; \quad \Omega = 1/2, \quad K = 5/2$$

and the wave function (1.2) is then a linear combination of the three functions

$$|5/2, 1/2, 1/2, 1/2\rangle = (3/8\pi^2)^{1/2}\{\varphi_{1/2} D^{5/2}_{M1/2} + \varphi_{-1/2} D^{5/2}_{M,-1/2}\},$$

$$|5/2, 1/2, -3/2, 1/2\rangle = (3/8\pi^2)^{1/2}\{\varphi_{1/2} D^{5/2}_{M,-3/2} + \varphi_{-1/2} D^{5/2}_{M,3/2}\},$$

$$|5/2, 1/2, 5/2, 1/2\rangle = (3/8\pi^2)^{1/2}\{\varphi_{1/2} D^{5/2}_{M5/2} + \varphi_{-1/2} D^{5/2}_{M,-5/2}\}.$$

The energy of the three rotational levels with spin $I = 5/2$ is determined by the cubic equation

$$\varepsilon^3 - \frac{9\varepsilon^2}{\sin^2 3\gamma} + \frac{9(9 + 2\sin^2 3\gamma)\varepsilon}{4\sin^4 3\gamma} - \frac{3(27 + \sin^2 3\gamma)}{4\sin^4 3\gamma} = 0.$$

The solutions to this equations for several values γ are listed in Table I.

In the same fashion one can show that the ener-

TABLE I

$\gamma°$	0	5	10	15	20	25	30
$\varepsilon_1(5/2)$	1.000	1.018	1.074	1.175	1.331	1.547	1.708
$\varepsilon_2(5/2)$	∞	65.91	16.78	7.682	4.525	3.131	2.646
$\varepsilon_3(5/2)$	∞	67.20	18.10	9.143	6.144	4.968	4.644

TABLE II

γ °	0	5	10	15	20	25	30
$\varepsilon_1\,(^7/_2)$	3.333	3.384	3.531	3.750	3.954	4.011	4.0
$\varepsilon_2\,(^7/_2)$	∞	67.17	18.10	9.10	6.10	4.923	4.5
$\varepsilon_3\,(^7/_2)$	∞	68.54	19.42	10.62	8.066	7.802	8.5
$\varepsilon_4\,(^7/_2)$		263.9	67.05	30.63	17.98	12.30	10.0
$\varepsilon_1\,(^9/_2)$	3.333	3.382	3.531	3.750	3.954	4.000	4.0
$\varepsilon_2\,(^9/_2)$	∞	68.52	19.43	10.63	8.070	7.800	8.5
$\varepsilon_3\,(^9/_2)$	∞	70.25	21.19	12.42	9.757	9.23	9.0
$\varepsilon_4\,(^9/_2)$	∞	263.6	66.92	30.64	18.00	12.30	10.0
$\varepsilon_5\,(^9/_2)$	∞	266.0	68.93	32.55	20.0	15.0	13.5

gies of the levels with spins $^7/_2$ and $^9/_2$ are given by the solutions of equations of the fourth and fifth degree in ϵ, respectively. The solutions to these equations for several values γ are listed in Table II.

It follows from (1.6) and Tables I and II that for $\gamma = 0$ (axially symmetric nucleus) the energies of the rotational levels coincide (up to an additive constant)* with the energy values given by the formula (in our units $\hbar^2/B\beta^2$)

$$\varepsilon_r = {}^1/_6\,\{I\,(I+1) - (-1)^{I-{}^1/_2}(I + {}^1/_2) + {}^3/_4\},$$

of Bohr and Mottelson (reference 6, p 22) for an axially symmetric nucleus, neglecting the nondiagonal matrix elements and assuming that $\Omega = K = \frac{1}{2}$.

The energies of the levels with spins $^3/_2$ and $^5/_2$, as well as those with spins $^7/_2$ and $^9/_2$, coincide for an axially symmetric nucleus. As γ (i.e., the deviation from axial symmetry) increases, the energies of the levels of the axially symmetric nucleus are raised somewhat, while at the same time the remaining rotational energy levels with the same spin value come down from infinity. For $\gamma > 20°$ the energy spectrum becomes rather complicated. Several levels with the same spin value correspond to one and the same internal state. Before, these states belonged to different rotational bands.

Figure 1 illustrates how the rotational states behave as γ changes in the interval 10 to 30°.

In Tables I and II we list the values of the rotational energy for various values γ in the interval 0, $\pi/6$. The values of the rotational energy for values of γ in the interval $\pi/6$, $\pi/3$ can be easily obtained by noting that the equations (1.6) to (1.9) are invariant under the transformation $\gamma \to \gamma' = \pi/3 - \gamma$.

We therefore have the equation

$$\varepsilon(I,\ \gamma) = \varepsilon(I,\ \pi/3 - \gamma).$$

2. COMPARISON WITH EXPERIMENT

Since we are employing a very idealized model (a core plus one outer nucleon in the state $j = \frac{1}{2}$),

*The rotational energies of the nucleus in the work of Bohr and Mottelson are not normalized to zero for the ground state.

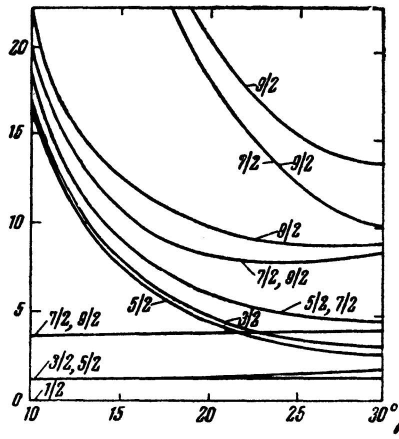

FIG. 1

the results obtained can be applied only to a few nuclei with the spin $\frac{1}{2}$ in the ground state. The deformed nucleus W^{183} apparently is such a nucleus. The right hand side of Fig. 2 represents the level scheme for this nucleus.[8] We indicate the spins and (in parentheses) the ratio of each level energy of the first excited state of the nucleus (46.5 kev). The left hand side of Fig. 2 represents the theoretical level scheme calculated on the basis of the results of the present paper with $\gamma = 27°$. It is seen from the figure that the theory reproduces satisfactorily the energy levels and the corresponding spins. To get a fuller understanding of the agreement with theory, we shall calculate in a future paper the probabilities for electromagnetic transitions between the rotational levels. Furthermore,

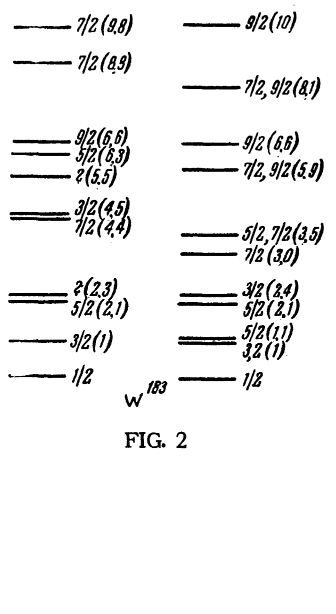

FIG. 2

we shall extend the theory to the case where the outer nucleon is in states with an angular momentum $j \neq 0$.

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